COMMUNICATIONS ENGINEERING

Essentials for Computer Scientists and Electrical Engineers

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Contents

Preface

It may be said that our world is witnessing an amazing development of communication technology. Billions of people can watch the World Soccer Game at the same time. We can watch more than 60 channels of TV broadcast in our homes. Our computers are almost all connected to the Internet and we can easily exchange information through it. Besides, we can listen to beautiful music broadcasts via our computers and our speakers may possibly be linked to the tuners wirelessly.

Traditionally, communication engineering is a field inside electrical engineering. It is still true that most of the critical technologies of communication are related to electrical engineering. Although we are dealing with digital data most of the time, we still have to know frequency and bandwidth which are entirely within the realm of analog signal processing. Besides, when we transmit digital signals, such as in the case of mobile phone, we still need the help of radio frequency technologies.

But, as communication technologies become more and more advanced, a communication system cannot be realized without the help of computer scientists. Not mentioning the large systems used in those telephone companies, even for producing a mobile phone, a large amount of software systems must be developed. Suddenly, there is a big problem, most of programmers do not know what the electrical engineers are talking about. When they talk about terms such as QPSK, RF and OFDM, the poor computer scientists just sit there and do nothing.

On the other hand, computer scientists will gradually feel the need to understand how communication works. In the old days, a computer was a computer. The facilities making a computer connecting to the outside world was not an integral part of the computer. These days, since most computers are connected to the Internet, communication capability becomes a part of the computer, not something added on. If a computer scientist is ignorant of communication technology, it will be hard for him to design computer.

But, we must admit that most computer science students do not understand communication technology very well. For instance, they do not know how bits can be mixed and sent out, why we still talk about bandwidth when we only send pulses and no carrier frequency is involved and why we need the so-called RF technology in wireless communication.

This book was written for both electrical engineering and computer science students. Communication is a very complicated engineering technology. Since this is introductory textbook, we only discuss the key concepts of communication technologies, namely those critical and basic ones.

These days, we seem to be digitizing all kinds of things. We have digital TV, digital radio and almost all of the data we send are digital data. Most of students think that analog signals are ancient and out-of-date while digital signals are modern and advanced. Yet, when we send these digital data wirelessly, we always send them with the help of radio frequency signals which are analog. Thus, we talk about carrier frequency and bandwidth. Even when we send them through wires in the form of pulses where no carrier frequency is involved, we still talk about bandwidth. After all, we know that in order to have high bit rate, we must have broadband system. Why?

This book, in some sense, brings the attention of students back to analog signals. We make clear to the students that digital signals are after all also analog signals. A pulse consists of a set of cosine functions. The narrower the pulse is, the more cosine functions it contains and thus the larger the bandwidth. Thus, even when we send pulses, we have to be concerned with bandwidth.

We have tried our best to give the reader physical meaning of many terms as much as possible. A typical example is the discussion of Fourier transform. Most students are totally confused about the existence of negative frequencies in Fourier transform. In this book, this is explained clearly. Analog modulation is another example. We not only illustrate the mechanism of analog modulation, we also explain clearly why analog modulation is necessary.

We realize that communication technologies are quite involved and by no means easy to understand. Therefore we took pains to explain every concept in detail. The Fourier transform is introduced in almost every textbook in communication. That amplitude modulation will lift the frequencies of a signal is explained by using four different methods; some of them rather easy to understand. We are proud that we have presented many figures and tables. There are 142 figures and 14 tables in this book.

Only basic trigonometry and calculus are required for reading this book. Some parts of this book may be found to be too difficult to some students. Then the instructor can ignore them.

To help the students have some feeling about existing communications systems, we tried to introduce briefly many such systems. For instance, AM and FM broadcasting, Ham radio, walkie-talkie, mobile phone, hand-held phone, T1 to T4, ADSL, digital radio, Blue-tooth and so on. In each system, we talked about the modulation scheme, carrier frequency and bandwidth which are important parameters.

It is our experience that this book can be used in a one-semester course. Since no sophisticated mathematics are required, this book can be taught to college students freshmen above.

Solutions manual, slides, lecture notes and an instructors manual can be found at: http://www.wiley.com/go/rctlee

1

An Overview of Computer Communications

Among all the modern technologies available to us, communications technology is perhaps one of the most puzzling. We use radio and television sets all the time but perhaps struggle to understand how these things work. We might also be quite puzzled when we use our mobile phones. When we make a call, messages are sent out to, say, a base station – which we vaguely know. Yet there must be many other people close by who are using the mobile phone system, so that base station must be receiving a mixture of signals. How can it recover all the *individual* signals? A lot of us use ADSL these days to connect a computer at home to the Internet. Again, it might be hard to understand what ADSL is and how it works.

Actually, communications technologies are not that hard. The first thing we have to understand is the nature of signals. We usually see a signal from the viewpoint of time, but it is important to know that a signal actually is composed of a set of *cosine functions*. How do we know? We know this through the use of the *Fourier transform*. This transforms a signal in the *time domain* into the *frequency domain*. It is an exceedingly powerful tool. In fact, we could say that modern telecommunications would not be possible without the Fourier transform.

With the Fourier transform we can view any signal from the viewpoint of its contituent frequencies. When the ordinary person talks about digital signals, he or she might not relate the pulses with frequencies. The person can easily understand the *data rate* of digital signals, but we will show that digital pulses are related to frequencies.

Let us consider radio broadcasting, which we are all familiar with. There are many radio transmitting stations. They all broadcast either the human voice or music, or both. We must give each radio station a unique frequency to broadcast. That frequency is called the *carrier frequency.* Compared with the frequencies associated with voices or music, these carrier frequencies are much higher. The radio stations, through some mechanism, attach the voice or music signals to the higher carrier frequencies. This mechanism is called *modulation*.

The air is thus filled with all kinds of signal corresponding to different frequencies. Our radio receives some of these. Actually we *tune* our radio so that at a particular time it picks out one particular carrier frequency. In this way we can hear the broadcast clearly. This process in the radio is the opposite of modulation and is called *demodulation*.

Through use of the Fourier transform, it can be shown that the human voice and music consist of relatively *low frequencies*. Can low-frequency signals be broadcast directly? No,

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because low-frequency signals have *large wavelength*. Unfortunately, the length of an antenna is proportional to the wavelengths of the signals it can handle. Thus, to broadcast low-frequency signals we would need a very large antenna, which is not practical. After introducing the much higher carrier frequency, we can then use reasonably sized antennas to broadcast and to receive.

In the above, our modulation technique is assumed to be applied to *analogue signals*, so this is called *analogue modulation*. The carrier frequencies are often called *radio frequencies* (RF for short). Actually, for reasons that need not bother us here, 'RF' today often refers to high frequencies that are no longer restricted to radio broadcasting.

What about *digital signals*? Data are almost always represented by digital signals. Digital signals are often thought of as a *sequence of pulses*. We must note that, in the wireless environment, we cannot transmit pulses except in very unusual cases. This is because, in the wireless environment, only *electromagnetic waves* are transmitted. What we can do is to transmit a *sinusoidal signal* within that short period of duration of a single pulse. This sinusoidal signal is a carrier signal and its frequency is called the carrier frequency. In this way, each user can again be uniquely identified by its carrier frequency. This kind of modulation is called *digital modulation*.

Digital modulation is more interesting than that. We can transmit more than one *bit* at the same time. This will make the communication more efficient because two bits can be sent. Note that the first bit may be 1 or 0 and the second may also be 1 or 0. Thus there are four possible cases for the receiver to determine at any instant. How can the receiver do the job of distinguishing the value of the bit i ($i = 1, 2$)? This is a very fundamental problem which we must be able to deal with.

Fortunately, there is a ready-made answer. Mathematics tells us that as long as the signals are *orthogonal* to each other, they can be mixed and later recovered easily by using the *inner-product operation*. The inner-product operation is thus very important, so in this book it is introduced quite early.

If two bits can be mixed together, a larger number of bits can also be mixed. Thus the *orthogonal frequency-division method* (OFDM) is capable of mixing 256 bits together. In this method, each bit is represented by a cosine function with a distinct frequency. It can be easily shown that these functions are orthogonal and thus can be recovered. For OFDM systems, we shall use the *discrete Fourier transform* in this book, so that the transmission can be done efficiently. Our ADSL system is based on this ODFM method.

In communications, it is natural to have the situation in which several users want to send signals to the same receiver simultaneously. A typical case is the mobile phone system whereby many callers around one base station are using the same base station. How can the receiver distinguish between the users? One straightforward method to distinguish data generated by different users is to use the *frequency-division multiple-access* (FDMA) method. We can also use the *time-division multiple-access* (TDMA) method, in which data are divided according to time slots. Another very interesting technique is the *code-division multiple-access* (CDMA) method. In the CDMA method, different users use different codes to represent the values of bits. Each code can be considered as a function, and they can be recovered because the functions corresponding to different codes are orthogonal.

A very important concept in communications is *bandwidth*. Consider the human voice case. Experimental results show that the human voice contains frequencies mainly from 0 to

5000 hertz (Hz), which means that the bandwidth of any system transmitting this voice signal must be larger than 5000 Hz. Let us denote the carrier frequency by f_c. After analogue modulation is done, the frequencies now range from $(f_c - 5)$ kHz to $(f_c + 5)$ kHz. We will then say that the bandwidth of the signal is 10 kHz. Unfortunately, a large bandwidth of a communication system occupies more resources and requires more sophisticated electronic circuitry. Thus we shall often be mentioning the concept of bandwidth.

Now consider a cable TV system. The cable is used to transmit a number of TV signals. Assume that there are N television stations and that the bandwidth of each TV signal is W. The bandwidth of the TV cable must therefore be larger than NW. For large N and W, the bandwidth must be quite large, and this is why we often call this kind of system a *broadband system*.

It is easy to understand the bandwidth of analogue signals, but digital signals also have a bandwidth associated with them. Every digital signal can be seen as a sequence of pulses. Each pulse has a *pulse width*. When we introduce the Fourier transform, we will show that a short pulse width will occupy a wide band of frequencies. Further, a high data rate will necessarily mean a short pulse width, and consequently a large bandwidth. Thus, we may say that if we want to transmit a large number of bits in a short time, we must have a communications system with a large bandwidth. So, even when we send pure digital data, the concept of bandwidth is still important.

We do not, of course, want a transmission mechanism that requires a very large bandwidth. That would be very costly. On the other hand, a very narrow bandwidth transmission has the disadvantage that it is easy for intruders to penetrate. For security reasons, sometimes we would like to widen the bandwidth. We will introduce the concept of *spread spectrum technology*, by which the bandwidth of a system is widened which will make it more secure.

Spread spectrum technology is designed not only to make a system more secure. *Frequency hopping*, for instance, is a spread spectrum technology that allows a transceiver (transmitter/receiver) to communicate with many other transceivers. Imagine, for example, that we have equipment in a laboratory which are connected to many devices. Each device sends data to the equipment from time to time, and the equipment will have to send instructions to these devices very often. Frequency hopping allows this to happen.

Finally, *coding* is something that we must understand. There are two kinds. Removing redundant data is called *source coding*, and adding redundant data to correct errors is called *channel coding*. Both are introduced in this book.

Further Reading

- For a detailed discussion of ADSL, see [B00].
- For the history of communications, see [L95].
- For a detailed discussion of communication networks, see [T95].

2

Signal Space Representation

This chapter discusses a very important topic. Suppose we have to mix signals together, how can we then extract them correctly? Since these signals are basically sinusoidal, it is important for the reader to consult the formulas in Appendix A (which contains most of the formulas frequently used in the theory of communications) whenever he or she is not familiar with the equations used in the derivations.

An important concept in communications will now be introduced: Let us consider the case where we have the following periodic function (the term 'periodic function' will be defined fully in the next chapter.):

$$f(t) = a\cos(2\pi f_c t) + b\sin(2\pi f_c t).$$

The period of this signal is $T = 1/f_c$. Suppose a receiver receives the above signal and, for some reason, it wants to determine the values of a and b. What can we do? To determine a, we may perform the following calculation:

$$\begin{aligned}
&\int_0^T f(t)\cos(2\pi f_c t)dt \\
&= a\int_0^T \cos^2(2\pi f_c t)dt + b\int_0^T \sin(2\pi f_c t)\cos(2\pi f_c t)dt \\
&= \frac{a}{2}\int_0^T (1+\cos(4\pi f_c t))dt + \frac{b}{2}\int_0^T \sin(4\pi f_c t)dt \\
&= \frac{aT}{2} + \frac{a}{2(4\pi f_c)}\sin(4\pi f_c t)\Big|_0^T - \frac{b}{2(4\pi f_c)}\cos(4\pi f_c t)\Big|_0^T \\
&= \frac{aT}{2} + \frac{a}{8\pi f_c}(0-0) - \frac{b}{8\pi f_c}(1-1) \\
&= \frac{aT}{2}.
\end{aligned}$$

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Thus:

$$a = \frac{2}{T}\int_0^T f(t)\cos(2\pi f_c t)dt.$$

Similarly, the following expression gives us the value of b:

$$\int_0^T f(t)\sin(2\pi f_c t)dt.$$

That the values of a and b can be determined in the above manner is due to the *orthogonality of cosine and sine functions*. This concept will be used throughout the book, so we devote this entire chapter to it. First, we shall discuss the vector space concept.

2.1 The Vector Space

In an n-dimensional space, a vector $\mathbf{v}$ is an n-tuple $(v_1, v_2, \cdots, v_n)$. Let us first define the inner product of two vectors. Let $\mathbf{v}_1 = (v_{11}, v_{12}, \cdots, v_{1n})$ and $\mathbf{v}_2 = (v_{21}, v_{22}, \cdots, v_{2n})$. The *inner product* of these two vectors is

$$\mathrm{v}_1 \cdot \mathrm{v}_2 = \sum_{i=1}^{n} v_{1i}v_{2i}. \tag{2-1}$$

The norm of a vector $\mathbf{v}$, denoted as $||\mathbf{v}||$, is defined as $(\mathbf{v}\cdot\mathbf{v})^{1/2}$. Two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are said to be *orthogonal* if the inner product between them is 0.

Example 2-1
Consider $\mathbf{v}_1 = (1,1)$ and $\mathbf{v}_2 = (1,-1)$. The inner product of these vectors, $\mathbf{v}_1 \cdot \mathbf{v}_2 = (1\times 1) + (1\times(-1)) = 1 - 1 = 0$. Therefore these vectors are orthogonal to each other.

A set of vectors are orthonormal if each pair of these vectors are orthogonal to each other and the norm of each vector is 1. For instance, (1,0) and (0,1) are orthonormal and ($1/\sqrt{2}$, $1/\sqrt{2}$) and ($1/\sqrt{2}$,$-1/\sqrt{2}$) are also orthonormal.

Let us consider two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ which are orthogonal and $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2$. Then a can be found by $\mathbf{v}\cdot\mathbf{v}_1$ because

$$\begin{aligned}\mathbf{v}\cdot\mathbf{v}_1 &= a(\mathbf{v}_1\cdot\mathbf{v}_1) + b(\mathbf{v}_2\cdot\mathbf{v}_1)\\ &= a||\mathbf{v}_1||^2 + b\cdot 0\\ &= a||\mathbf{v}_1||^2\end{aligned}$$

Thus:

$$a = \frac{\mathbf{v}\cdot\mathbf{v}_1}{||\mathbf{v}_1||^2}.$$

The value of b can be found in a similar way.

Example 2-2
Consider vectors $\mathbf{v}_1 = (1/\sqrt{2}, 1/\sqrt{2})$ and $\mathbf{v}_2 = (1/\sqrt{2}, -1/\sqrt{2})$. Let $\mathbf{v} = (7/\sqrt{2}, -1/\sqrt{2})$. Suppose that $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2$. Then a can be found as follows:

$$a = \frac{\mathbf{v} \cdot \mathbf{v}_1}{||\mathbf{v}_1||^2} = \frac{\left(\frac{7}{2} - \frac{1}{2}\right)}{1} = 3.$$

Similarly, b can be found by

$$b = \frac{\mathbf{v} \cdot \mathbf{v}_2}{||\mathbf{v}_2||^2} = \frac{\left(\frac{7}{2} + \frac{1}{2}\right)}{1} = 4.$$

In general, if we have $\mathbf{v} = \sum_{i=1}^{n} a_i \mathbf{v}_i$, where the v_i are orthogonal with one another, a_i can be found as follows:

$$a_i = \frac{\mathbf{v} \cdot \mathbf{v}_i}{||\mathbf{v}_i||^2}. \tag{2-2}$$

2.2 The Signal Space

Signals are functions of time. Let us denote a signal by $\phi(t)$. Given two signals $\phi_i(t)$ and $\phi_j(t)$, the inner product of these signals, denoted as $\langle \phi_i(t), \phi_j(t) \rangle$ is defined as follows:

$$\langle \phi_i(t), \phi_j(t) \rangle = \int_a^b \phi_i(t)\phi_j(t)dt. \tag{2-3}$$

Note that the inner product of two signals, defined as in (2-3), is also a function of the two parameters a and b. Different values of a and b will give different results. To emphasize this point, in this book, whenever it is needed, we shall sometimes say that this is an inner product of $\phi_1(t)$ and $\phi_2(t)$ with respect to interval $\{a, b\}$.

Two signals are said to be *orthogonal* if their inner product is 0. The norm of a signal $\phi(t)$, denoted as $||\phi(t)||$, is defined as $(\langle \phi(t), \phi(t) \rangle)^{1/2}$. A set of signals are called *orthonormal* if they are orthogonal to one another and their norms are all equal to 1.

In the rest of this section, whenever parameters k and n are used as coefficients, they denote positive integers, unless otherwise stated. Let us now give several examples.

Example 2-3
Let $\phi_1(t) = \sin(2\pi k f_c t)$ and $\phi_2(t) = \sin(2\pi n f_c t)$. The inner product of these signals will depend on the interval of integration and whether $k = n$ or $k \neq n$. In the following, we assume that the interval of integration is $\{0, T\}$ where $T = 1/f_c$.

Case 1: $k \neq n$. In this case, we use Equation A-10:

$$\begin{aligned} \langle \phi_1(t), \phi_2(t) \rangle &= \int_0^T \sin(2\pi k f_c t) \sin(2\pi n f_c t) dt \\ &= \int_0^T \frac{1}{2} [\cos(2\pi(k-n) f_c t) - \cos(2\pi(k+n) f_c t)] dt \\ &= \frac{1}{2} \left[\frac{\sin(2\pi(k-n) f_c t)}{(2\pi(k-n) f_c)} - \frac{\sin(2\pi(k+n) f_c t)}{(2\pi(k+n) f_c)} \right] \Bigg|_{t=0}^{t=T} \\ &= 0 \end{aligned} \tag{2-4}$$

Case 2: $k = n \neq 0$. In this case, we use Equation A-17:

$$\langle \phi_1(t), \phi_2(t) \rangle = \frac{1}{2} \int_0^T (1 - \cos(2\pi(2k) f_c t)) dt = \frac{1}{2} \Bigg|_{t=0}^{t=T} = \frac{T}{2}. \tag{2-5}$$

In summary, we have:

$$\langle \sin(2\pi k f_c t), \sin(2\pi n f_c t) \rangle = 0 \quad \text{if} \quad k \neq n, \tag{2-6}$$

$$\langle \sin(2\pi k f_c t), \sin(2\pi n f_c t) \rangle = \frac{T}{2} \quad \text{if} \quad k = n \neq 0. \tag{2-7}$$

Identical results hold for $\phi_1(t) = \cos(2\pi k f_c t)$ and $\phi_2(t) = \cos(2\pi n f_c t)$.

Example 2-4
Let $\phi_1(t) = \sin(2\pi k f_c t)$ and $\phi_2(t) = \cos(2\pi n f_c t)$.

Case 1: $k \neq n$. We use Equation A-11:

$$\begin{aligned} \langle \phi_1(t), \phi_2(t) \rangle &= \int_0^T \sin(2\pi k f_c t) \cos(2\pi n f_c t) dt \\ &= \int_0^T \frac{1}{2} (\sin(2\pi(k-n) f_c t) + \sin(2\pi(k+n) f_c t)) dt \\ &= -\frac{1}{2} \left[\frac{\cos(2\pi(k-n) f_c t)}{(2\pi(k-n) f_c)} + \frac{\cos(2\pi(k+n) f_c t)}{(2\pi(k+n) f_c)} \right]_{t=0}^{t=T} \\ &= 0. \end{aligned} \tag{2-8}$$

Case 2: $k = n \neq 0$. We use Equation A-13:

$$\langle \phi_1(t), \phi_2(t) \rangle = \frac{1}{2} \int_0^T (\sin(2\pi(2k) f_c t)) dt = 0. \tag{2-9}$$

Thus we have:

$$\langle \sin k(2\pi f_c t), \cos n(2\pi f_c t)\rangle = 0 \quad \text{for all } k \text{ and } n. \tag{2-10}$$

We may summarize the results from Examples 2-3 and 2-4 as follows:

$$\langle \sin(2\pi k f_c t), \sin(2\pi n f_c t)\rangle = 0 \text{ if } k \neq n \tag{2-11}$$

$$\langle \sin(2\pi k f_c t), \sin(2\pi n f_c t)\rangle = \frac{T}{2} \text{ if } k = n \neq 0 \tag{2-12}$$

$$\langle \cos(2\pi k f_c t), \cos(2\pi n f_c t)\rangle = 0 \text{ if } k \neq n \tag{2-13}$$

$$\langle \cos(2\pi k f_c t), \cos(2\pi n f_c t)\rangle = \frac{T}{2} \text{ if } k = n \neq 0 \tag{2-14}$$

$$\langle \sin(2\pi k f_c t), \cos(2\pi n f_c t)\rangle = 0 \text{ for all } k \text{ and } n. \tag{2-15}$$

Table 2-1 summarizes the above equations. As shown, sine functions, or cosine functions, are not orthogonal only to themselves.

Some examples are summarized in Figure 2-1, which illustrates four cases: $\sin(x)\cos(3x)$, $\sin(x)\sin(3x)$, $\sin(x)\cos(x)$ and $\sin(x)\sin(x)$. The reader should examine each case carefully. For the first three cases, the total area above the x-axis is equal to the total area below the x-axis. Consequently, integration of the functions over the period from 0 to 2π yields 0 for these three cases. For the last case, there is no area below the x-axis. Thus, the integration yields a non-zero result. These results are compatible with Equations (2-11) to (2-15).

The above results can be explained from another point of view. Note that the integration of a sinusoidal function over its period is 0. An examination of Equations (A-9) to (A-13) in Appendix A shows that $\cos\alpha\cos\beta$, $\sin\alpha\sin\beta$, $\sin\alpha\cos\beta$ and $\cos\alpha\sin\beta$ are all equivalent to summations of cosine and sine functions. If we perform an integration on them over the period, the result will be zero. On the other hand, Equations (A-17) and (A-18) show that there are constants in the formulas for $\sin^2\alpha$ and $\cos^2\alpha$. Thus integration over them will produce a non-zero result.

Suppose that our signal is represented by the following formula:

$$T = 1/S_c, f(t) = \sum_{k=1}^{n} a_k \cos(2\pi k f_c t) + \sum_{k=1}^{n} b_k \sin(2\pi k f_c t) \tag{2-16}$$

Table 2-1 The orthogonality of sinusoidal functions

Inner product	$\cos(2\pi k f_0 t)$	$\cos(2\pi n f_0 t)$	$\sin(2\pi k f_0 t)$	$\sin(2\pi n f_0 t)$
$\cos(2\pi k f_0 t)$	$\frac{T}{2}$	0	0	0
$\cos(2\pi n f_0 t)$	0	$\frac{T}{2}$	0	0
$\sin(2\pi k f_0 t)$	0	0	$\frac{T}{2}$	0
$\sin(2\pi n f_0 t)$	0	0	0	$\frac{T}{2}$

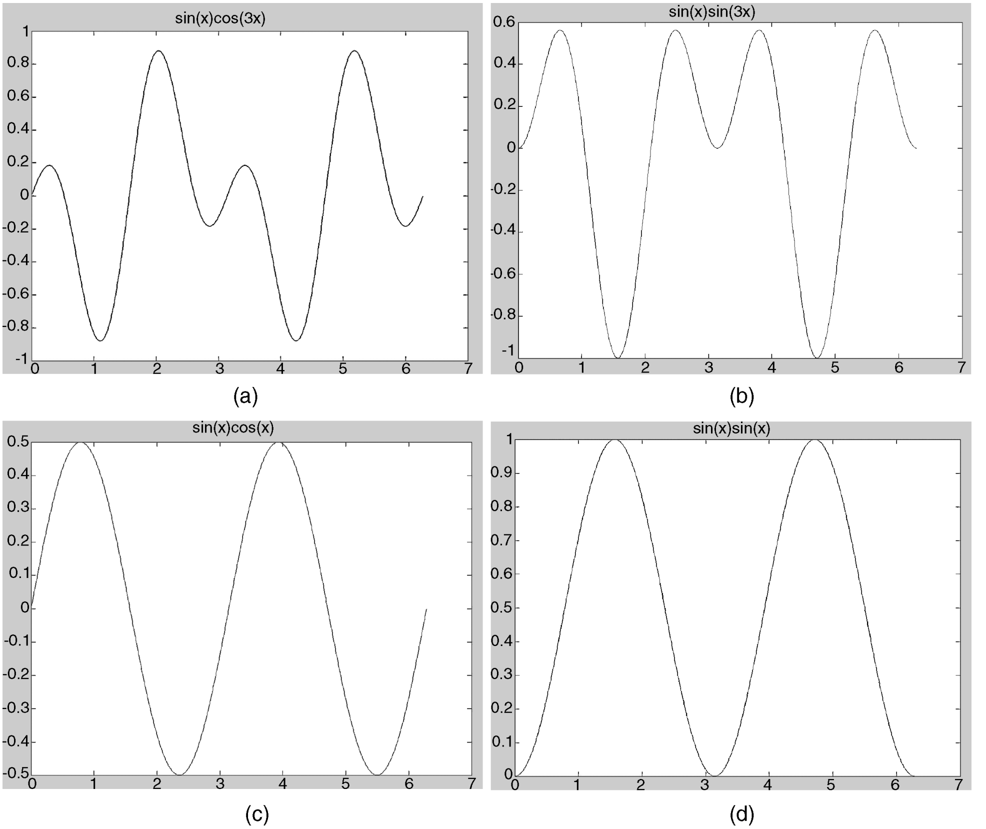

Figure 2-1 Illustration of four functions: (a) $\sin(x)\cos(3x)$, (b)$\sin(x)\sin(3x)$, (c)$\sin(x)\cos(x)$, and (d)$\sin(x)\sin(x)$

and we are asked to determine a_k and b_k. To do this, we may use the above equations concerned with inner products of cosine and sine functions. That is, to determine a_k, we perform the following:

$$\langle f(t), \cos(2\pi k f_c t)\rangle = \int_0^T f(t)\cos(2\pi k f_c t)dt. \tag{2-17}$$

From Equations (2-11) to (2-15), we can easily see that all terms in Equation (2-17) vanish except a_k. Thus, from Equation (2-14), we have:

$$a_k = \frac{2}{T}\langle f(t), \cos(2\pi k f_c t)\rangle = \frac{2}{T}\int_0^T f(t)\cos(2\pi k f_c t)dt. \tag{2-18}$$

Similarly, b_k can be found by the following formula:

$$b_k = \frac{2}{T}\langle f(t), \sin(2\pi k f_c t)\rangle = \frac{2}{T}\int_0^T f(t)\sin(2\pi k f_c t)dt. \quad (2\text{-}19)$$

The above result will be very useful when we introduce the concepts of Fourier series, Fourier transform, amplitude modulation, double sideband modulation, single sideband modulation, QPSK and OFDM etc. in later chapters.

Example 2-5
Consider the case where $\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_1 t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_2 t)$ under the condition that $0 \le t < T$ and

$$f_i = \frac{n_c + i}{T} \quad \text{for some integer } n_c \text{ and } i = 1, 2. \quad (2\text{-}20)$$

$$\begin{aligned}\langle \phi_1(t), \phi_2(t)\rangle &= \frac{2}{T}\int_0^T \cos(2\pi f_1 t)\cos(2\pi f_2 t)dt \\ &= \frac{1}{T}\int_0^T (\cos(2\pi(f_1 + f_2)t) + \cos(2\pi(f_1 - f_2)t))dt. \end{aligned} \quad (2\text{-}21)$$

Substituting Equation (2-20) into Equation (2-21), we have:

$$\langle \phi_1(t), \phi_2(t)\rangle = \frac{1}{T}\left(\int_0^T \cos\left(2\pi\left(\frac{2n_c + 3}{T}t\right)\right)dt + \int_0^T \cos\left(2\pi\left(\frac{-1}{T}\right)t\right)dt\right). \quad (2\text{-}22)$$

In the above equation, for both the first and the second integrations, an integer number of periods are involved. Therefore both terms are zero. We thus have:

$$\langle \phi_1(t), \phi_2(t)\rangle = \langle \cos(2\pi f_1 t), \cos(2\pi f_2 t)\rangle = 0 \quad (2\text{-}23)$$

under the condition that Equation (2-20) is satisfied.

We can also prove that, for $i = 1{,}2$:

$$\langle \phi_i(t), \phi_i(t)\rangle = 1. \quad (2\text{-}24)$$

There is another way of looking at the problem. Let $f_c = 1/T$. Then $f_1 = (n_c + 1)f_c$ and $f_2 = (n_c + 2)f_c$. We may say that $f_1 = kf_c$, $f_2 = nf_c$ and $k \neq n$. Thus, according to Equation (2-13), we can easily derive Equation (2-23).

Suppose that a signal is represented as $f(t) = a\phi_1(t) + b\phi_2(t)$, where $\langle \phi_1(t), \phi_2(t) \rangle = 0$ and $\langle \phi_i(t), \phi_i(t) \rangle = 1$ for $i = 1, 2$. Then, it can be easily seen that a and b can be found through the following formulas:

$$
\begin{aligned}
a &= \int_0^T f(t)\phi_1(t)dt \\
b &= \int_0^T f(t)\phi_2(t)dt.
\end{aligned}
\tag{2-25}
$$

The above result will be useful when we introduce binary frequency-shift keying in Chapter 5.

Example 2-6
Let us consider the two signals illustrated in Figure 2-2(a) and 2-2(b). The multiplication of these two signals is shown in Figure 2-2(c), and the integration of the signal in 2-2(c) is shown in 2-2(d). One can see that these two signals are orthogonal because $\int_0^T \phi_1(t)\phi_2(t)dt = 0$.

Example 2-7
Consider two signals similar to those in Example 2-6. As shown in Figure 2-3, these two signals are again orthogonal.

In the above examples, we can also consider these two signals as vectors because they assume discrete values. In Example 2-6, the two signals can be represented as follows:

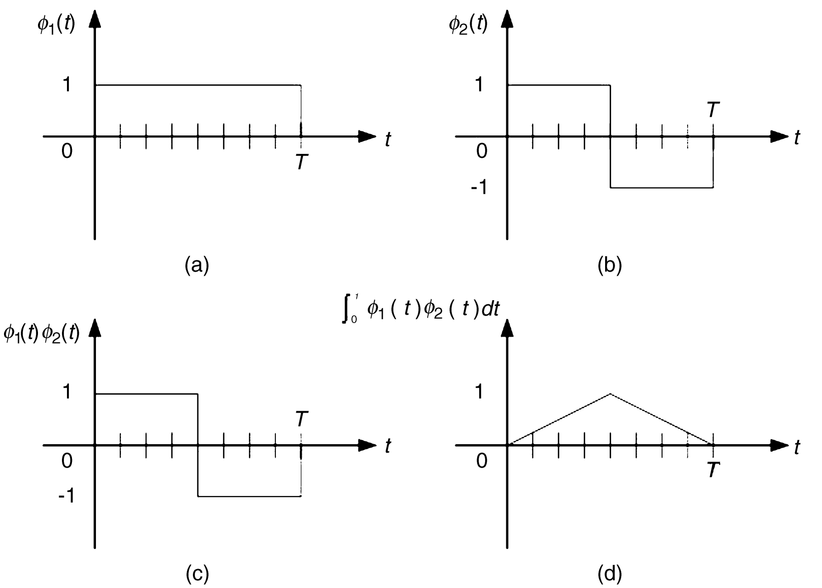

Figure 2-2 Two orthogonal signals: (a) $\phi_1(t)$, (b) $\phi_2(t)$, (c) $\phi_1(t)\phi_2(t)$, and (d) $\int_0^t \phi_1(t)\phi_2(t)dt$

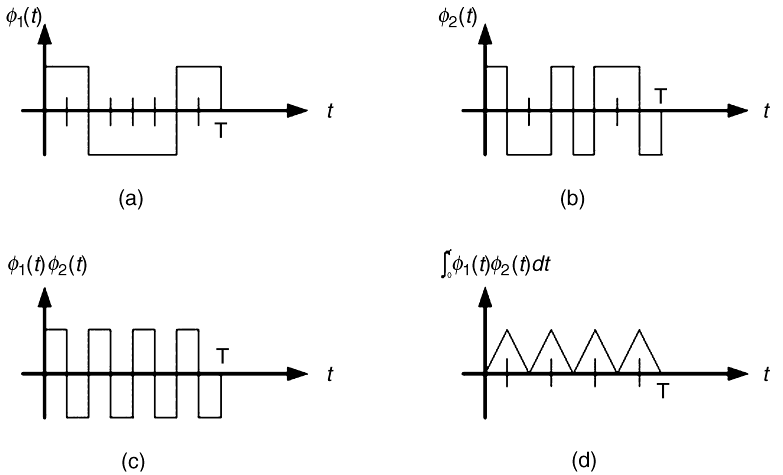

Figure 2-3 Another two signals: (a) $\phi_1(t)$, (b) $\phi_2(t)$, (c) $\phi_1(t)\phi_2(t)$, and (d) $\int_0^t \phi_1(t)\phi_2(t)dt$

$\mathbf{v}_1 = (1,1)$ and $\mathbf{v}_2 = (1,-1)$. The inner product of these vectors is

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = (1 \times 1) + (1 \times (-1)) = 1 + (-1) = 0.$$

Similarly, the signals in Example 2-7 can be represented as $\mathbf{v}_1 = (1, 1, -1, -1, -1, -1, 1, 1)$ and $\mathbf{v}_2 = (1, -1, -1, 1, -1, 1, 1, -1)$. The inner product of these two vectors is

$$\begin{aligned}
&\mathbf{v}_1 \cdot \mathbf{v}_2 \\
&= (1 \times 1) + (1 \times (-1)) + ((-1) \times (-1)) + ((-1) \times 1) \\
&\quad + ((-1) \times (-1)) + ((-1) \times 1) + (1 \times 1) + (1 \times (-1)) \\
&= 1 + (-1) + 1 + (-1) + 1 + (-1) + 1 + (-1) \\
&= 0
\end{aligned}$$

It is interesting to ask whether the signal space concept and the vector space concept are entirely different. Consider two signals denoted as $\phi_1(t)$ and $\phi_2(t)$. The inner product between $\phi_1(t)$ and $\phi_2(t)$ is defined to be $\int_a^b \phi_1(t)\phi_2(t)dt$. Suppose we do not have analytical formulas for $\phi_1(t)$ and $\phi_2(t)$. We can sample these two signals at time $t_1, t_2, \cdots, t_n$ from $t = a$ to $t = b$. Thus $\phi_1(t)$ is characterized as $\phi_1(t_1), \phi_1(t_2), \cdots, \phi_1(t_n)$ and $\phi_2(t)$ is characterized as $\phi_2(t_1), \phi_2(t_2), \cdots, \phi_2(t_n)$. Let $A = (\phi_1(t_1), \phi_1(t_2), \cdots, \phi_1(t_n))$ and $B = (\phi_2(t_1), \phi_2(t_2), \cdots, \phi_2(t_n))$. Then we can easily see that:

$$A \cdot B = \frac{b-a}{n}\sum_{i=1}^{n} \phi_1(t_i)\phi_2(t_i) = \int_a^b \phi_1(t)\phi_2(t)dt \quad \text{as} \quad n \to \infty.$$

Let us redraw Figure 2-3(c) as in 2-4(a). We can see that the integration of $\phi_1(t)\phi_2(t)$ is equivalent to finding the total area covered by the function $\phi_1(t)\phi_2(t)$. From $t = 0$ to $t = 1$, the area is +1. From $t = 1$ to $t = 2$, the area is –1. Thus the total area from $t = 0$ to $t = 2$ is

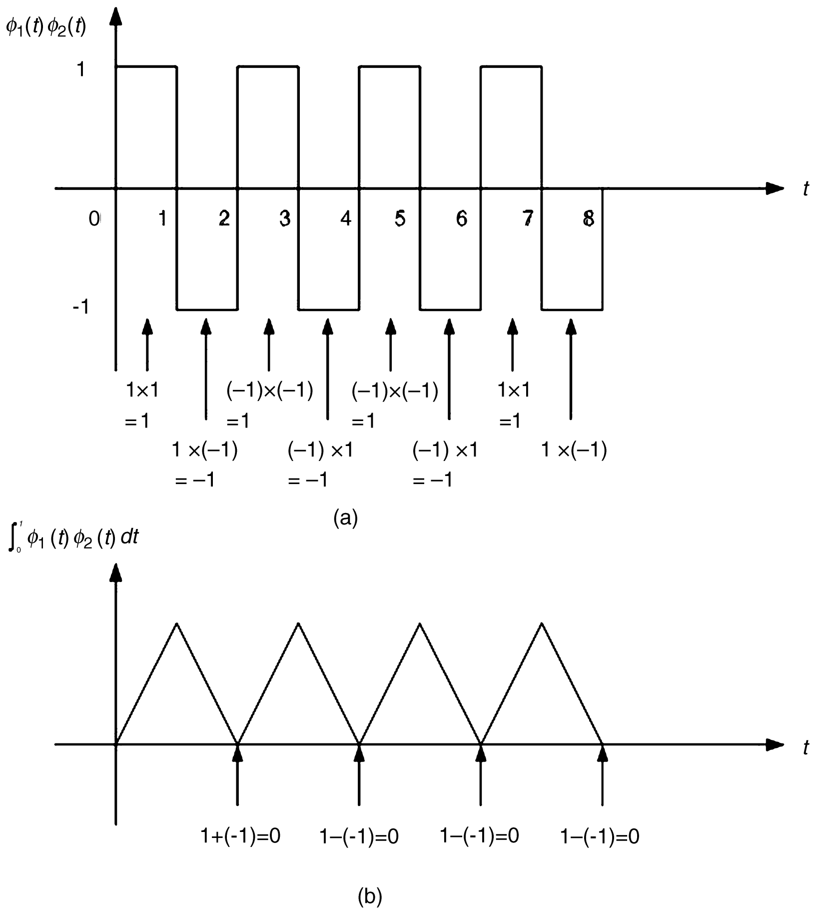

Figure 2-4 Comparison of the signal space inner product and vector space inner product: (a) vector space inner product, (b) signal space inner product

$+1 + (-1) = 0$. This exactly corresponds to the first two steps of the calculation of $\mathbf{v}_1 \cdot \mathbf{v}_2$ as shown above. Note that the first step of $\mathbf{v}_1 \cdot \mathbf{v}_2$ is $1 \times 1 = 1$ and the second step is $1 \times (-1) = -1$. The sum of the results of the first two steps is therefore $1 + (-1) = 0$. This process continues as indicated in Figure 2-4(b). Note that at the end, the result is 0.

The above result is useful when we introduce the CDMA mechanism which will be presented later.

2.3 Summary

In general, as will be shown later, a signal is often represented as follows:

$$f(t) = \sum_{i=1}^{n} a_i \phi_i(t)$$

where the ϕ_i are orthogonal. Our job is often to find the a_i. Because of the orthogonal properties of the ϕ_i, a_i can be found as follows:

$$a_i = \frac{1}{||\phi_i(t)||^2} \int_0^T f(t)\phi_i(t)dt.$$

This mechanism is used again and again in many chapters of this book.

Further Reading

- For more details about the background of linear algebra, see [AR05].
- For more details about the relations between signal and systems, see [C98].

Exercises

2.1 (a) Plot $\cos(2\pi(2)t)$ and $\sin(2\pi(3)t)$.
(b) Explain why $\int_0^1 \cos(2\pi(2)t)dt = 0$ and $\int_0^1 \sin(2\pi(3)t)dt = 0$.

2.2 (a) Plot $\cos^2(2\pi t)$ and $\sin^2(2\pi t)$.
(b) Explain why $\int_0^1 \cos^2(2\pi t)dt \neq 0$ and $\int_0^1 \sin^2(2\pi t)dt \neq 0$.

2.3 Complete the following equations:
$\cos(\alpha + \beta) =$
$\cos(\alpha - \beta) =$
$\sin(\alpha + \beta) =$
$\sin(\alpha - \beta) =$

2.4 Complete the following equations:
$\cos\alpha \cos\beta =$
$\sin\alpha \sin\beta =$
$\sin\alpha \cos\beta =$
$\cos\alpha \sin\beta =$

2.5 Complete the following equations:
$\sin(2\alpha) =$
$\cos(2\alpha) =$
$1 - 2\sin^2(\alpha) = 2\cos^2(\alpha) - 1 =$
$2\sin(\alpha)\cos(\alpha) =$

2.6 Complete the following:
$\sin^2\alpha =$
$\cos^2\alpha =$

2.7 Let $\mathbf{v}_1 = \left(\cos(2\pi(0)), \cos\left(2\pi\left(\frac{1}{4}\right)\right), \cos\left(2\pi\left(\frac{2}{4}\right)\right), \cos\left(2\pi\left(\frac{3}{4}\right)\right)\right)$
and $\mathbf{v}_2 = \left(\sin(2\pi(0)), \sin\left(2\pi\left(\frac{1}{4}\right)\right), \sin\left(2\pi\left(\frac{2}{4}\right)\right), \sin\left(2\pi\left(\frac{3}{4}\right)\right)\right)$.
Compute $\mathbf{v}_1 \cdot \mathbf{v}_2$. Explain the result.

2.8 (a) Let $\mathbf{v}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\mathbf{v}_2 = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Show that these two vectors are orthonormal.
(b) Let $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2 = \left(\frac{2+3\sqrt{3}}{2}, \frac{2\sqrt{3}-2}{2}\right)$. Find a and b.

2.9 Let $\phi_1(t) = \cos(2\pi t)$ and $\phi_2(t) = \cos\left(2\pi t - \frac{\pi}{4}\right)$. Are $\phi_1(t)$ and $\phi_2(t)$ orthogonal to each other? Why?

2.10 Let $\phi_1(t) = \cos(2\pi t)$ and $\phi_2(t) = \cos\left(2\pi t - \frac{\pi}{2}\right)$. Are $\phi_1(t)$ and $\phi_2(t)$ orthogonal to each other? Why?

2.11 Let $V_1 = (1, -1, 1, -1, -1, 1, -1, 1)$ and $V_2 = (1, -1, -1, 1, -1, 1, 1, -1)$.
(a) Prove that these two vectors are orthogonal to each other.
(b) Let $V = (0, 0, 2, -2, 0, 0, -2, 2) = aV_1 + bV_2$. Determine a and b.

2.12 Consider the following vectors:

$$
\begin{aligned}
V_1 &= (1, 1, 1, 1, 1, 1, 1, 1)\\
V_2 &= (1, -1, 1, -1, 1, -1, 1, -1)\\
V_3 &= (1, 1, -1, -1, 1, 1, -1, -1)\\
V_4 &= (1, -1, -1, 1, 1, -1, -1, 1).
\end{aligned}
$$

(a) Prove that these four vectors are orthogonal to each other.
(b) Let $V = (-2, -2, 4, 0, -2, -2, 4, 0) = aV_1 + bV_2 + cV_3 + dV_4$. Determine a, b, c and d.

2.13 Let $\phi_1(t)\cos(2\pi t)$ and $\phi_2(t)\cos(6\pi t)$. Are these signals orthogonal to each other? Explain.

3

Fourier Representations of Signals

In this chapter, the concept of representing a signal in both time-domain and frequency-domain spaces is introduced and applied. Usually, a signal is described as a function of time. However, there are some amazing advantages if a signal can be expressed in the frequency domain. To represent a signal in both time and frequency domains results from the introduction of a series of harmonically related sinusoids. The study of signals and systems using sinusoidal representations is termed *Fourier transform analysis* after Jean Baptiste Joseph Fourier (1768–1830), a great French mathematician, for his contributions to the theory of representing functions as weighted superpositions of sinusoids. Fourier methods including Fourier series and Fourier transform for both continuous-time and discrete-time cases have widespread applications in almost every branch of engineering and science.

Before giving formal concepts related to the Fourier transform, let us first consider Figure 3-1. Imagine that we want to transmit this signal, which is denoted as $f(t)$. A straightforward way to do so is to perform a sampling and then to transmit the sampled points. That is, we transmit $f(t_1), f(t_2), \cdots, f(t_n)$. It can be easily seen that n should be as large as possible. In order to make sure that $f(t)$ is received accurately, a large amount of data has to be transmitted. This puts a heavy burden to the communications system.

The signal in Figure 3-1 is not sinusoidal. Suppose we apply the discrete Fourier transform to the signal in Figure 3-1 after sampling. The result is presented in Figure 3-2.

From Figure 3-2, it can be seen that the signal consists of two sinusoidal functions. One is of frequency 7 and with a small amplitude. The other is of frequency 13 and with a large amplitude. We shall show later that the frequencies 51 and 57 should be ignored. In fact, the discrete Fourier transform indicates that the signal in Figure 3-1, denoted as $f(t)$, can be expressed by the following equation:

$$f(t) = \cos(2\pi(7)t) + 3\cos(2\pi(13)t). \tag{3-1}$$

In other words, instead of transmitting $f(t_1), f(t_2), \cdots, f(t_n)$, we only have to send the frequencies 7 and 13, and their corresponding amplitudes, of the two cosine functions. The receiving end, after receiving these parameters, can reconstruct $f(t)$ by using Equation (3-1).

Let us now consider Figure 3-3. The result of applying discrete the Fourier transform to the signal is shown in Figure 3-4. From this we can see that the signal shown in Figure 3-3 is

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