LECTURE NOTES
ON
STRUCTURAL MECHANICS

(3rd Semester)

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Th1. STRUCTURAL MECHANICS

Name of the Course: Diploma in Civil Engineering			
Course code:		Semester	3^{rd}
Total Period:	75	Examination	3 hrs
Theory periods:	5P/week	Internal Assessment:	20
Maximum marks:	100	End Semester Examination:	80

A. RATIONALE

The course aims to prepare the students to comprehend the design principles associated with the structural members. The students will develop competency in calculating necessary dimensions and material properties so that the members can withstand the loading conditions.

B. COURSE OBJCTIVES

On completion of the course, students will be able to -

1. Comprehend, define, compute and interpret major mechanical properties demonstrated by solid materials.
2. Analyze solid states under uniaxial loading and plane stress conditions.
3. Draw shear force and bending moment diagrams of simple statically determinate and statically indeterminate structural members subject to transverse loading.
4. Obtain slope and deflection profiles of statically determinate simple structural members.
5. Comprehend buckling as a failure mode in column and determine crippling loads for columns using Euler's theory.
6. Compute forces in members of a truss

C. TOPIC WISE DISTRIBUTION

Chapter	Name of topics	Periods
1	Review of Basic Concepts	04
2	Simple and Complex Stress, Strain	15
3	Stresses in Beams	10
4	Columns and Struts	04
5	Shear Force and Bending Moment	12
6	Slope and Deflection	10
7	Indeterminate Beams	10
8	Trusses and Frames	10

D. Course Contents:

1 Review Of Basic Concepts

1.1 Basic Principle of Mechanics: Force, Moment, support conditions, Conditions of equilibrium, C.G & MI, Free body diagram

1.2 Review of CG and MI of different sections

2 Simple And Complex Stress, Strain

2.1 Simple Stresses and Strains

Introduction to stresses and strains: Mechanical properties of materials – Rigidity, Elasticity, Plasticity, Compressibility, Hardness, Toughness, Stiffness, Brittleness, Ductility, Malleability, Creep, Fatigue, Tenacity, Durability, Types of stresses -Tensile, Compressive and Shear stresses, Types of strains - Tensile, Compressive and Shear strains, Complimentary shear stress - Diagonal tensile / compressive Stresses due to shear, Elongation and Contraction, Longitudinal and Lateral strains, Poisson's Ratio, Volumetric strain, computation of stress, strain, Poisson's ratio, change in dimensions and volume etc, Hooke's law - Elastic Constants, Derivation of relationship between the elastic constants.

2.2 Application of simple stress and strain in engineering field:

Behaviour of ductile and brittle materials under direct loads, Stress Strain curve of a ductile material, Limit of proportionality, Elastic limit, Yield stress, Ultimate stress, Breaking stress, Percentage elongation, Percentage reduction in area, Significance of percentage elongation and reduction in area of cross section, Deformation of prismatic bars due to uniaxial load, Deformation of prismatic bars due to its self weight.

2.3 Complex stress and strain

Principal stresses and strains: Occurrence of normal and tangential stresses, Concept of Principal stress and Principal Planes, major and minor principal stresses and their orientations, Mohr's Circle and its application to solve problems of complex stresses

3 Stresses In Beams and Shafts

3.1 Stresses in beams due to bending: Bending stress in beams – Theory of simple bending – Assumptions – Moment of resistance – Equation for Flexure– Flexural stress distribution – Curvature of beam – Position of N.A. and Centroidal Axis – Flexural rigidity – Significance of Section modulus

3.2 Shear stresses in beams: Shear stress distribution in beams of rectangular, circular and standard sections symmetrical about vertical axis.

3.3 Stresses in shafts due to torsion: Concept of torsion, basic assumptions of pure torsion, torsion of solid and hollow circular sections, polar moment of inertia, torsional shearing stresses, angle of twist, torsional rigidity, equation of torsion

3.4 Combined bending and direct stresses: Combination of stresses, Combined direct and bending stresses, Maximum and Minimum stresses in Sections, Conditions for no tension, Limit of eccentricity, Middle third/fourth rule, Core or Kern for square, rectangular and circular sections, chimneys, dams and retaining walls

4 Columns and Struts

4.1 Columns and Struts, Definition, Short and Long columns, End conditions, Equivalent length / Effective length, Slenderness ratio, Axially loaded short and long column, Euler's theory of long columns, Critical load for Columns with different end conditions

5 Shear Force and Bending Moment

5.1 Types of loads and beams:

Types of Loads: Concentrated (or) Point load, Uniformly Distributed load (UDL), Types of Supports: Simple support, Roller support, Hinged support, Fixed support, Types of Reactions: Vertical reaction, Horizontal reaction, Moment reaction, Types of Beams based on support conditions: Calculation of support reactions using equations of static equilibrium.

5.2 Shear force and bending moment in beams:

Shear Force and Bending Moment: Signs Convention for S.F. and B.M, S.F and B.M of general cases of determinate beams with concentrated loads and udl only, S.F and B.M diagrams for Cantilevers, Simply supported beams and Over hanging beams, Position of maximum BM, Point of contra flexure, Relation between intensity of load, S.F and B.M.

6 Slope and Deflection

6.1 Introduction: Shape and nature of elastic curve (deflection curve); Relationship between slope, deflection and curvature (No derivation), Importance of slope and deflection.

6.2 Slope and deflection of cantilever and simply supported beams under concentrated and uniformly distributed load (by Double Integration method, Macaulay's method).

7 Indeterminate Beams

7.1 Indeterminacy in beams, Principle of consistent deformation/compatibility, Analysis of propped cantilever, fixed and two span continuous beams by principle of superposition, SF and BM diagrams (point load and udl covering full span)

8 Trusses

8.1 Introduction: Types of trusses, statically determinate and indeterminate trusses, degree of indeterminacy, stable and unstable trusses, advantages of trusses.

8.2 Analysis of trusses: Analytical method (Method of joints, method of Section)

E. Course Coverage Upto Internal Assessment: Chapters 1,2,3,4

F. Recommended Books

Sl. No	Name of Authors	Titles of Book	Name of Publisher
1	R.Subramanian	Strength of Materials	Oxford Publication
2	S.Rammrutham,	Theory of structure	Dhanpat Rai Publications
3	V.N.Vazirani&M.M. Rathwani	Analysis of Structures-Vol.I&II	Khanna Publication

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UNIT-I

SIMPLE STRESSES, STRAIN AND DEFORMATION OF SOLIDS

1. LOAD OR FORCE :

It is the external energy or agent which causes changes in a body at rest is called as force or load.

Unit of Force is $N \Rightarrow kg\, m/s^2$

2. TYPES OF LOAD :

Load → Point load, UDL, UVL

3. Nature of load :

Tensile load

(pulling load)

compressive load

(pushing load)

4. Stress :

As the internal resistance offered by the body due to the application of external force per unit cross sectional area is known as stress.

5. Types of stresses :

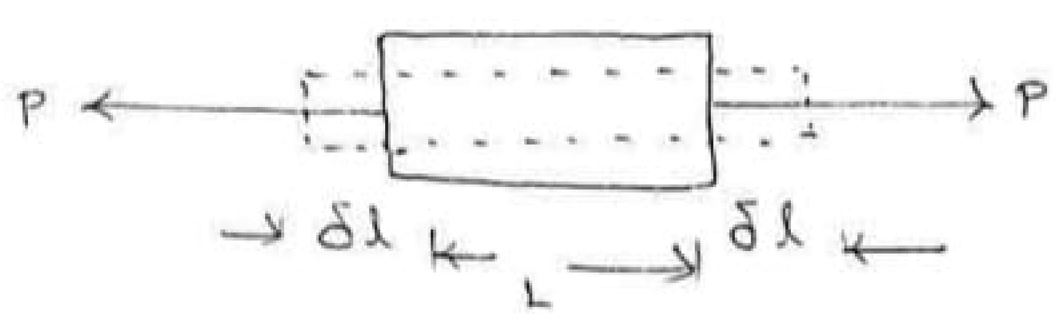
1. Simple Stress (or) Direct Stress

Tensile stress | Compressive stress | Shear stress

2. Indirect stress: Bending, Torsion.

6. Tensile stress :

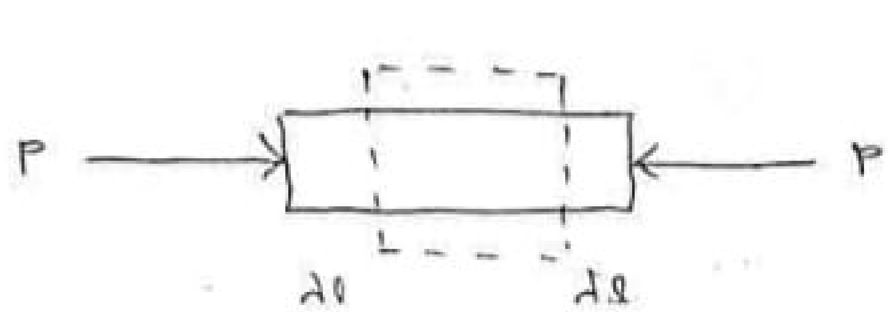
When a material is subjected to tensile force at two ends the internal resistance offered by the body per unit cross sectional area is known as tensile stress.



$$\sigma_t = \frac{\text{Tensile load}}{\text{C/s Area}}$$

7. Compressive stress :

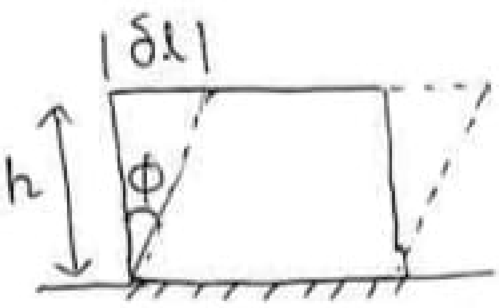
When a material is subjected to compressive force at two ends the internal resistance offered by the body per unit cross sectional area is called compressive stress.



$$\sigma_c = \frac{\text{comp. load}}{\text{C/S Area.}}$$

⑧. Shear stress : (τ)

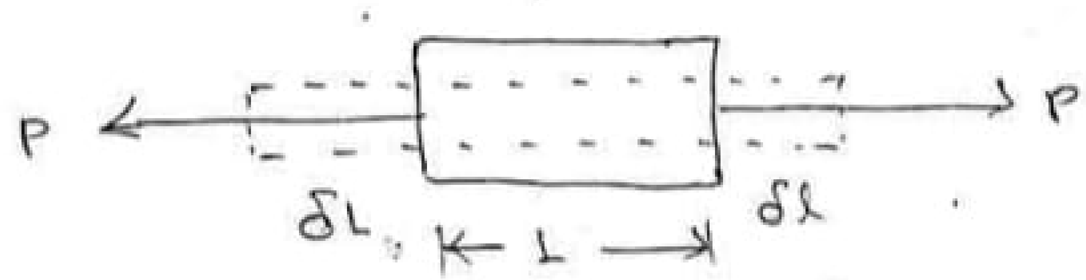
when a material is subjected to load which acts tangentially the resistance offered is called as shear stress



$$\tau = \frac{\text{shear load}}{\text{C/S Area.}}$$

⑨. Strain : (ε)

The ratio of change in dimension of a the body to the original dimension is known as strain.



$$\text{Shain} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{\delta l}{L}$$

(4)

(10) Types of strain :

i) Tensile strain :

The ratio of increase in length to the original length when tensile load is applied is called as tensile strain.

$$\varepsilon_t = \frac{\text{Increase in length}}{\text{original length.}}$$

ii) Compressive strain :

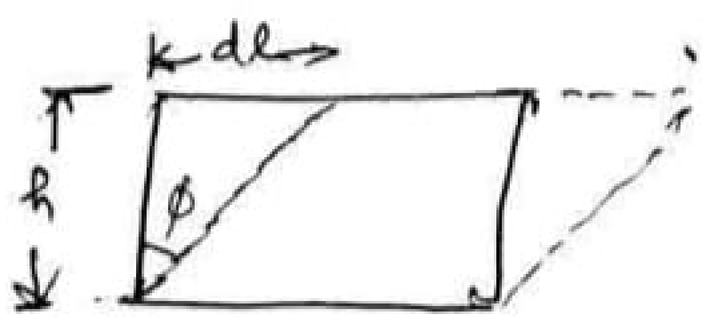
The ratio of decrease in length to the original length when compressive load is applied is called as compressive strain.

$$\varepsilon_c = \frac{\text{Decrease in length}}{\text{original length}}$$

iii) Shear strain :

The ratio of change in dimension to the distance moved from the fixed end to the height of the object.

$$\tan\phi = \frac{\delta l}{h}$$



$$\phi = \tan^{-1}\frac{\delta l}{h}$$

$\phi \Rightarrow$ transverse angle.

iv) Volumetric strain :

It is the ratio of change in volume to the original volume

$$\varepsilon_v = \frac{\delta v}{V}$$

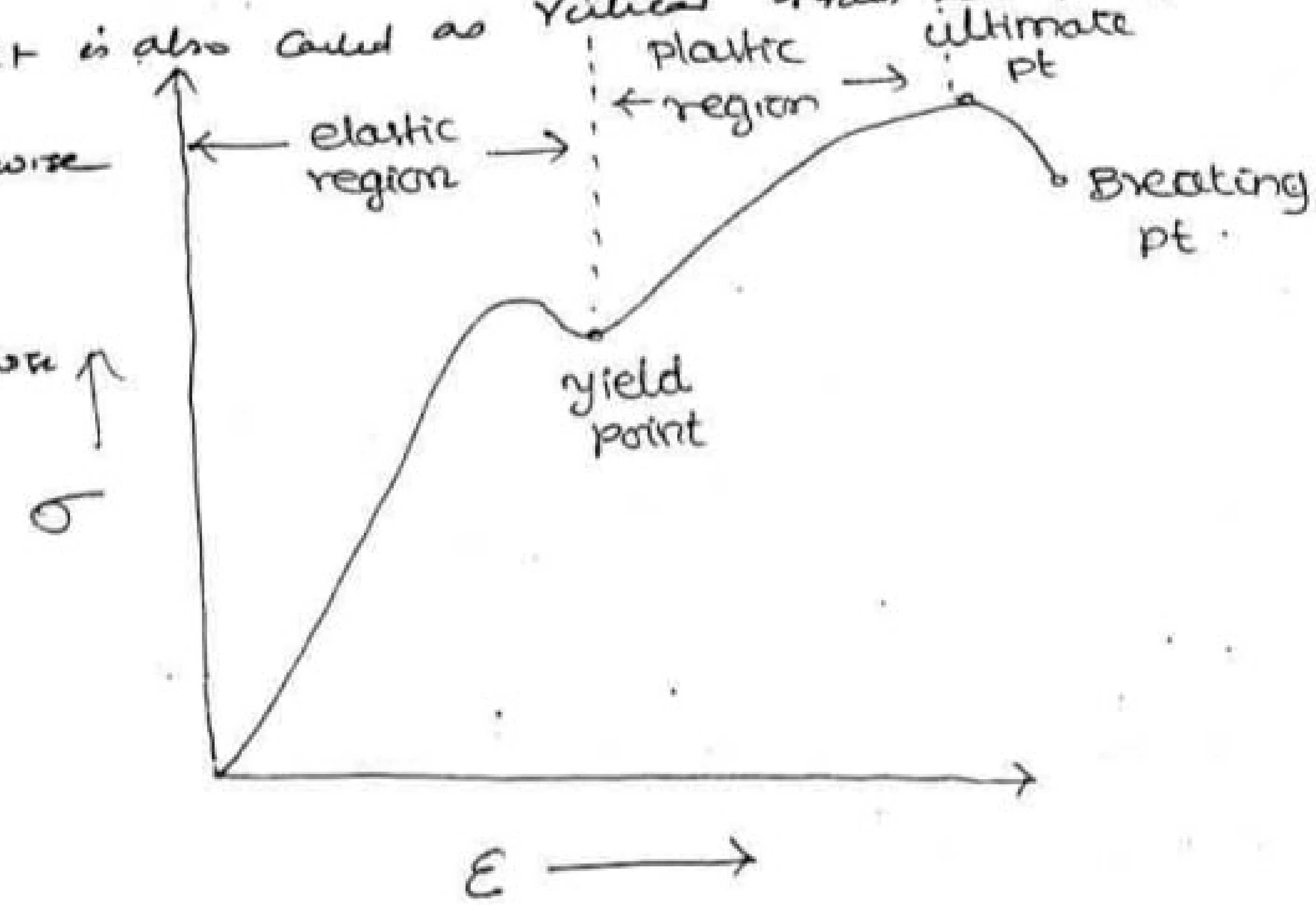
Nature of Strain: 1. Longitudinal strain (or) Linear strain (or) strain parallel to the direction of force. $\xi = \frac{\delta l}{L}$

(ii). Stress strain curve for ducktail material :

2. Lateral strain : It is also called as vertical strain ; It is ⊥r to the direction of force.

Lateral strain in breadth wise direction $= \frac{\delta b}{b}$

Lateral strain in depth wise direction $= \frac{\delta d}{d}$.



i) Elastic region :

Elastic region Materials regains its original shape after the removal of load. (temporary deformation).

ii) Plastic region :

Material does not regain its original shape after the removal of load. (permanent deformation)

iii) Yield point :

It is the end of elastic region after which permanent deformation occurs.

iv) Ultimate point :

It is a point where maximum load is applied before the material starts to fail.

v). Breaking point :

The point where the material fails (or) ruptures.

(2). Hooke's law :
(X).

It states that when a material is loaded within the elastic limit the stress is directly proportional to strain. This constant is known as modulus of elasticity (or) modulus of rigidity (or) Elastic moduli.

(13). Modulus of Elasticity (or) Young's modulus :

It is the ratio of tensile (or) compressive stress to the tensile (or) compressive strain.

N/mm^2 (or) MPa

E =

14. Modulus of rigidity (or) Shear modulus :

It is the ratio of shear stress to the shear strain.

$$C \text{ (or) } G \text{ (or) } N = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi} \ N/mm^2$$

15. Bulk modulus : K

It is the ratio of direct stress to the volumetric strain.

$$K = \frac{\text{direct stress}}{\text{volumetric strain}} = \frac{\sigma}{\left(\frac{\delta v}{v}\right)}$$

16. <u>Expression for young's modulus in terms of Bulk modulus :</u>

$$E = 3K(1-2\mu)$$

μ = poisson ratio.

K = Bulk modulus.

i) Modular ratio :

$$\text{Modular ratio } (m) = \frac{\text{Linear strain}}{\text{lateral strain}}$$

ii) Poisson ratio :

$$\text{Poisson ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{1}{m}$$

Factor of Safety:

Units: Large Quantities: Kilo — 10^3
Mega — 10^6
Giga — 10^9
Tera — 10^{12}

Small Quantities: milli —
Micro — 10^{-6}
Nano — 10^{-9}
Pico — 10^{-12}

(8)

1. A circular cross sectional bar of diameter 10 mm having a length of 1m is subjected by an axial pull of 10 KN Determine i) stress, strain & Youngs modulus of the material. The change in length due to the load acting is 0.05 mm.

GIVEN :

$d = 10mm = 10 \times 10^{-3} m$

$l = 1 m$

tensile load → $F = 10 KN \Rightarrow 10 \times 10^3 N$

$dl = 0.05 mm$

$= 0.05 \times 10^{-3} m$

TO FIND :

i) Stress , σ = ?

ii) Strain , e = ?

iii) Young's modulus : = ?

SOLUTION :

Stress $\sigma = \frac{\delta L}{L} = \frac{\text{Load}}{\text{Area}}$

$$A = \frac{\pi}{4} d^2$$

$$= \frac{3.14 \times (10 \times 10^{-3})^2}{4}$$

$$= 7.85 \times 10^{-5} m^2$$

$$\therefore \sigma = \frac{10 \times 10^3}{7.85 \times 10^{-5}} = 127.38 \ N/mm^2$$

ii) Strain : (ε)

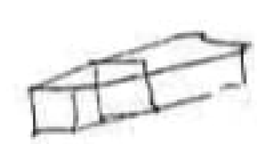
$$\varepsilon = \frac{\delta l}{L} = \frac{0.01 \times 10^{-3}}{1 \times 10^{3}} = 1 \times 10^{-8}$$

iii) Young's modulus : (E)

$$E = \frac{\sigma}{\varepsilon} = \frac{127.38}{1 \times 10^{-5}} \Rightarrow 127.38 \times 10^{5}\ N/mm^2$$

② - A rectangular bar of 10 mm breadth and 12 mm depth is having a length of 1500 mm. It is subjected by a tensile load of 20 kN. If the Young's modulus is 2×10^5 N/mm². Determine the change in length of the bar.

Cross section :



Given :

$A = \cancel{l \times} b \times d$

$= 10 \times 12 = 120\ mm$

$= \cancel{120 \times}$

$L = 1500\ mm$

Load $= 20\ kN$

$= 20 \times 10^3\ N$

Young's $E = 2 \times 10^5\ N/mm^2$

$\delta L = ?$

Solution :

Stress $\sigma = \frac{Load}{Area} = \cancel{150}\ \frac{20 \times 10^3}{120} = 166.66\ N/mm$

Young's $E = \frac{\sigma}{\varepsilon}$

$$2\times10^5 = \frac{166.66}{\varepsilon}$$

$$\varepsilon = 8.33\times10^{-4}\ N/mm^2$$

Strain : $\varepsilon = \frac{\delta L}{L}$

$$\delta L = \varepsilon . L$$

$$\delta L = 8.33\times10^{-4} \times 1500$$

$$\delta L = 1.25\ mm$$

(3) Determine the diameter of a circular bar if the bar is having a length of 1m subjected to a tensile load of 10 kN. The Young's modulus of the material is 2×10^5 N/mm². and the elongation of the material is 0.001 mm.

GIVEN :

$d = ?$

$l = 1\times10^3\ mm$

load $= 10\ kN \Rightarrow 10\times10^3\ N$

$E = 2\times10^5\ N/mm^2$

$\delta L = 0.001\ mm.$

Stress $\sigma = \dfrac{\text{load}}{\text{area}}$

$$\sigma = \frac{10 \times 10^3}{\pi \times r^2 \times 1 \times 10^3}$$

2 $\pi r^2 L$

Strain $\varepsilon = \dfrac{\delta L}{L}$

$$\varepsilon = \frac{0.001}{1 \times 10^3} = 1 \times 10^{-6}$$

Young's $E = \dfrac{\sigma}{\varepsilon}$

$$2 \times 10^5 = \frac{\sigma}{1 \times 10^{-6}}$$

$$\sigma = 0.2$$

$\dfrac{\pi d^2}{4}$

$$\therefore \sigma = \frac{10 \times \cancel{10^3}}{\pi \times r^2 \times 1 \times \cancel{10^3}}$$

$$\frac{0.2 \times 3.14}{10} = \frac{1}{r^2}$$

$$r^2 = \frac{10}{0.628} \Rightarrow 15.92$$

$$r = 3.99$$

$$= 7.98$$

25 2.37

22/6/14 (12)

(Not Required)

Factor of Saftey : (FOS)

It is defined as the ratio of yield stress to the safe working stress.

$$FOS = \frac{\text{Yield stress}}{\text{safe working stress}}$$

Ultimate

4) A bar 30 mm diameter is subjected to a pull of 60 kN the measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm. Calculate. i) Young's modulus, ii) Poison's ratio, iii) Bulk modulus.

GIVEN :

$D = 30mm$

$F = 60 kN = 60\times10^3 N$

$L = 200mm$

$dl = 0.1mm$

$\partial D = 0.004mm.$

Solution :

Youngs $E = \frac{\sigma}{\varepsilon}$

$\sigma = \frac{Load}{Area} = \cancel{\frac{60\times10^3}{\pi\times15^2\times200}} \quad \frac{60\times10^3}{\frac{\pi}{4}(30)^2}$

$= \frac{60\times10^3}{\frac{\pi}{4}(30)^2} = \cancel{0.434}$

$= \cancel{84.882\ N/mm^2}$

$= 84.882\ N/mm^2.$

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$$\text{Strain}: \varepsilon = \frac{\delta L}{L} = \frac{0.1}{200}$$

$$= 5 \times 10^{-4} \Rightarrow (\text{Linear strain})$$

$$\text{Young's } E = \frac{\sigma}{\varepsilon} = \frac{84.882}{5 \times 10^{-4}} = 1.697 \times 10^{5} \text{ N/mm}^2$$

ii) ~~Young's~~ Poisson

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} \quad \begin{matrix} d & \frac{\delta d}{d} \\ L & \frac{\delta L}{L} \end{matrix}$$

$$\varepsilon_{lat} = \frac{\delta d}{d} = \frac{0.004}{30} = 1.33 \times 10^{-4}$$

m

$$\mu = \frac{1.33 \times 10^{-4}}{5 \times 10^{-4}}$$

$$= 0.26$$

iii) Bulk modulus:

$$K = \frac{\text{direct stress}}{\text{volumetric strain}} = \frac{\sigma}{\left(\frac{\delta V}{V}\right)}$$

(or)

Eq:

$$E = 3K(1-2\mu)$$

$$1.697 \times 10^{5} = 3K(1-2(0.26))$$

$$1.697 \times 10^{5} = 3K(0.48)$$

$$1.697 \times 10^{5} = 1.44K$$

$$K = 1.22 \times 10^{5} \text{ N/mm}^2$$

② A circular bar of 20mm diameter is subjected by an axial tensile load of 10 kN, the length of the bar is 900mm. Due to the application of the load, the length increases by 0.0125 mm and the dia ... Modular ratio, Poisson ratio, strain strain

5). The Following data refer to a mild steel specimen in a Laboratry.

i) diameter of specimen = 25 mm

ii) Length of specimen = 300 mm

iii) Extension of bar under a load of 15 kN = 0.045 mm

iv) Load at yield point = 127.65 kN

v). Max. Load = 208.6 kN

vi) Length of the specimen after Failure = 375 mm

vii) Neck diameter = 17.75 mm

Find out young's modulus, yield pt. stress, ultimate stress, % of elongation, percentage reduction in area, safe stress adapting a factor of saftey of 2.

GIVEN :

$d = 25\,mm$

$L = 300\,mm$

$\delta l = 0.045\,mm$

$L = 15\,kN$

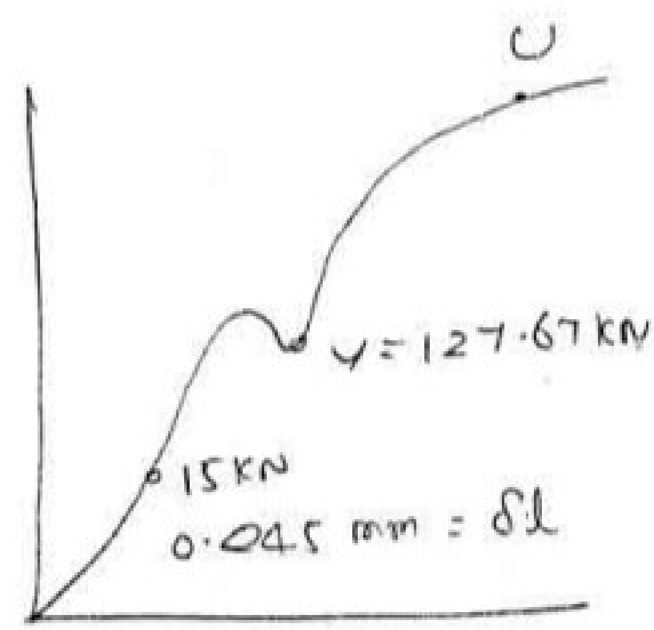
$L_y = 127.65\,kN$

$L_{max} = 208.6\,kN$

$L_{Fail} = 375\,mm$

$d_{neck} = 17.75$

$FOS = 2$



Solution :

i) Youngs. $E = \dfrac{\text{Stress}}{\text{Cores Strain}}$

$\sigma = \dfrac{\text{Load}}{A} = \dfrac{15 \times 10^3}{A} = \dfrac{15 \times 10^3}{\pi \times 25^2} = 30.55\ N/mm$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.045}{300} = 1.5 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{30.55}{1.5 \times 10^{-4}} = 2.03 \times 10^{5}\ N/mm^2$$

ii) Yield stress :- (σ_y)

$$\sigma_y = \frac{\text{Yield pt load}}{\text{Area}} = \frac{127.65 \times 10^3}{\frac{\pi}{4} \times 25^2} = 260.17\ N/mm^2$$

iii) Ultimate stress (σ_u)

$$\sigma_u = \frac{\text{max (or) ultimate load}}{\text{Area}}$$

$$= 425.17\ N/mm^2$$

iv) % of elongation :

$$= \frac{\text{Final length} - \text{original length}}{\text{Original length}} \times 100$$

$$= \frac{375 - 300}{300} \times 100$$

$$= \frac{75}{300} \times 100$$

$$= 25.\%$$

v). Reduction in area :

$$= \frac{\text{Original area} - \text{reduced area}}{\text{original area}} \times 100$$

$$= 247.32 \times 100$$

vi). Safe working stress :

$$F.O.S = \frac{\text{Yield stress}}{\text{Safe stress}}$$

$$2 = \frac{260.17}{S_s}$$

Safe stress = ~~130.085~~

$= 130.085\ N/mm^2$

Young's modulus of different materials :

$$E = \frac{\sigma}{\varepsilon}\ (N/mm^2)$$

1) Steel = 2.1×10^5

2) cast iron = 1.3×10^5

3) wrought iron = 1.9×10^5

4) Brass = 0.8×10^5

5) Timber = 0.1×10^5

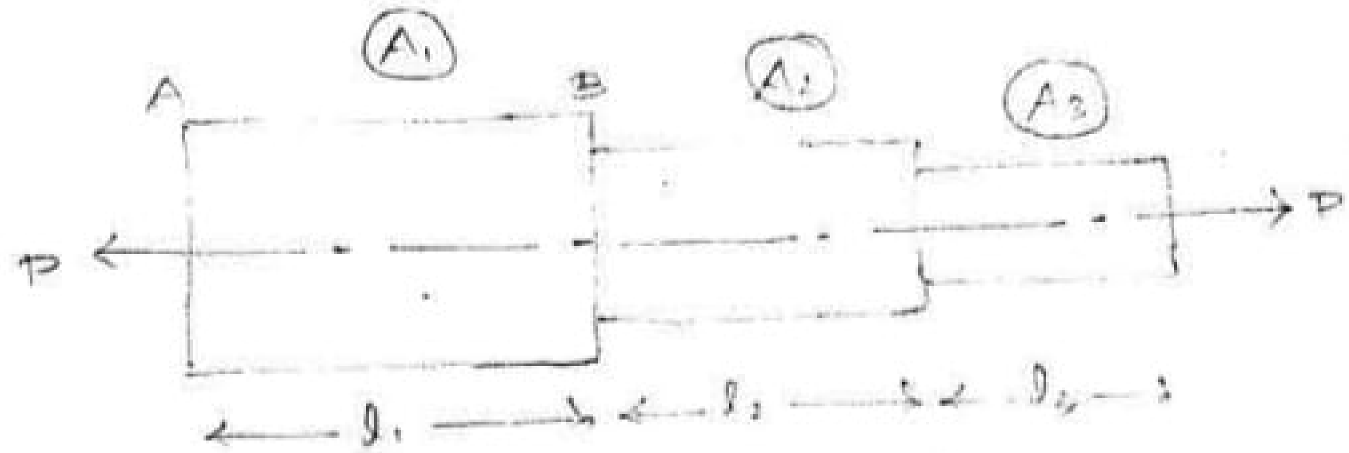
6) Copper = 1×10^5

7) Aluminium = 0.7×10^5

UNITS :

Kilo	$- 10^3$	milli	$= 10^{-3}$
mega	$- 10^6$	micro	$= 10^{-6}$
Giga	$- 10^9$	nano	$= 10^{-9}$
Terra	$- 10^{12}$	Pico	$= 10^{-12}$

Bars of varying section :



We know that,

$$\text{Young's } E = \frac{\sigma}{\varepsilon}$$

$$E = \frac{P/A}{\delta l / L}$$

$$E = \frac{P}{A} \times \frac{L}{\delta l}$$

$$\delta l_1 = \frac{P \times L_1}{A_1 E_1}$$

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{P \times L_1}{A_1 E_1} + \frac{P \times L_2}{A_2 E_2} + \frac{P \times L_3}{A_3 E_3}$$

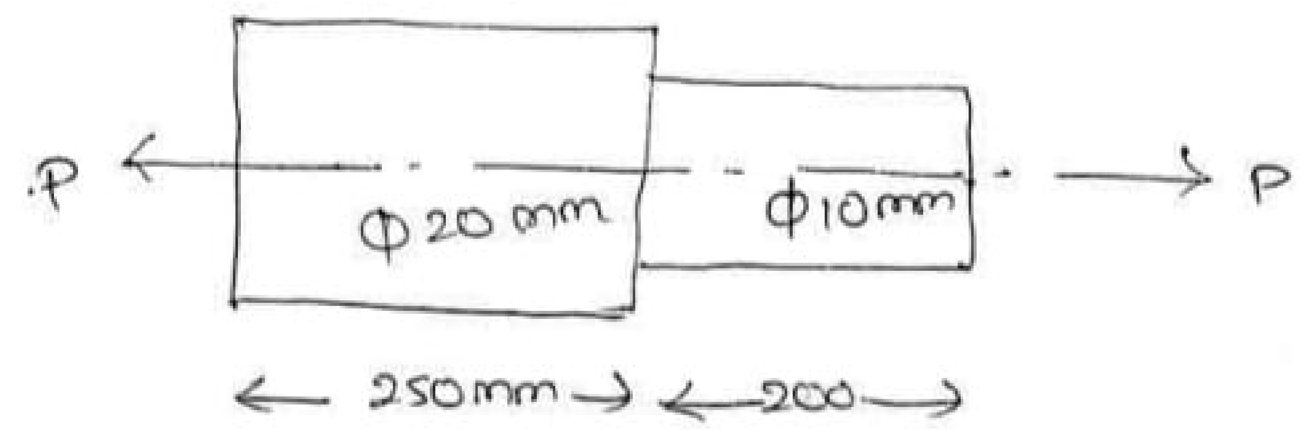
for same material,

$(E_1 = E_2 = E_3 = E)$

NOTE :
If the material is subjected to compressive load use -ve sign for reduction in length (or) in the final answer write reduction in length.

1. A straight bar 450 mm long is 20 mm in diameter for the first 250 mm length and 10 mm diameter for the remaining length. If the bar is subjected to an axial pull of 10kN. Find the extension of the bar. Take $E = 2\times10^5$ N/mm².

Given:



$d_1 = 20$ mm $\qquad\qquad P = 10\times10^3$ N

$d_2 = 10$ mm

$L_1 = 250$ mm

$L_2 = 200$ mm

$E = 2\times10^5$ N/mm²

$$\delta l = \frac{P\times L_1}{A_1 E_1} + \frac{P\times L_2}{A_2 E_2}$$

$$= \frac{10\times10^3\times250}{\frac{\pi}{4}\times20^2\times2\times10^5} + \frac{10\times10^3\times200}{\frac{\pi}{4}\times10^2\times2\times10^5}$$

$$= \frac{25\times10^5}{\frac{\pi}{4}\times400\times2\times10^5} + \frac{20\times10^5}{\frac{\pi}{4}\times10^2\times2\times10^5}$$

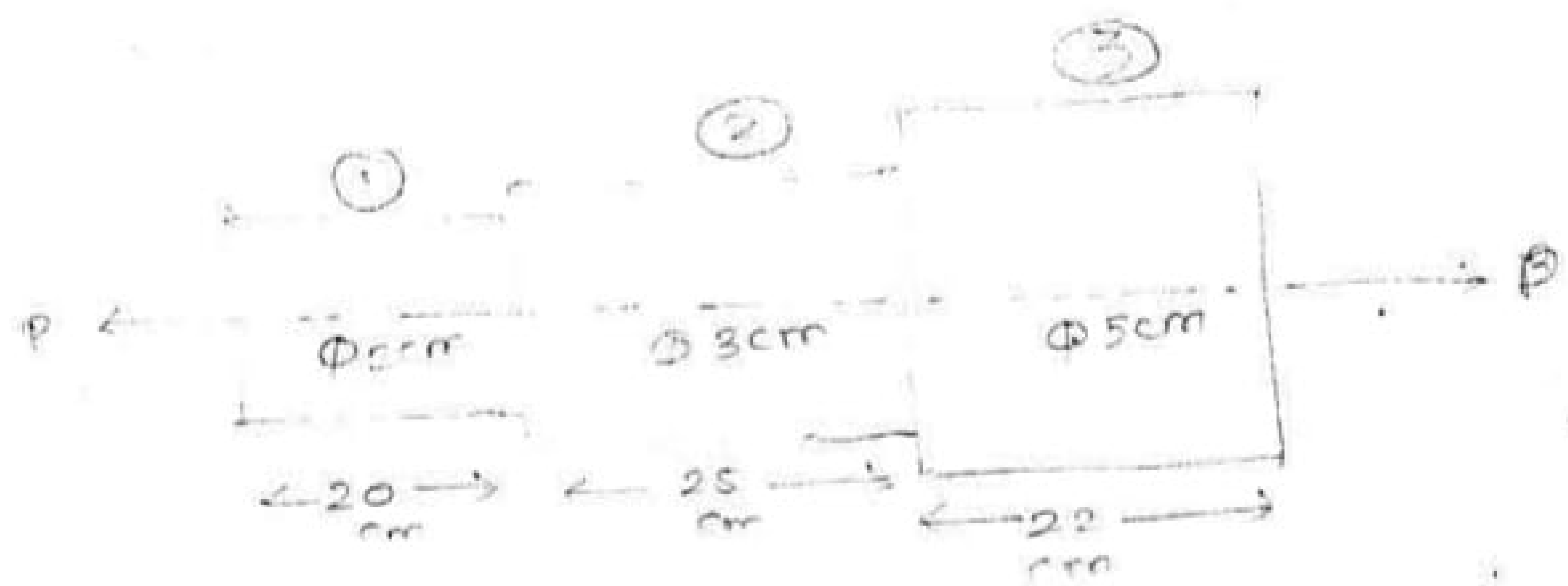
$$= 0.039 + 0.127$$

$\delta l = 0.166$ mm

② An Axial pull of 35 KN is acting on a bar consisting of three sections 20 cm length with 2 cm diameter, 25 cm length with 3 cm dia and 22 cm length with 5 cm dia. If the Young's modulus $= 2.1 \times 10^5 \ N/mm^2$ Determine.

i) Stress in each section

ii) Total extension of the bar.



GIVEN :

$P = 35 \ KN$

$= 35 \times 10^3 N$

$d_1 = 2 \ cm = 2 \times 10 \ mm \qquad l_1 = 20 cm = 200 mm$

$d_2 = 3 \ cm = 30 \ mm, \qquad l_2 = 25 \ cm = 250 \ mm$

$d_3 = 5 \ cm = 50 \ mm, \qquad l_3 = 22 \ cm = 220 \ mm$

$E = 2.1 \times 10^5 \ N/mm^2$

ii) $$\delta l = \frac{P \times L_1}{A_1 E_1} + \frac{P L_2}{A_2 E_2} + \frac{P L_3}{A_3 E_3}$$

$$= \frac{35 \times 10^3 \times 200}{\frac{\pi}{4} \times 20^2 \times 2.1 \times 10^5} + \frac{35 \times 10^3 \times \cancel{300}\ 250}{\frac{\pi}{4}\ 30^2 \times 2.1 \times 10^5} +$$

$$\frac{35 \times 10^3 \times 220}{\frac{\pi}{4} \times 50^2 \times 2.1 \times 10^5}$$

$\delta l = 0.1061 + 0.0589 + 0.0186$

$\delta l = 0.1836$ mm.

i) Stress in section :

Section ① :

$$\sigma_1 = \frac{\text{Load}}{\text{Area}} = \frac{35 \times 10^3}{\frac{\pi}{4} \cdot 20^2} = 111.46 \text{ N/mm}^2$$

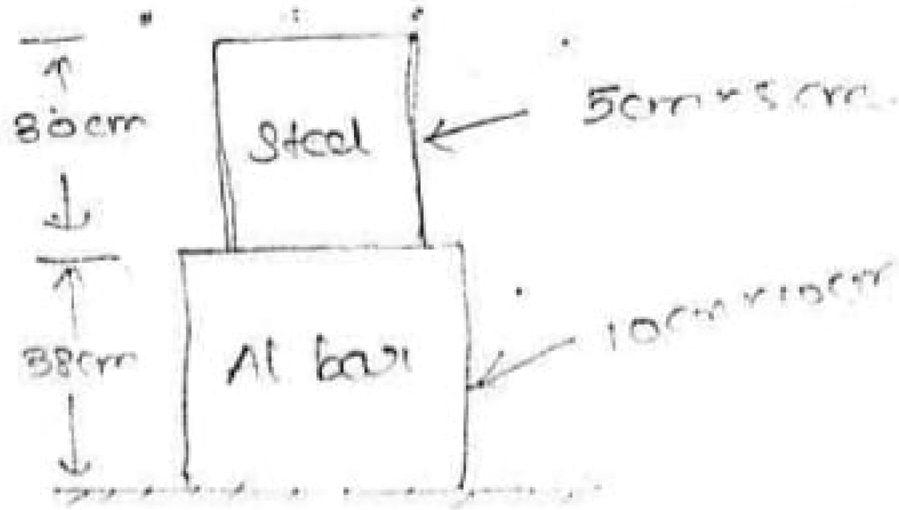
Section ② :

$$\sigma_2 = \frac{\text{load}}{\text{area}} = \frac{35 \times 10^3}{\frac{\pi}{4} \cdot 30^2} = 49.53 \text{ N/mm}^2$$

Section ③ :

$$\sigma_3 = \frac{\text{load}}{\text{area}} = \frac{35 \times 10^3}{\frac{\pi}{4} \cdot 50^2} = 17.83 \text{ N/mm}^2$$

③ A member formed by connecting a steel bar to an aluminium bar as shown in the figure. Assuming that the bars are prevented from buckling. side base ways calculate the magnitude of Force P that will cause total length of the member to decrease by 0.25 mm. The length values of elastic modulus of steel and alumnium are 2.1×10^5 N/mm² and 7×10^4 N/mm².



$\delta l_s = \delta l_a = 0.25$

GIVEN :

$$\delta l = \frac{P L_{Al}}{A_{Al} E_{Al}} + \frac{P L_{st}}{A_{st} E_{st}}$$

$$0.25 = \frac{P \times 38 \times 10}{(10 \times 10) \times 2.1 \times 10^5} + \frac{P \times 300}{(5 \times 5) \times 7 \times 10^4}$$

$$0.25 = P(1.809 \times 10^{-5}) + P(1.7142 \times 10^{-5})$$

$$0.25 = P(3.523 \times 10^{-5})$$

$$0.25 - \delta l_s = P(3.523 \times 10^{-5}) \quad \text{—} ①$$

$$0.25 - \delta l_A = P(1.714 \times 10^{-5}) \quad \text{—} ②$$

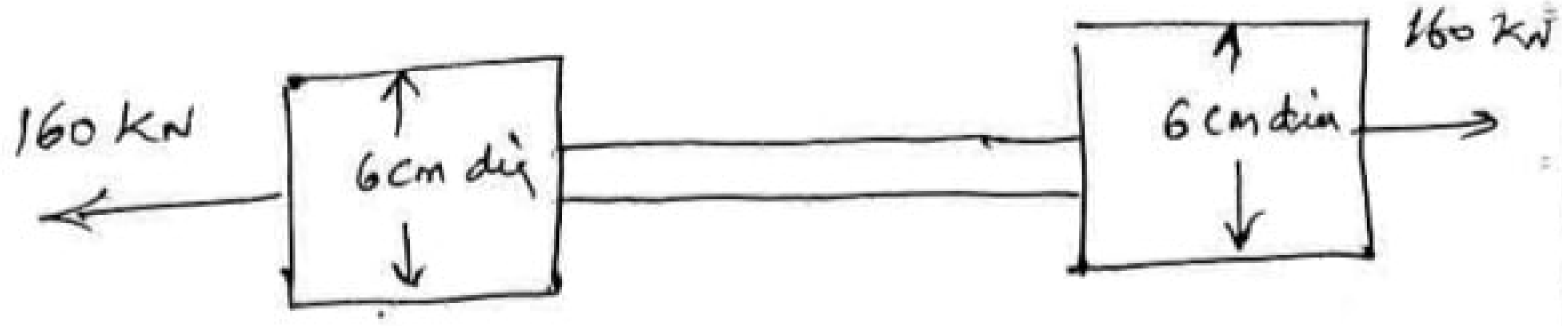
$$P = 2.243 \times 10^5 \text{ N}$$

$$\boxed{P = 224.37 \text{ KN}}$$

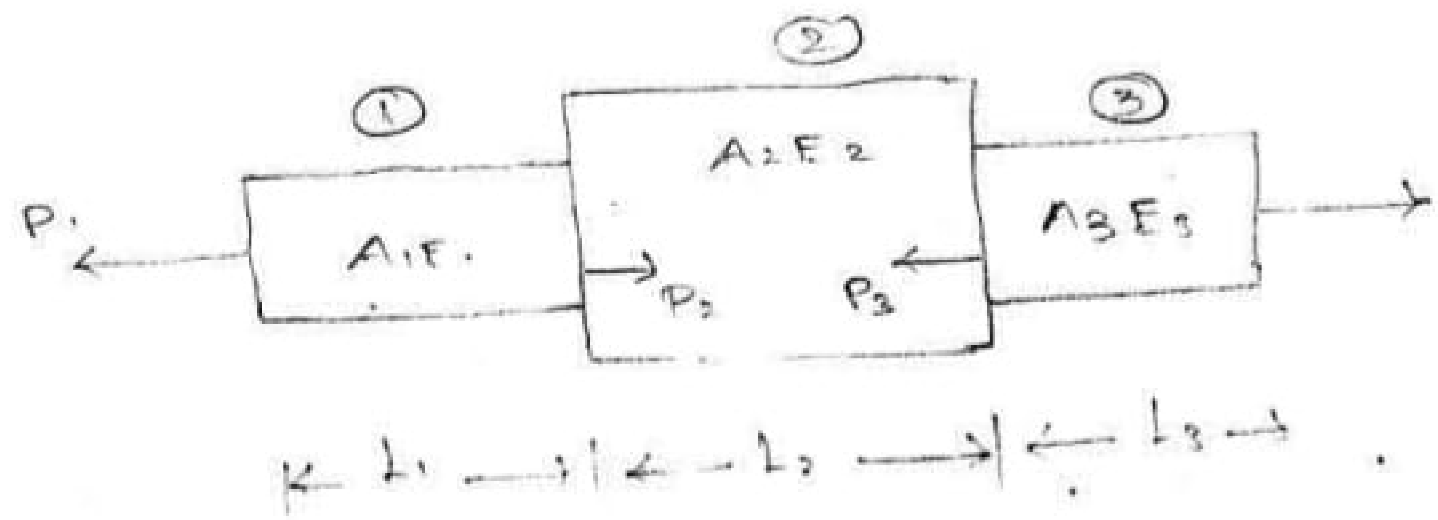
(22)

(4) A bar shown in Fig subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm², Det the dia of middle portion; find also the length of middle portion if the total elongation of the bar is to be 0.2 mm. Young modulus 2.1×10^5 N/mm².

$D_2 = 36.85$ mm
$L_2 = 20.714$ cm.

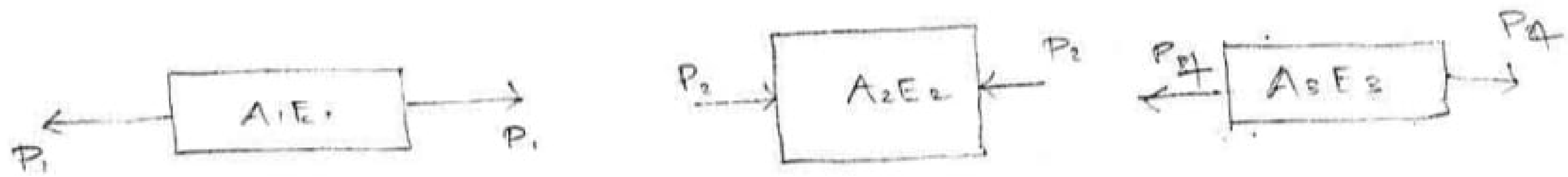


PRINCIPLE OF SUPER POSITION :



When number of loads are acting on a body the resulting strain according to the principle of superposition will be algebric sum of strains caused by individual loads.

The free body diagram for individual section is drawn and the deformation of each section is obtained



$$E = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\sigma}{E}$$

$$\delta l = \frac{P \times L}{AE}$$

Deformation, $\delta l = \delta l_1 + \delta l_2 + \delta l_4$

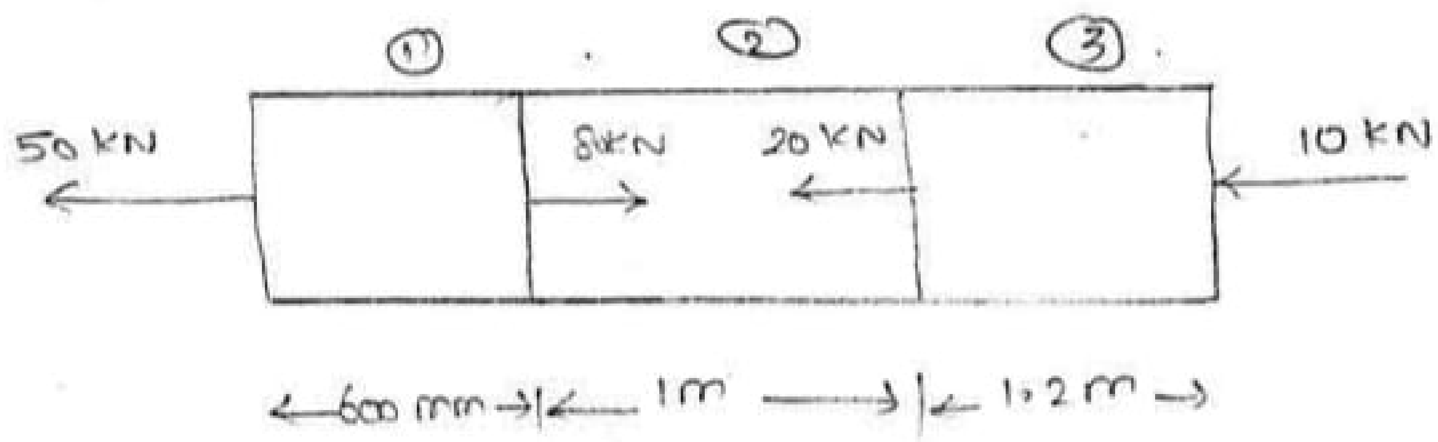
$$= \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_4}{A_3 E_4}$$

NOTE :

1. The negative sign for compression and positive sign for tension has to be used for deformation.

2. Nature and magnitude of force at two ends should be same.
nature (Tensile (or) compression). &
Magnitude (Force or load values).

1. A brass bar having cross sectional area of 1000 mm^2 is subjected to axial forces as shown in the figure. Find the total elongation of the bar. Take $E = 1.05 \times 10^5$ N/mm^2.



Given :

$L_1 = 600$ mm

$L_2 = 1 \times 10^{+3}$ mm

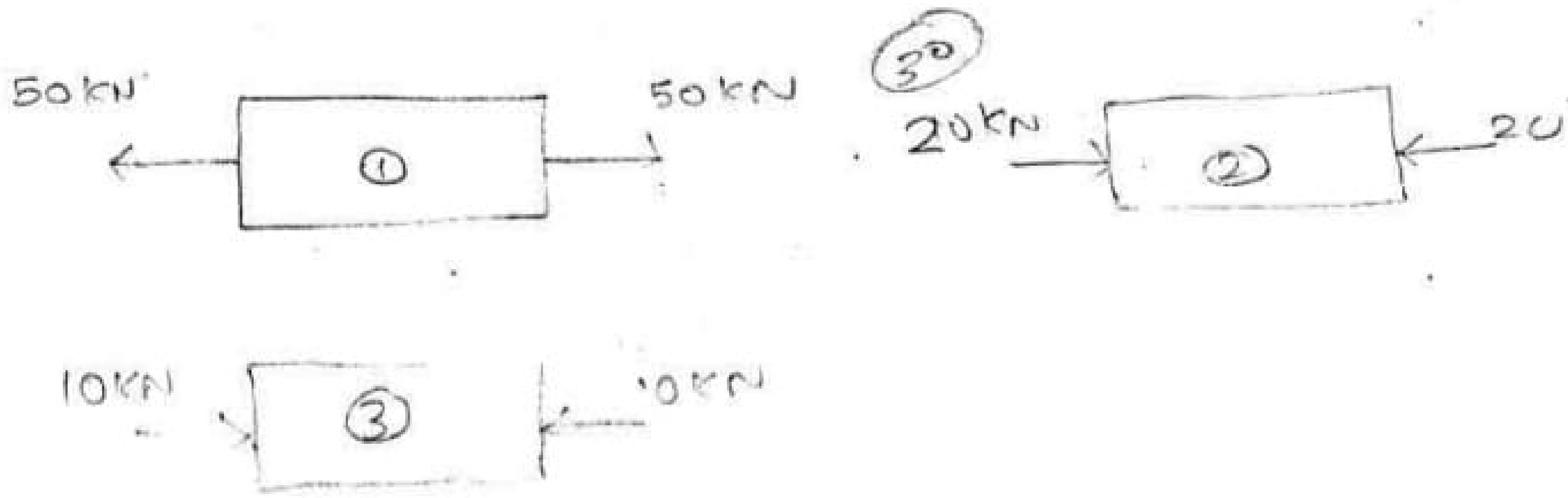
$L_3 = 1.2 \times 10^3$ mm.

$A = 1000$ mm^2

$\therefore A_1 = A_2 = A_3$ (Uniform bar)

Solution :

Free body diag :



$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l_1 = \frac{P_1 L_1}{AE} , \quad \delta l_2 = \frac{P_2 L_2}{AE} , \quad \delta l_3 = \frac{P_3 L_3}{AE}$$

$$\delta l = \frac{P_1 L_1}{AE} - \frac{P_2 L_2}{AE} - \frac{P_3 L_3}{AE}$$

$$\delta l = \frac{50 \times 600 \times 1000}{1000 \times 1.05 \times 10^5} - \frac{20 \times 1000 \times 10^3}{1000 \times 1.05 \times 10^5} - \frac{10 \times 1000 \times 1.2 \times 10^3}{1000 \times 1.05 \times 10^5}$$

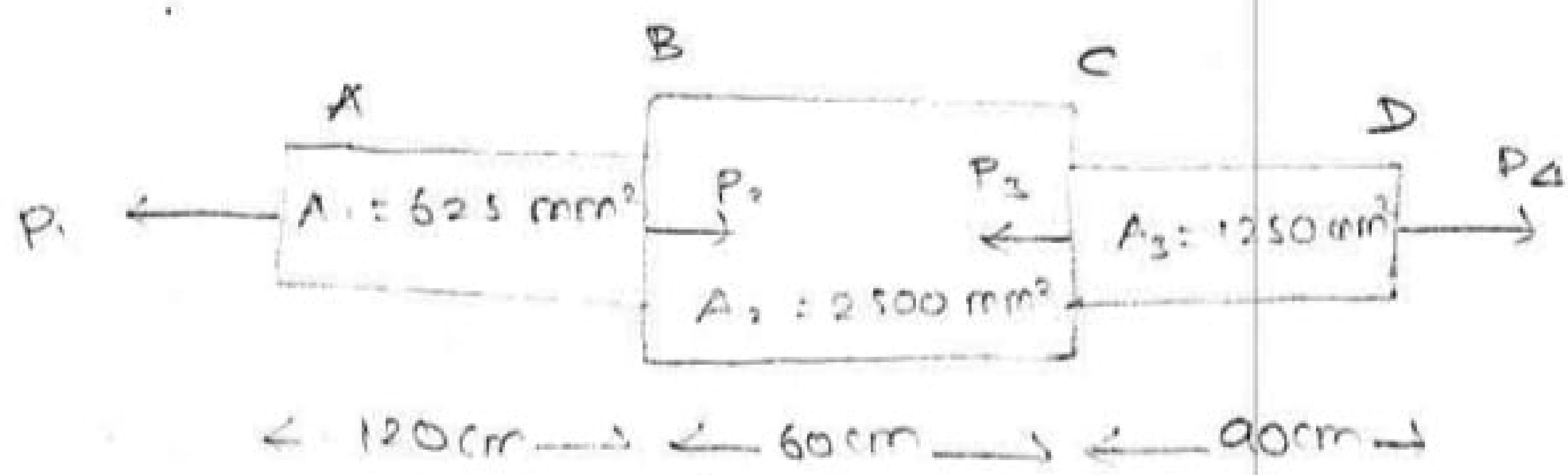
$$\delta l = 0.2857 - 0.1904 - 0.1142$$

$$\delta l = -0.0189 \text{ mm}$$

The −ve sign indicates the compression.

(26)

②. A member ABCD. is subjected to point loads P_1, P_2, P_3 & P_4. as shown in fig. Calc. the forces P_2 necessary for equilibrium. If $P_1 = 45$ KN, $P_3 = 450$ kN, $P_4 = 130$ kN. Determine the total elongation of the member. assuming $E = 2.1 \times 10^5$ N/mm²



GIVEN :

$A_1 = 625$ mm²　　　　$P_1 = 45 \times 10^3$ N

$A_2 = 250$ mm²　　　　$P_3 = 450 \times 10^3$ N

$A_3 = 1250$ mm²　　　　$P_4 = 130 \times 10^3$ N

$L_1 = 120 \times 10$ mm　　　　$P_2 = ?$

$L_2 = 600$ mm

$L_3 = 900$ mm

Free body diagram :

W.K.T, Force acting towards left = Force acting towards right.

$$\therefore \quad P_1 + P_3 = P_2 + P_4$$

$$45 + 450 = P_2 + 130$$

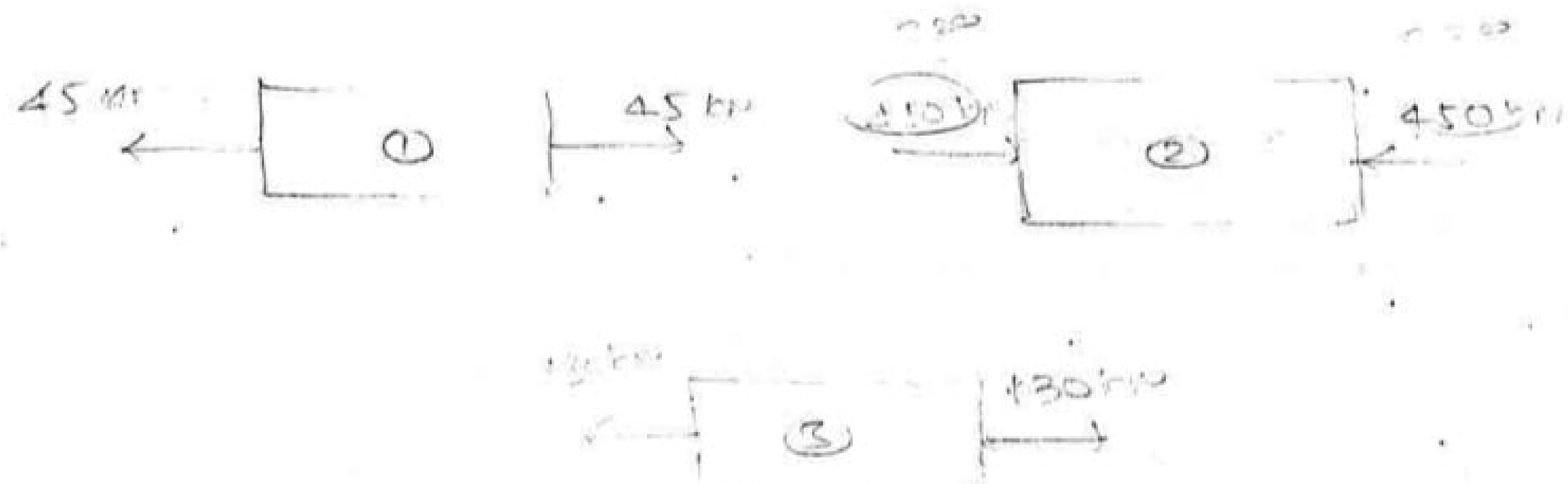
$$495 = P_2 + 130$$

$$P_2 = 495 - 130$$

$$P_2 = 365 \text{ kN}$$

Total elon

Free body diag :



Total elongation :

$$\delta l = \frac{P_1 L_1}{A_1 E} - \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

$$\delta l = \frac{45 \times 10^3 \times 1200}{625 \times 2.1 \times 10^5} - \frac{365 \times 10^3 \times 600}{250 \times 2.1 \times 10^5} + \frac{450 \times 10^3 \times 900}{1250 \times 1.2 \times 10^5}$$

$$\delta l = 0.4114 - 4.1714 + 2.7$$

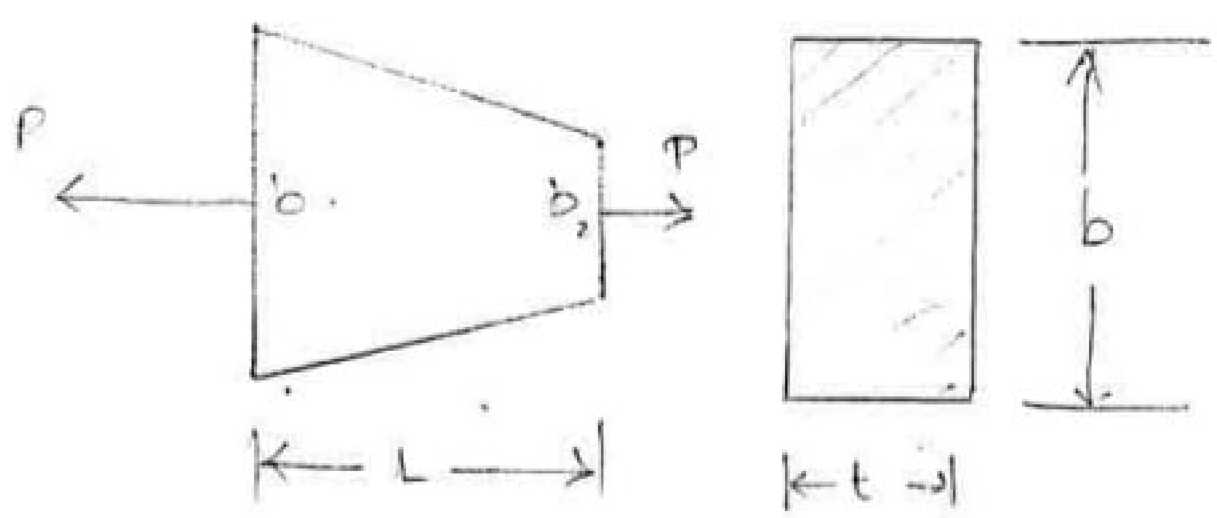
$$\delta l = -1.06$$

-3.76

The - sign indicates compression.

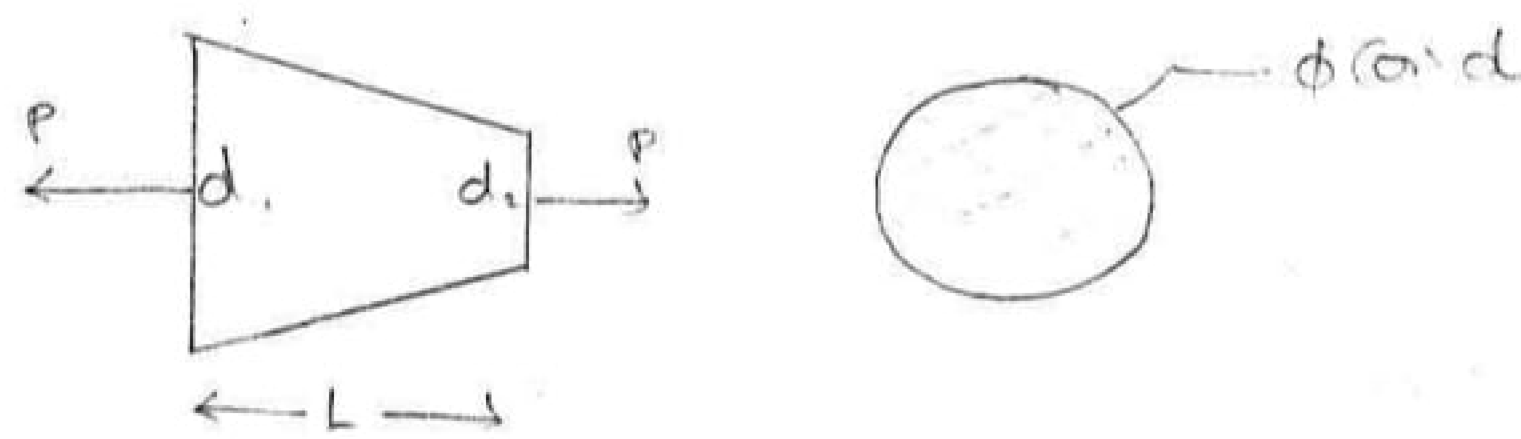
(3). Analysis of uniformly tapering bar :

(1). Rectangular bar :



$$\delta l = \frac{PL}{tE\,(b_1 - b_2)} \times \log\left(\frac{b_1}{b_2}\right)$$

(2). circular bar :



$$\delta l = \frac{4PL}{\pi E d_1 d_2} \text{ (mm)}$$

d_1 = Larger dia.

d_2 = Smaller dia.

(1). A rectangular steel bar of length 400 mm and thickness 10 mm having $E = 2\times10^5$ N/mm. The extension due to load is 0.21 mm. The bar tapers uniformly in width from 100 mm to 50 mm. Determine axial load on the bar.

GIVEN :

$l = 400$ mm

$t = 10$ mm

$E = 2\times10^5$ N/mm^2

$\delta L = 0.2$ mm

$b_1 = 100$ mm

$b_2 = 50$ mm.

$P = ?$

$$\delta L = \frac{PL}{tE(b_1-b_2)} \times \log\left(\frac{100}{50}\right)$$

$$0.21 = \frac{P \times 400}{10\times2\times10^5(50)} \times \log(2)$$

$$50000 = P \times \log 2$$

$$P = 174401.225$$

$$P = 1.7440 \times 10^5 \text{ N}$$

(2). Find the modulus of Elasticity of a rod which tapers uniformly from 30 mm to 15 mm diameter in a length of 350 mm. The rod is subjected to an axial load

load of 5.5 kN and extension of the rod is 0.025 mm.

GIVEN:

$E = ?$

$d_1 = 30$ mm

$d_2 = 15$ mm

$L = 350$ mm

$P = 5.5$ kN

$= 5.5 \times 10^3$ N

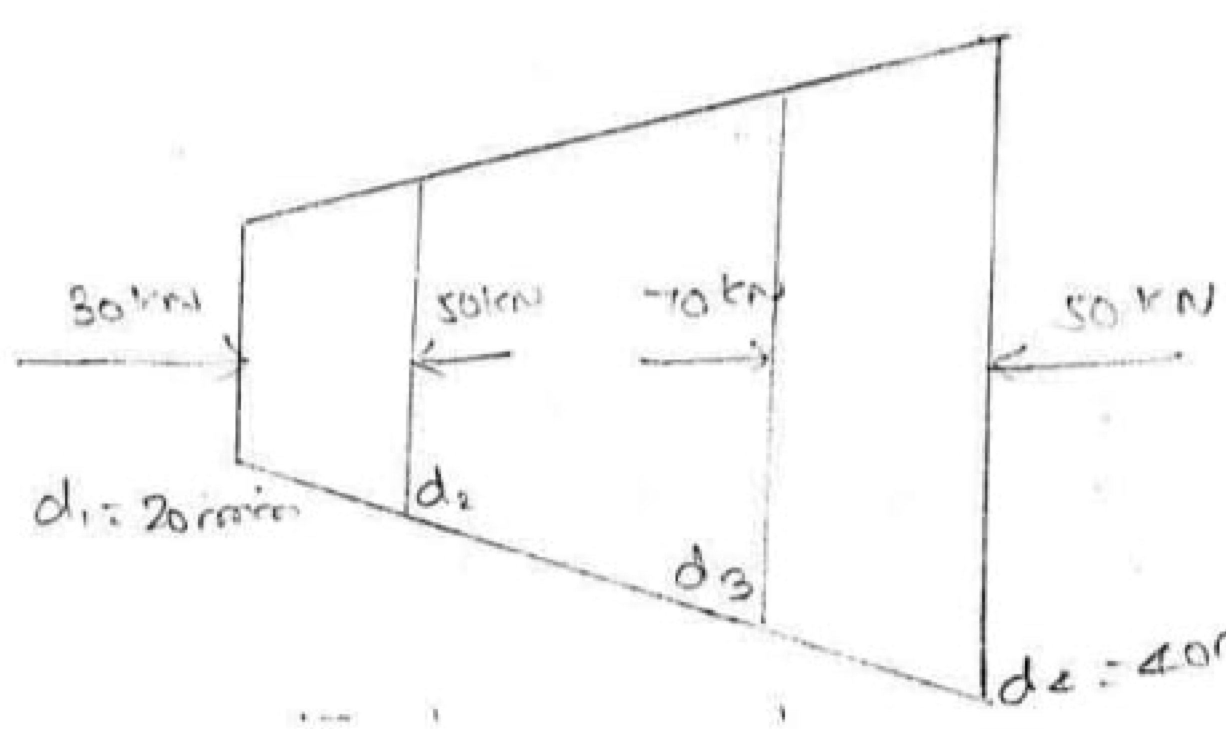
$\delta l = 0.025$ mm.

$$\delta l = \frac{4PL}{\pi E d_1 d_2}$$

$$0.025 = \frac{4 \times 5.5 \times 10^3 \times 350}{3.14 \times E \times 30 \times 15}$$

$$E = 2.17975 \times 10^5 \ N/mm^2$$

(3). Determine the diameter d_2 and d_3 and the total deformation for the given diagram.



$E = 2 \times 10^5 \ N/mm^2$

GIVEN :

$d_1 = 20\,mm$

$d_4 = 40\,mm$

$L_1 = 1m = 1\times10^3\,mm$

$L_2 = 2m = 2\times10^3\,mm$

$L_3 = 1m = 1\times10^3\,mm$

$E = 2\times10^5\,N/mm^2$

Sol :

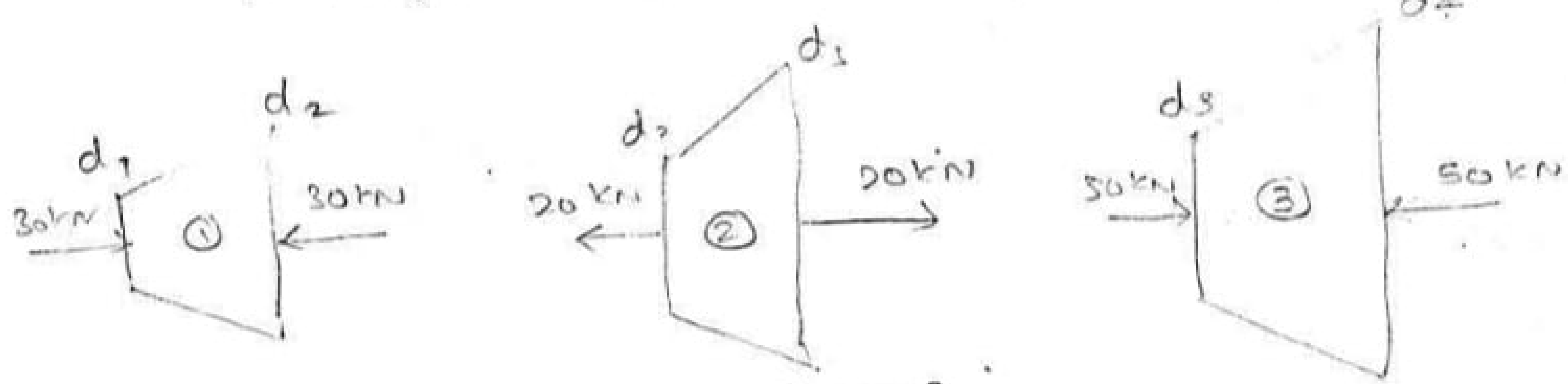
i) To find d_2 & d_3 :

Since it is uniformly tapering circular section for every one meter of length there is an increase of 5 mm diameter. So

$d_2 = 25\,mm$

$d_3 = 35\,mm$

ii) Deformation in length (δl):



$$\delta l_1 = \frac{4\times30\times1\times10^3\times10^3}{3.14\times2\times10^5\times20\times25}$$

$= 0.38\,mm$ (compression -ve).

$$\delta l_2 = \frac{4 \times 20 \times 2\times10^3 \times 10^3}{3.14 \times 2\times10^5 \times 25 \times 35}$$

$= 0.29$ (mm) (extension $= +ve$)

$$\delta l_3 = \frac{4 \times 50 \times 10^3 \times 1\times10^3}{3.14 \times 2\times10^5 \times 35 \times 40}$$

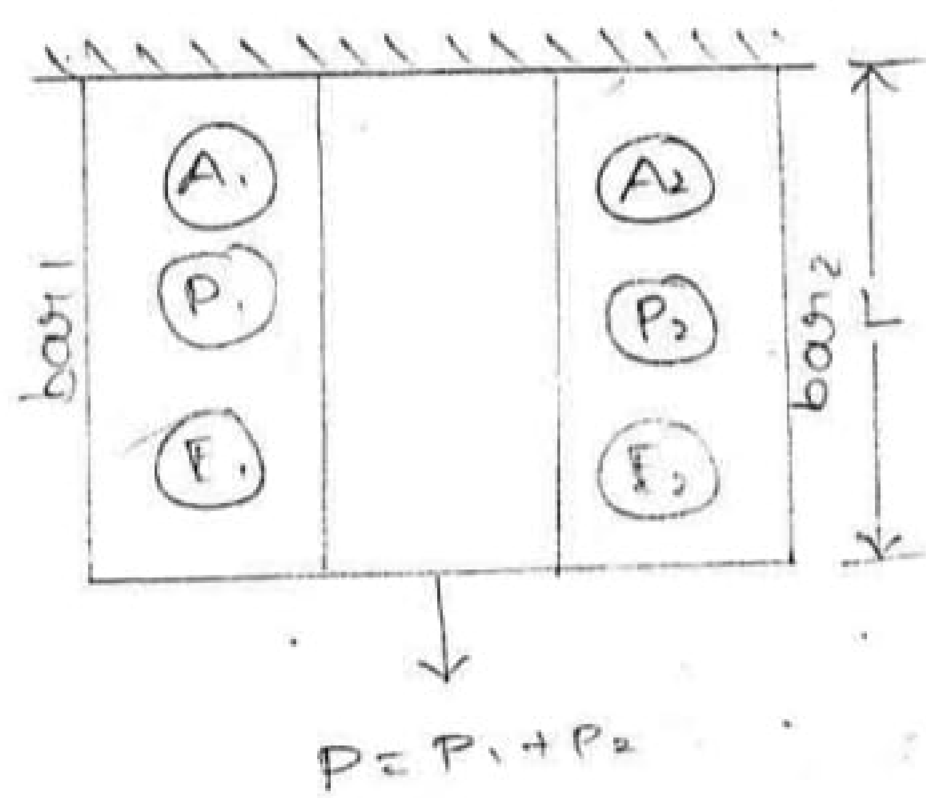
$= 0.22$ mm (compression $= (-ve)$)

$\delta l = \delta l_1 + \delta l_2 + \delta l_3$

$= -0.31$

$= 0.31$ (compression.

COMPOUND (OR) COMPOSITE BARS:



Bar made of two different materials rigidly fixed as one unit is called compound (or) composite bar

Condition :

1). Elongation (or) contraction is always equal. i.e). strain in each bar is constant.

2). Total load = Sum of loads acting at each material.

$$P = P_1 + P_2$$

Formulas :

① $P = P_1 + P_2$

$P = \sigma_1 A_1 + \sigma_2 A_2$

② $\delta l_1 = \delta l_2$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

③. modular ratio :

$$E = \frac{\sigma}{\varepsilon}$$

$$\boxed{\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2}}$$

Formula :

(i) Strain Constant

$$e_1 = e_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$e_1 = \frac{\sigma_1}{E_1} \qquad e_2 = \frac{\sigma_2}{E_2}$$

$$\frac{\delta l_1}{L_1} = \frac{\sigma_1}{E_1} \qquad \delta l_2 = \frac{\sigma_2 L_2}{E_2}$$

$$\delta l_1 = \frac{\sigma_1}{E_1} \times L_1$$

$$\therefore \delta l_1 = \delta l_2$$

$$\frac{\sigma_1 L_1}{E_1} = \frac{\sigma_2 L_2}{E_2}$$

(ii) Load Constant $\left[\because P = \sigma A\right]$

$P = P_1 + P_2$

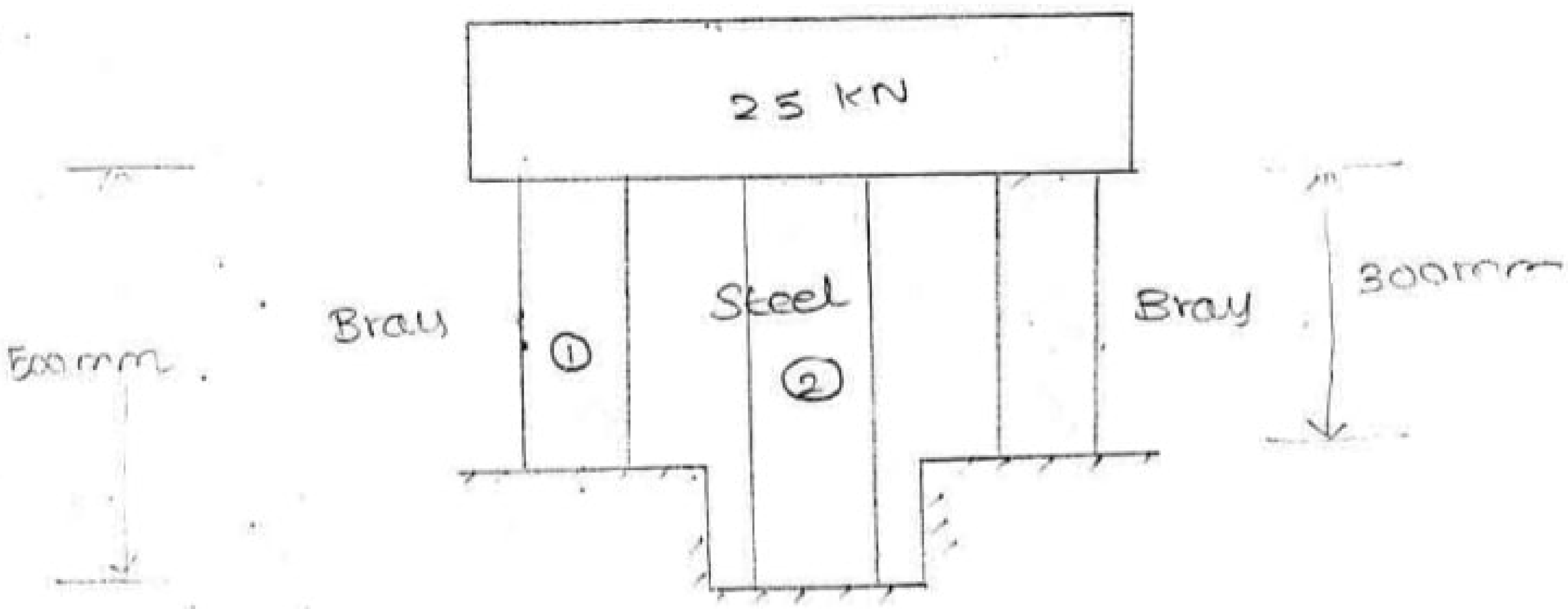
$= \sigma_1 A_1 + \sigma_2 A_2$

3A

Problems :

① Determine the magnitude of stress developed in a composite bar

$E_s = 2\times10^5\ N/mm^2$, $E_b = 1\times10^5 N/mm^2$. Area of Steel & brass are 800 mm^2 and 500 mm^2



GIVEN :

$E_s = 2\times10^5\ N/mm^2$

$E_b = 1\times10^5\ N/mm^2$

$A_s = 800\ mm^2$

$A_b = 500\ mm^2$

$P = 25\ KN$

$= 25\times10^3\ N$

To find : σ_s, σ_B

Solution :

1) Strain constant :

$$\delta l_s = \delta l_B$$

$\frac{PL}{AE}$

$$\frac{\sigma_s L_s}{E_s} = \frac{\sigma_b L_b}{E_b}$$

$$\sigma_s = \frac{F}{A} = \frac{25\times10^3}{800} = 31.25$$

$$\sigma_b = \frac{F}{A} = \frac{25\times10^3}{500} = 50$$

$$\frac{31.25 \times 500}{}$$

$$\frac{\sigma_s \times 500}{2\times10^5} = \frac{\sigma_b \times 300}{1\times10^5}$$

$$\boxed{\sigma_s = 1.2\,\sigma_b}$$

ii) Total load = sum of load.

$$P = P_1 + P_2$$

$$P = \sigma_s A_s + \sigma_b A_b$$

$$P = \sigma_s \times 800 + \sigma_b (2\times500)$$

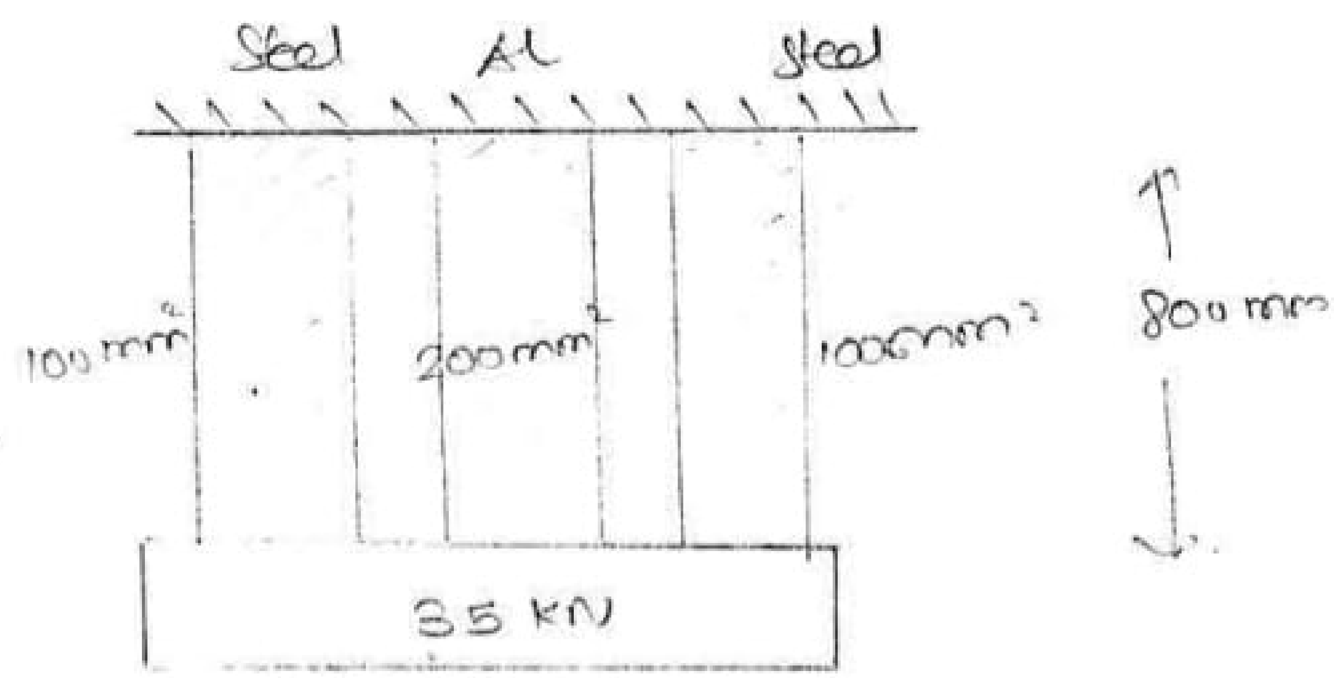
$$25\times10^3 = (1.2(\sigma_b)\times800) + (\sigma_b \times 1000)$$

$$\sigma_b = 12.755 \text{ N/mm}^2$$

$$\sigma_s = 15.30 \text{ N/mm}^2$$

(2) Determine the total elongation of a composite bar loaded as shown in the figure. Youngs modulus of aluminium & steel are 80 GPA and 200 GPA.

(26)



$P = 35 \times 10^3 N$ $A_s =$

$E_s : 200 \times 10^9$

$E_{al} : 80 \times 10^9$

Strain constant

$$\delta L_s = \delta L_{al}$$

$$\frac{\sigma_s L_1}{E_s} = \frac{\sigma_{al} L_2}{E_{al}}$$

$$\frac{\sigma_s \times 800}{20 \times 10^{10}} = \frac{\sigma_{al} \times 800}{8 \times 10^{10}}$$

$$\sigma_s = \frac{20 \times 10 \, \sigma_{al}}{8 \times 10^{10}}$$

$$\sigma_s = 2.5 \sigma_{al}$$

$$P = P_1 + P_2$$

$$35 \times 10^3 = \sigma_s A_s + \sigma_{al} A_{al}$$

$$35 \times 10^3 = 1.5 \times 200 \times \sigma_{al} + \sigma_{al} \times 200$$

$$35 \times 10^3 = \sigma_{al} (500 + 200)$$

$$7000 \, al = 35 \times 10^3$$

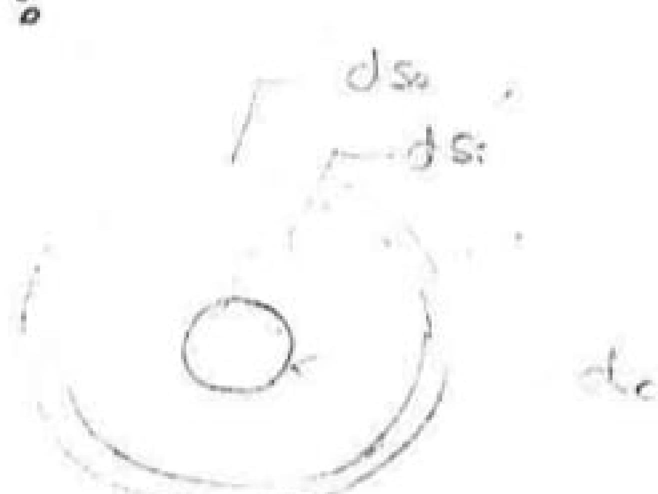
$$\sigma_{al} = \frac{35000\cancel{0}}{700\cancel{0}}$$

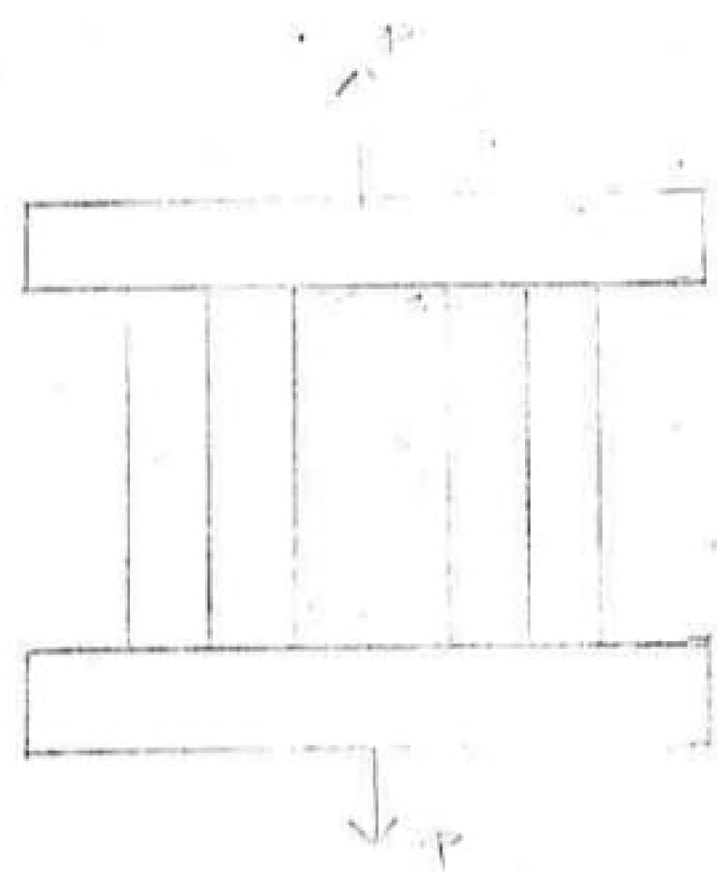
$$\sigma_{al} = 50\ N/mm^2$$

$$\sigma_s = 12.5\ N/mm^2.$$

(3) A solid copper rod 36 mm diameter is rigidly fixed at both the ends inside a steel tube of 45 mm inside dia and 50 mm outer dia. A compound section is subjected to an axial pull of 98 KN. Determine stress induced in the rod and tube and total elongation of composite section in a length of 1m. E for copper & steel is $1.1 \times 10^5\ N/mm^2$ and $2 \times 10^5\ N/mm^2$.

GIVEN :





GIVEN :

$d_c = 36mm$

$d_{si} = 45\ mm$

$d_{so} = 50\ mm$

$E_s = 2 \times 10^5\ N/mm^2$

$E_c = 1.1 \times 10^5\ N/mm^2$

$L = 1m \times 10^3 mm$

Solution :

i) Strain constant :

$\varepsilon_s = \varepsilon_c$

$\frac{\sigma_s}{E_s} L = \frac{\sigma_c}{E_c} L$

$\sigma = \frac{P}{A}$

$\frac{\sigma_s A_s L}{A E}$

$\sigma_s = 1.9 \sigma_c$

ii) Total load = sum of load

$P = \sigma_s A_s + \sigma_c A_c$

$A_c = \frac{\pi}{4} d_c^2 = 1017.36\ mm^2$

$A_s = \frac{\pi}{4}(d_{os}^2 - d_{is}^2) = \frac{\pi}{4}(50^2 - 45^2)$

$= 372.8\ mm^2$

$98 \times 10^3 = \sigma_s \times A_s + \sigma_c A_c$

$98 \times 10^3 = (1.9 \sigma_c \times 372.8) + (\sigma_c \times 1017.36)$

$98 \times 10^3 = 708.32\ \sigma_c + \sigma_c \times 1017.36$

$\sigma_c = 56.77\ N/mm^2$

$\sigma_s = 1.9 \sigma_c$

$= 1.9 (56.77)$

$\sigma_s = 107.89 \ N/mm^2$

ii) δ Total elongation:

$$\delta l = \delta l_1 + \delta l_2$$

$$= \frac{\sigma_s L}{E_s} + \frac{\sigma_c L}{E_c}$$

$$= \frac{107.89 \times 1 \times 10^3}{2 \times 10^5} + \frac{56.77 \times 10^3}{2 \times 10^5}$$

$$= 0.539 + 0.283$$

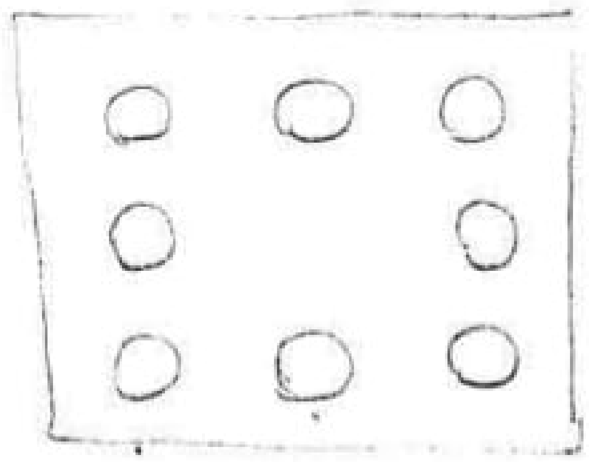
$$= 0.51 \ mm$$

(4). A reinforced short ~~complete~~ concrete column $250 \times 250 \ mm^2$ in section is reinforced with 8 steel bars. The total area of steel bar is $2500 \ mm^2$. The column carries a load of 400 kN. If the modulus of elasticity of steel is 15 times of concrete. Find the stress in concrete and steel. Also find area of steel required so that column can support 500 kN load. The max stress in concrete is $4.5 \ N/mm^2$.

(10)

GIVEN :

Total area = 250×250 mm²



Area of steel bar $a_s = 2500$ mm²

Area of concrete = ~~ar~~ Total area − area of steel bar

= 60,000 mm²

$E_s = 15 E_c$ i) Load = 400×10^3 N

$$\frac{E_s}{E_c} = 15$$

Soln :

case i) σ_s, σ_c ?

Cond i) : Strain constant

$$\varepsilon_s = \varepsilon_c$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\frac{E_s}{E_c} = \frac{\sigma_s}{\sigma_c}$$

$$15 = \frac{\sigma_s}{\sigma_c}$$

$$\boxed{\sigma_s = 15\sigma_c}$$

$\delta l = \frac{PL}{AE}$

$\varepsilon = \frac{\delta l}{L} = \frac{\sigma}{E}$

$\varepsilon = \frac{\sigma}{E}$ σ/E

$\delta l / L = \sigma L/E$

$\delta L =$

Cond ii) : Total load = Sum of load.

$$P = P_s + P_c$$

$$P = \sigma_s A_s + \sigma_c A_c$$

$$400 \times 10^3 = 15\sigma_c(2500) + \sigma_c(60{,}000)$$

$$400000 = 37500\,\sigma_c + 60{,}000\,\sigma_c$$

$$40{,}0000 = 97500\,\sigma_c$$

$$\sigma_c = 4.10 \text{ N/mm}^2$$

$$\sigma_s = 61.5 \text{ N/mm}^2$$

<u>case ii)</u> : Area of steel required :

$$\sigma_{c\,max} = 4.5 \text{ N/mm}^2$$

$$P = 500 \text{ kN} = 500 \times 10^3$$

Cond ii) $P = P_s + P_c$

If area of steel is changed there will be a change in area of concrete.

∴ Area of concrete.

A_c = Total area − Area of steel.

$$500 \times 10^3 = \sigma_s A_s + \sigma_c A_c$$

$$500 \times 10^3 = \sigma_s A_s + \sigma_c (A - A_s)$$

6/4 //

$$500 \times 10^3 = 15\sigma_c A_s + \sigma_c (\overset{62500}{\cancel{+25000}} + A_s)$$

$$500 \times 10^3 = 168750 + (\underset{281250}{\cancel{-11250}} + 4.5A)$$

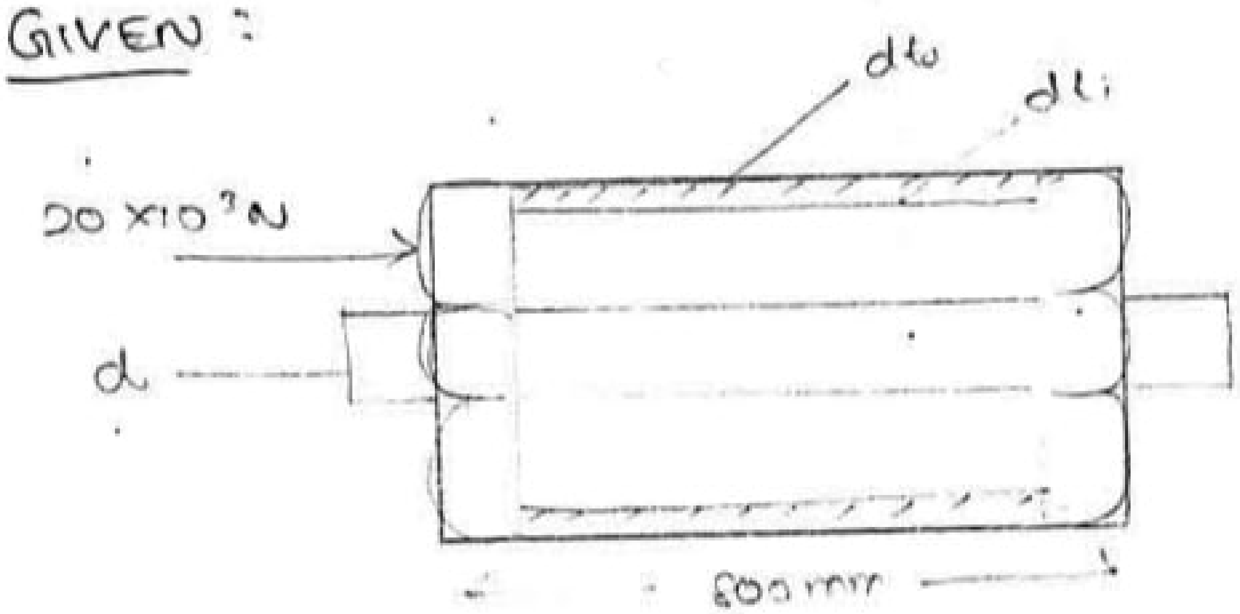
6/4 //

A_s = 2172.2 mm²

42

(5) A steel rod 20mm in diameter passes centrally through a steel tube of 25 mm internal diameter and 30 mm external diameter. The tube is 800 mm long and is closed by a rigid washer of neglegible thickness which are fastened by nut threaded on the rod. until the compressive load on the tube is 20 KN. Calculate the stresses in the rod and the tube. also find the increase in the stresses when 1 nut is tighted by 1 quarter of turn relative to the other. There are four threads per 10 mm. Take $E = 2 \times 10^5\ N/mm^2$.

GIVEN :



$d = 20mm$

$d_{t_o} = 30mm$

$d_{t_i} = 25mm$

$L = 800mm$

$P = 20KN \Rightarrow 20 \times 10^3 N$

$E = 2 \times 10^5\ N/mm^2$

No. of threads = 4 threads per 10 mm

Sol:

when the bolt is tighten the compressive load acting on the steel tube is equal to the tensile load acting on the rod.

Compressive load on tube (P_t) = Tensile load on the rod (P_r)

$$\sigma_t \cdot A_t = \sigma_r A_r$$

$$A_t = \frac{\pi}{4}(d_{to}^2 - d_{ti}^2)$$

$$= \frac{\pi}{4}(30^2 - 25^2)$$

$$= 215.87$$

$$A_r = \frac{\pi}{4} d_r^2 = \frac{\pi}{4} \times 20^2$$

$$= 314.$$

$$\sigma_t (215.87) = \sigma_r (314)$$

$$\boxed{\sigma_t = \sigma_r (1.454)} \quad \text{--- (1)}$$

$$\sigma_t = \frac{P_t}{A_t}$$

$$\sigma_t = 92.64 \text{ N/mm}^2$$

~~a2~~ sub σ_t in (1)

$$92.64 = \sigma_r (1.454)$$

$$\boxed{\sigma_r = 63.8 \text{ N/mm}^2}$$

(14)

ii) Stress in rod and tube when nut is tighted by 1 quarter of turn:

Distance moved by nut = Decrease in length of tube + Increase in length of the rod.

$\delta l_t + \delta l_r$

$$= \frac{\sigma_t L}{E} + \frac{\sigma_r L}{E} \quad — (2)$$

Distance moved by nut = one quarter of turn × pitch

$$= \frac{10}{4} \times \frac{1}{4}$$

$$= 0.625$$

W.K.T. $\boxed{\sigma_t = 1.45\,\sigma_r}$

From (2),

$$\text{Nut distance} = \frac{\sigma_t L}{E} + \cancel{\frac{\sigma_t L}{1.45 \times E}} \; \frac{\sigma_r L}{E}$$

$$0.625 = \frac{1.45\sigma_r L}{E} + \frac{\sigma_r L}{E}$$

$$0.625 = \frac{1.45 \times \sigma_r \times 800}{2 \times 10^5} + \frac{\sigma_r \times 800}{2 \times 10^5}$$

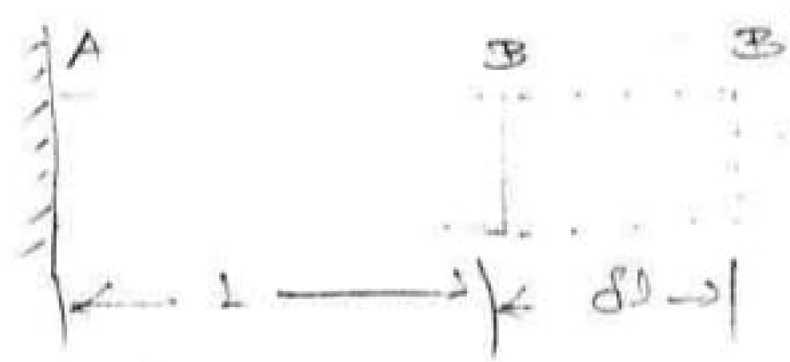
$$0.625 = 5.8 \times 10^{-4}\,\sigma_r + 4 \times 10^{-3}\,\sigma_r$$

~~$0.625 = 4.58 \times 10^{-3}\,\sigma_r$~~

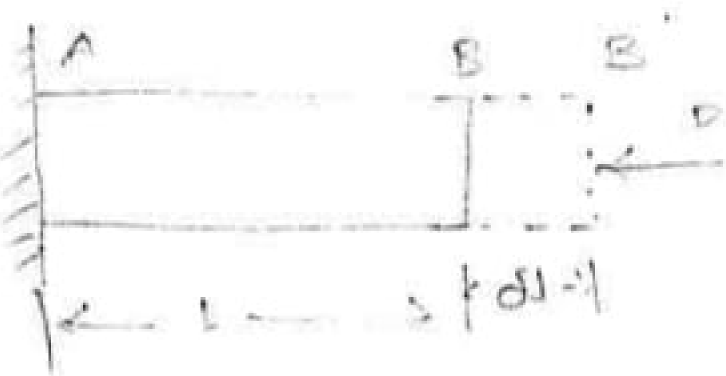
$$\sigma_r = 63.77.$$

Thermal stresses :

The thermal stresses are the stresses induced in a body due to change in temperature. when the thermal expansion of the material is not restricted then no stresses acts on the material. If the expansion is restricted by some means then thermal stresses acts on the material.



No stresses acts.



Thermal stresses acts

FORMULA FOR THERMAL STRESSES :
(Homogenous material).

i) when expansion is allowed :

$$\text{strain}, \ \varepsilon = \frac{\alpha T L - \delta}{L} = \frac{\alpha (T_1 - T_2) - \delta}{L}$$

$$\text{stress}, \ \sigma = \left(\frac{\alpha T L - \delta}{L}\right) E = \left(\frac{\alpha (T_1 - T_2) L - \delta}{L}\right) E$$

ii) when expansion is restricted :

$$\text{Strain} = \alpha T = \alpha (T_1 - T_2)$$

$$\text{Stress} = \alpha T E = \alpha (T_1 - T_2) E$$

iii) Expansion of bar (or) change in length δl :

$$\delta l = \alpha T L = \alpha (T_1 - T_2) L.$$

$T_1 \Rightarrow$ ~~increase~~ Initial in temperature, T_2 = Final Temp.
(or) $T_1 - T_2$ = Increase in Temp.
$\alpha \Rightarrow$ coefficient of thermal expansion.

$\delta \Rightarrow$ deformation at yield point or permanent deformation allowed.

<u>Thermal stresses in composite bar :</u>

Two different materials are rigidly fixed with each other hence composite bar has a hole. will expand by the same amount.

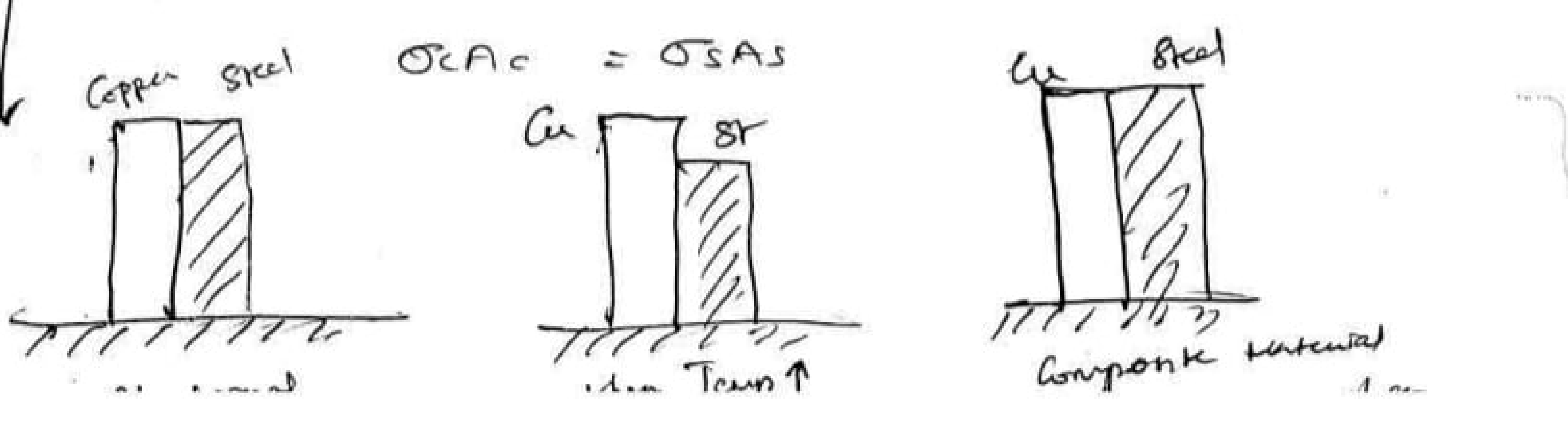
Assumptions or conditions:

1). When temperature increases :

Compressive load on copper = Tensile load on ~~steel~~ steel.

$$P_c = P_s$$

$$\sigma_c A_c = \sigma_s A_s$$



ii). Strain constant :

$$\delta l_c = \delta l_s$$

$$\delta l_c = \cancel{\alpha_c T L}$$

Expansion of steel = free expansion + expansion due to tensile load.

$$\delta l_s = \alpha_s (T_1 - T_2) L + \frac{\sigma_s L}{E_s}$$

Expansion of copper = free expansion − contraction due to compressive load.

$$\delta l_c = \alpha_c (T_1 - T_2) L - \frac{\sigma_c L}{E_c}$$

iii) $\sigma_c > \sigma_s$ when temperature increases.

When temperature decreases

i) $P_c = P_s$

$$\sigma_c A_c = \sigma_s A_s$$

ii) $\delta l_c = \delta l_s$

a) Contraction ~~Expansion~~ of steel = free Contraction ~~expansion~~ + Contraction ~~expansion~~ due to Compressive ~~tensile~~ load

$$\delta l_s = \alpha_s (T_1 - T_2) S + \frac{\sigma_s L}{E_s}$$

(b) Contraction of brass = Free Contraction − Expansion due to tensile load.

$$\delta l_c = \alpha_c (T_1 - T_2) L - \frac{\sigma_c L}{F_c}$$

1. A steel rod 3 cm diameter and 5 m long if it is connected to two grips and rod is maintained at a temperature of 95°c determine stress, strain and pull exerted when temperature falls to 30°c. $E = 2\times10^5\ N/mm^2$, $\alpha = 12\times10^{-6}/°C$ find.

i) ends are not yield.

ii) ends are yield by 0.12 cm.

<u>GIVEN</u> :

$d = 3\ cm \Rightarrow 300\ mm$

$L = 5m \Rightarrow 5\times10^3\ mm$

$T_1 = 95°C$

$T_2 = 30°C$

$E = 2\times10^5\ N/mm^2$

$\alpha = 12\times10^{-6}\ /°C$

Increase in temp $= 95°C - 30°C$

$= 65°C$

$= 338\ K$

$\delta = 0.12\ cm$

$= 0.12\times10^2\ mm$

<u>Soln</u> :

i) When ends are not yield.

Stress, $\sigma = \alpha T E$

$= 12\times10^{-6} \times \overset{65}{\cancel{338}} \times 2\times10^5$

$= 156\ N/mm^2$

Strain $\varepsilon = \alpha T$

$= 12\times10^{-6}\times65$

$= 7.8\times10^{-4}$

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Pull (or) Load $= \sigma \times A = 156 \times \frac{\pi (300)^2}{4}$

$= 1102.14 \times 10^{+4}$ N

$= 110$ kN

ii) When ends are allowed yield :

Stress , $\sigma = \left(\frac{\alpha T \cancel{E} L - \delta}{L} \right) E$

$\sigma \ \cancel{156} = \left(\frac{12 \times 10^{-6} \times 65 \times \cancel{2 \times 10^{-5}} 5 \times 10^{3} - 0.12 \times 10^{2}}{5 \times 10^{3}} \right) 2 \times 10^{5}$

$\sigma = 108$ N/mm²

°C

Strain , $\epsilon = \frac{\alpha T L - \delta}{L} = 0.54 \times 10^{-3}$

Pull or load $= \sigma \times A$

$= 108 \times \frac{\pi}{4} (300)^2$

$= 7.630 \times 10^{6} \times 10^{-3}$

$= 76$ kN

composite bar.

②. A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm ext dia , 40 mm internal dia. The tube is closed at each end by rigid plates of negligible thickness.

The nuts are tightened slightly on the projecting parts of the rod. If the temperature of the assembly is raised by 50°C. Calculate stress developed in cu and Steel. Take $E_s = 200$ Giga N/m^2 and $E_c = 100$ Giga N/m^2 and α_s & α_c = $12 \times 10^{-6} /°C$ and $18 \times 10^{-6} /°C$.

GIVEN :

$d_s = 20 mm$ $\qquad E_s = 200 \; GN/m^2$

$d_{co} = 50 mm$

$d_{ci} = 40 mm$ $\qquad E_c = 100 \; GN/m^2$

$T_2 = 50 °C$ $\qquad \alpha_s = 12 \times 10^{-6} /°C$

$\sigma = ?$ $\qquad \alpha_c = 18 \times 10^{-6} /°C$

Soln :

1). Compressive load on copper = Tensile load on steel.

$$P_c = P_s$$

$$\sigma_c A_c = \sigma_s A_s$$

$$A_c = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$= \frac{\pi}{4} (50^2 - 40^2) \Rightarrow 706.85 \; mm^2$$

$$A_s = \frac{\pi}{4} d_s^2 \Rightarrow \frac{\pi}{4} (20)^2 = 314.15 \; mm^2$$

$$\sigma_c = \sigma_s (0.44)$$

(ii) $\delta l_c = \delta l_s$

δl_c = Free expansion - contraction due to steady process.

$$\delta L_c = \alpha_c T L - \frac{\sigma_c}{E_c} L$$

$$\left(\alpha_c T - \frac{\sigma_c}{E_c}\right) \cancel{L} = \left(\alpha_s T + \frac{\sigma_s}{E_s}\right) \cancel{L}$$

$$\delta L_s = \sigma_s T L - \frac{\sigma_s L}{E_c}$$

$$18 \times 10^{-6} \times 50 - \frac{\sigma_s}{100 \times 10^9} = 1$$

$$\delta l_s = \delta l_c$$

$$\alpha_s (T_1 - T_2) + \frac{\sigma_s}{E_s} = \alpha_c (T_1 - T_2) - \frac{\sigma_c}{E_c}$$

$\sigma_c = 14.117 \ N/mm^2$

$\sigma_s = 31.76 \ N/mm^2$

$e_c = \delta l / l$.

$e_s = \delta l / l$

(52)

(3): A steel tube of 30 mm external diameter and 25 mm internal diameter encloses a gun metal rod of 20 mm dia to which it is rigidly joined at each end. The temp. of whole assembly is raised to 140°C and the nuts on the rod are then screwed tightly at the ends of the tube. Find the intensity of rod when the common temp has fallen to 30°C the value of $E_s = 2.1\times10^5$ N/mm², $E_G = 1\times10^5$ N/mm², The Coeff. of steel & gun metal is $12\times10^{-6}/°C$ & $20\times10^{-6}/°C$.

$G\,N/m^2 \Rightarrow \times10^9\,N/mm^2$

$= 10^{-6}\,N/mm^2$

$10^3\,N/mm^2$

Sol:

$D = 30$ mm $T = 140°C$

$D_i = 25$ mm $\alpha_s = 12\times10^{-6}$

$D_o = 20$ mm $\alpha_g = 20\times10^{-6}$

$$A_s = \frac{\pi}{4}\times 5^2$$

$$= 19.625\ mm^2$$

$$A_g = \frac{\pi}{4}(20)^2$$

$$= 314\ mm^2$$

$$\sigma_s = 16\sigma_c$$

ii) $\delta L_s = \delta L_c$

$$\delta L_c = 2800\times10^{-6}L - 0.03125L\,\sigma_s$$

$$\delta L_s = 1680\times10^{-6}L + \sigma_s L.$$

$$\delta L_c = \delta L_s$$

$$2800 \times 10^{-6} - 0.03125\,\sigma_s = 1680 \times 10^{-6} + \sigma_s$$

$$1120 \times 10^{-6} = 1.03125\,\sigma_s$$

$$\sigma_s = 108.6\ N/mm^2$$

$$\sigma_c = 6.78\ N/mm^2$$

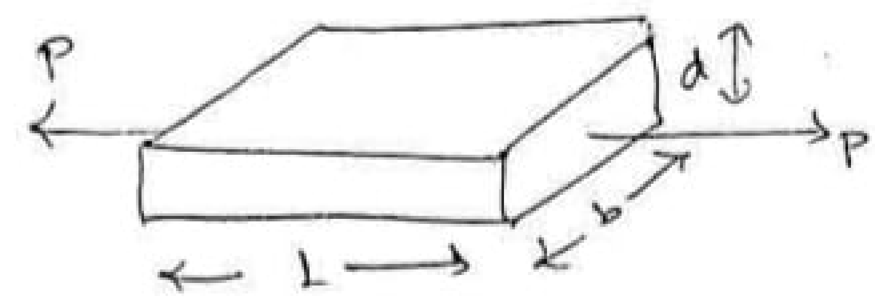
(51)
22/7/14

(44) Volumetric strain of rectangular bar subjected to axial load P in the direction of its length.

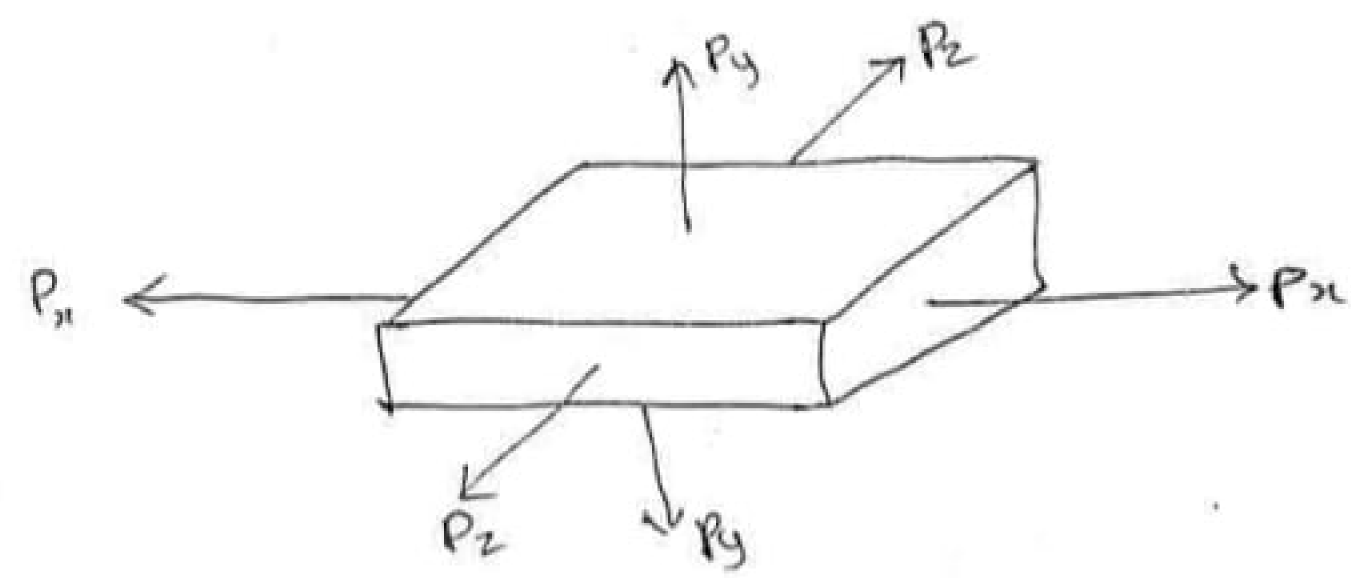
$$\varepsilon_v = \frac{\delta V}{V}$$

$$= \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta d}{d}$$

$$\varepsilon_v = \frac{\delta L}{L}(1-2\mu)$$



* Volumetric strain of bar where three forces acting perpendicular to each other



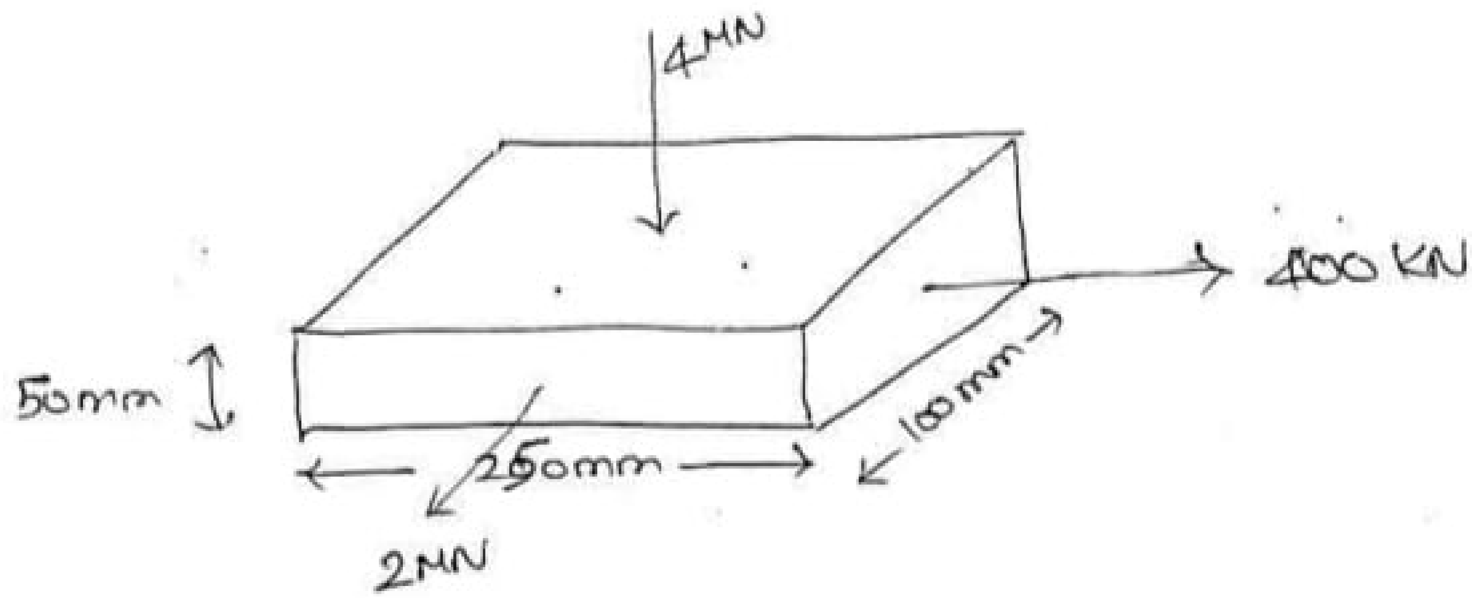
$$\varepsilon_v = \frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)$$

NOTE : 1) If any force is compressive that has to be considered as -ve force. The above eqn is gn for tensile forces

2) If $\frac{\delta V}{V}$ is +ve then it represents the volume is increasing

If -ve volume is decreasing

4. A metallic bar 250mm x 100mm x 50mm is loaded as shown in the fig. Find the change in volume. Take $E = 2\times10^5$ N/mm^2 and poisson ratio = 0.25. and also find the change in the force of 4MN that there should be no change in the vol. of the bar



GIVEN :

$l = 250$ mm

$b = 100$ mm

$d = 50$ mm

$E = 2\times10^5$ N/mm^2

$\mu = 0.25$

$P_x = 400 \times 10^3$ N

$P_y = 4\times10^6$ N

$P_z = 2\times10^6$ N

Sol :

Change in volume (δV).

W.K.T, $\varepsilon_v = \dfrac{\delta V}{V} = \dfrac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)$

$V = l \times b \times d$

$= 250 \times 100 \times 50$

$= 1250000$

$\frac{1}{\mu} = p$

$$\sigma_x = \frac{P_x}{A_x} = \frac{400 \times 10^3}{100 \times 50} \quad \text{(Tensile)}$$

$$= 80 \, N/mm^2$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{4 \times 10^6}{250 \times 100} \quad \text{(Compressive)}$$

$$= 160 \, N/mm^2$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{2 \times 10^6}{250 \times 50} \quad \text{(Tensile)}$$

$$= 160 \, N/mm^2$$

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_x - \sigma_y + \sigma_z)(1 - 2\mu)$$

$$\frac{\delta V}{V} = \frac{1}{2 \times 10^5}(80 - 160 + 160)(1 - 2(0.25))$$

$$\delta V = 250 \, mm^3$$

ii) change in 4 MN load for no change in volume.

since there is no change in vol.

$$\frac{\delta V}{V} = 0.$$

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z).(1 - 2\mu)$$

$$0 = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

$$\sigma_x + \sigma_y + \sigma_z = 0$$

$$\sigma_y = 240 \, N/mm^2 \quad \text{(Compression)}$$

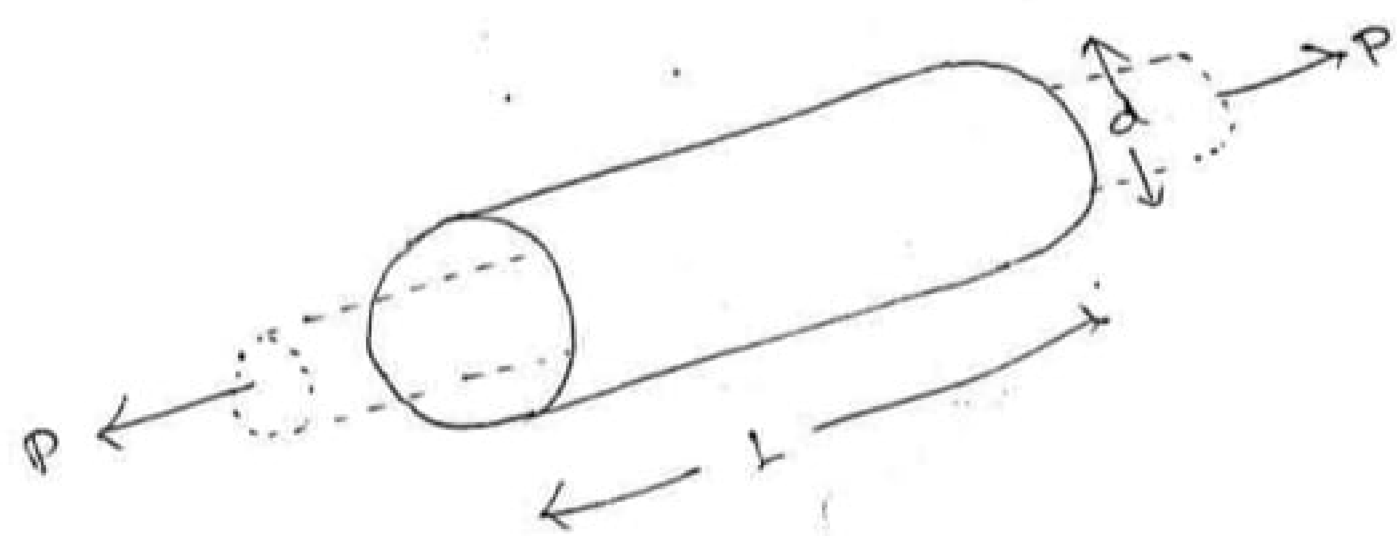
$$\sigma_y = \frac{P_y}{l \times b}$$

$$P_y = 6MN$$

- Additional ~~force~~ Load required $\Big\} = 6MN - 4MN$ (compressive)

$$= 2MN$$

(5). Volumetric strain of cylindrical rod :



$$\varepsilon_v = \frac{\delta L}{L} - \frac{2\delta d}{d}$$

6. A steel rod 5m long and 30 mm in diameter is subjected to an axial tensile load of 50 kN determine the change in length, diameter and volume of the rod. Take $E = 2\times10^5$ N/mm² and poisson ratio $\mu = 0.25$.

GIVEN :

$l = 5m$ $\qquad$ $E = 2\times10^5\ N/mm^2$

$d = 30\ mm$ $\qquad$ $\mu = 0.25$

$P = 50\ kN$

$= 50\times10^3 N$

$\delta L = ?$

$\delta d = ?$

change in length (δl)

$$E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\sigma}{E}$$

$$\frac{\delta l}{L} = \frac{\sigma}{E}$$

$$\delta l = \frac{\sigma}{E}\times L$$

$$= \frac{P}{A}\cdot\frac{1}{E}\times L$$

$$= \frac{50\times10^3}{\frac{\pi}{4}\cdot30^2}\cdot\frac{1}{2\times10^5}\times 5\times10^3 m$$

$$= 1.75\ mm$$

le
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t.

Change in diameter : (δd) :

$$\mu = \frac{\text{Lateral strain.}}{\text{Longitudinal strain}}$$

$$\text{Lateral strain} = \frac{\delta d}{d}$$

$$\frac{\text{Load}}{\text{Area}} = \frac{\delta d}{d}$$

$$\frac{50 \times 10^3}{\frac{\pi}{4} \times 30^2} = \frac{\delta d}{30}$$

$$\delta d = 2.65 \times 10^{-3} \text{ mm.}$$

change in volume : (δV).

$$\varepsilon_v = \frac{\delta L}{L} - \frac{2\,\delta d}{d}$$

$$\frac{\delta V}{V} = \frac{\delta L}{L} - \frac{2\,\delta d}{d}$$

$$\delta V = 618 \text{ mm}^3$$

Relationship between Young's modulus & Bulk modulus :

$$E = 3K(1 - 2\mu)$$

Relationship between young's modulus & modulus of rigidity :

$$E = 2C(1 + \mu)$$

1. A bar of cross section 80 mm × 80mm is subjected to an axial load of 7000 N. The lateral dimension of the bar is found to be changed to 7.9885mm × 7.9885 mm. If the modulus of rigidity is 0.8×10^5 N/mm². Determine the value of poisson ratio and young's modulus.

<u>GIVEN</u> :

$$\text{Area} = 8 \times 8 = 64\ mm^2$$

$$P = 7000\ N$$

Lateral dimension $= 7.9885\ mm \times 7.9885\ mm$

$$C = 0.8 \times 10^5\ N/mm^2$$

To find E & μ :

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\varepsilon_L = \frac{\delta b}{b} = \frac{8-\cancel{7.9885}^{\,7.9985}}{8} = \cancel{1.4375\times10^{-3}}\ 1.875\times10^{-4}$$

$$\varepsilon_a = \frac{\sigma}{E} = \frac{P}{A\cdot E} = \frac{7000}{64\times E}$$

$$\varepsilon = \frac{109.375}{E}$$

$$\mu = \frac{1.437\times10^{-3}}{\frac{109.375}{E}}$$

$$\boxed{\mu = 1.313\times10^{-5}\cdot E} \text{ — (1)} \quad 1.714\times10^{-6}$$

WKT.

$$E = 2C(1+\mu)$$

$$E = 2\times0.8\times10^{5}\ (1+\cancel{1.313\times10^{-5}}E)$$

$$E = 1.6\times10^{5}\ \cancel{+}\ \cancel{2.1008}E\ 0.27424$$

$$0 = 1.6\times10^{5} + \cancel{1.10}E\ -0.725E$$

$$E = -1.45\times10^{5}\ \text{N/mm}^2$$

$$\mu = -1.9$$

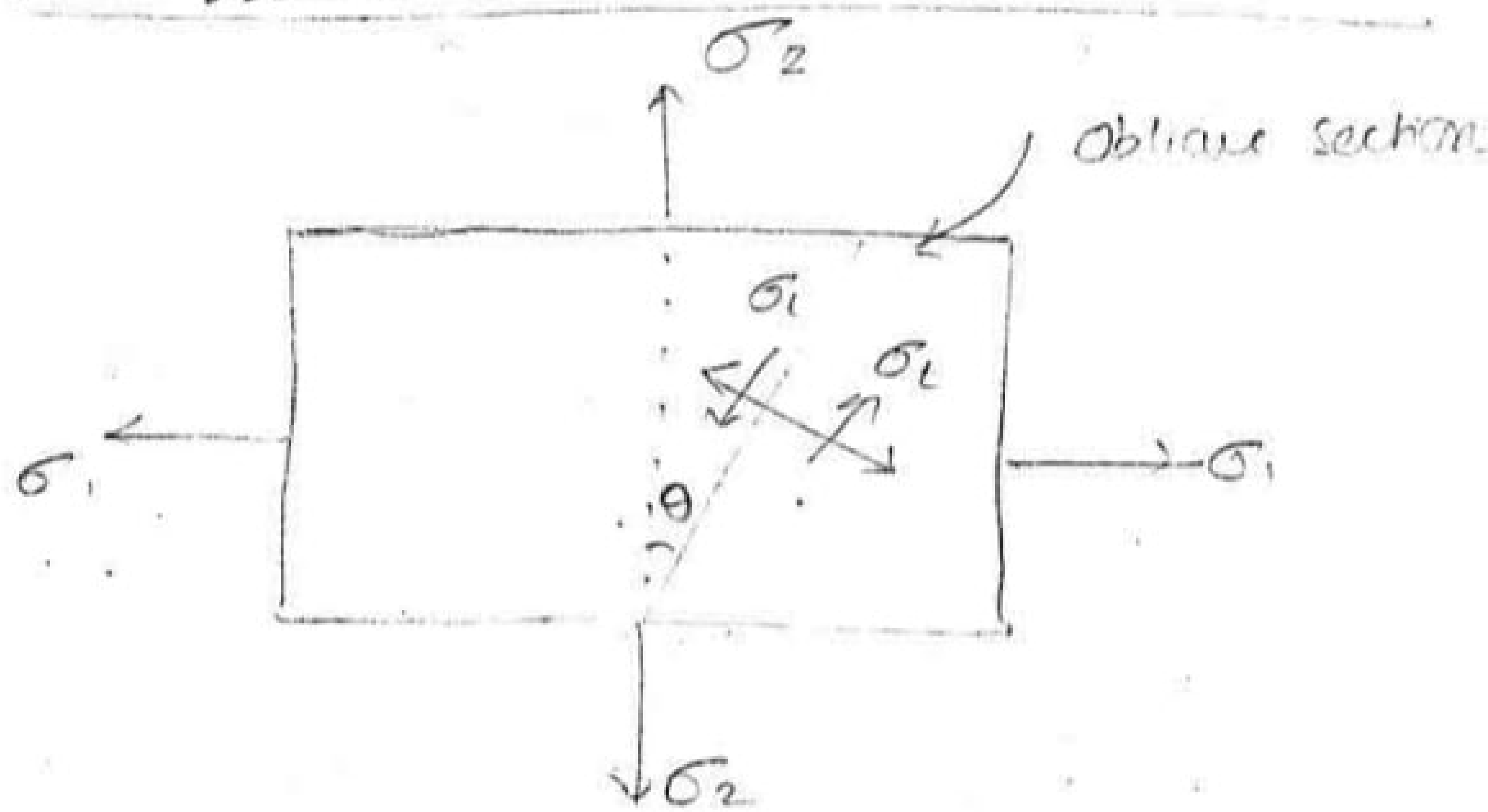
Principle planes & principle stress :
(Shear stress = 0)

The planes which have no shear stress are known as principal planes these planes carry only normal stresses.

Principle stresses :

The normal stresses acting on the principle plane is known as principle stresses.

<u>Oblique section (or) inclined section :</u>



It is the angle made by the resultant stress with normal of the oblique plane. It is donoted by ϕ.

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

where, $\sigma_n \Rightarrow$ normal stress $\quad \phi$ – obliquity angle (or) Inclined angle

$\sigma_t \Rightarrow$ tangential stress

$\theta \Rightarrow$ is inclination of obliq sec.

σ_1, σ_2 = direct stress.

Condition 1 :

Member subjected to direct stress in one plane

Direct Stress $\sigma = \dfrac{\text{Load}}{\text{C/S Area}}$

Normal stress $\sigma_n = \sigma \cos^2\theta$

Tangential (or) Shear stress σ_t $= \dfrac{\sigma}{2} \sin 2\theta$

Resultant stress $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$$\tan\varphi = \frac{\sigma_t}{\sigma_n}$$

When $\theta = 0$

σ_n = maximum

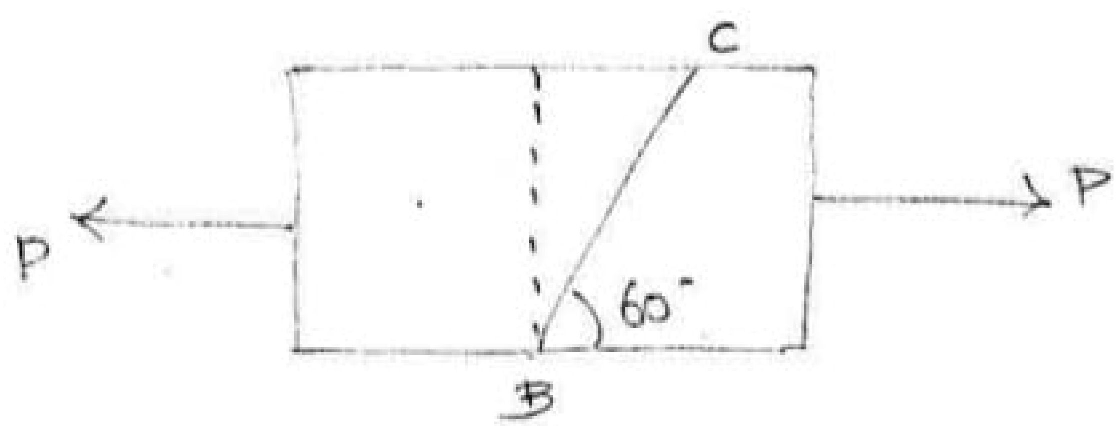
When $\theta = 90°$

σ_n = minimum.

When $\theta = 45°$

σ_t = maximum

1. A rectangular bar of C/s area of 11000 mm² is subjected to tensile load P as shown in the fig. The permissible normal shear stress on the oblique plane BC is given as 7 N/mm² and 3.5 N/mm². Determine the safe value of P.



Given :

$\sigma_n = 7 \, N/mm^2$

$\sigma_t = 3.5 \, N/mm^2$

$A = 11000 \, mm^2$

$\theta = 90° - 60°$ 2

$= 30°$

Soln: Safe load.

σ

$\sigma_n = \sigma \cos^2\theta$

$7 = \sigma \cos^2 30$

$\sigma = \frac{7}{\cos^2 30}$

$\boxed{\sigma = 9.33 \, N/mm^2}$

$\sigma = \frac{P}{A}$

$9.33 = \frac{P}{11000}$; $\boxed{P = 102 \, KN}$

$$\sigma_t = \frac{\sigma}{2} \sin 2\theta$$

$$3.5 = \frac{\sigma}{2} \sin 2(30)$$

$$7 = \sigma \sin 60$$

$$\boxed{\sigma = 8.08 \ N/mm^2}$$

$$\sigma = \frac{P}{A}$$

$$8.08 = \frac{P}{11000}$$

$$\boxed{P = 88.9 \ KN}$$

Safe load is minimum of the above value.

② Two wooden pieces 10cm x 10cm in cross section are glued together along the line AB as shown in the figure. What maximum axial force can be applied if with the allowable shearing stress is 1.2 N/mm².

Given :

Area = $10^4 \ mm^2$

$$\sigma_t = \frac{\sigma}{2} \sin 2\theta$$

$$\sigma = \frac{2 \times 1.2}{\sin 120}$$

$$\sigma = 2.77 \, N/mm^2$$

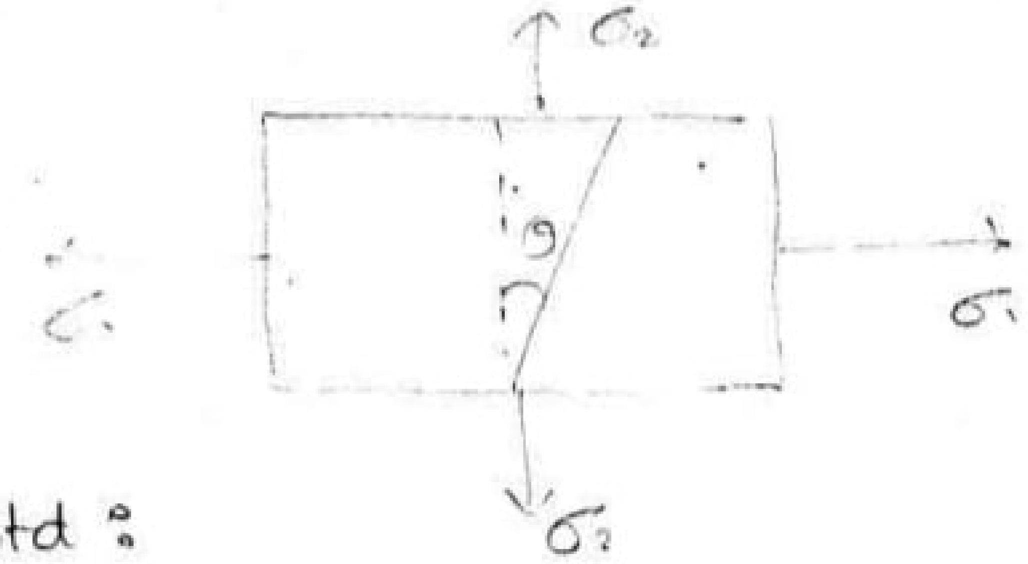
$$\sigma = P/A$$

$$P = 2.77 \times 10^4$$

$$P = 27.7 \, kN$$

(b). Condition 2:

Member subjected to direct stresses in two mutually perpendicular direction.



Analytical mtd:

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \cdot \sin 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

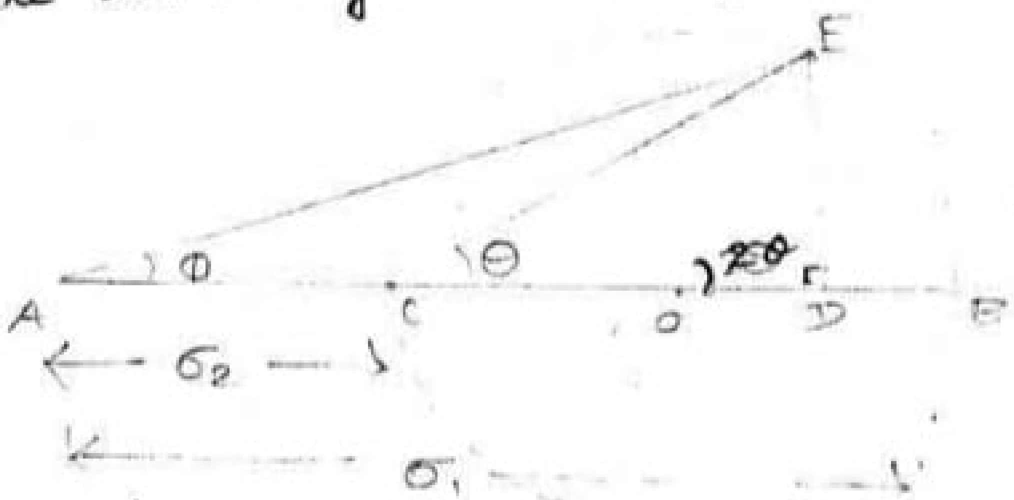
$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

Max shear stress, $(\sigma_t)_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$. (at $\theta = 45°$ $= 135°$)

GRAPHICAL METHOD : (Mohr's circle method)

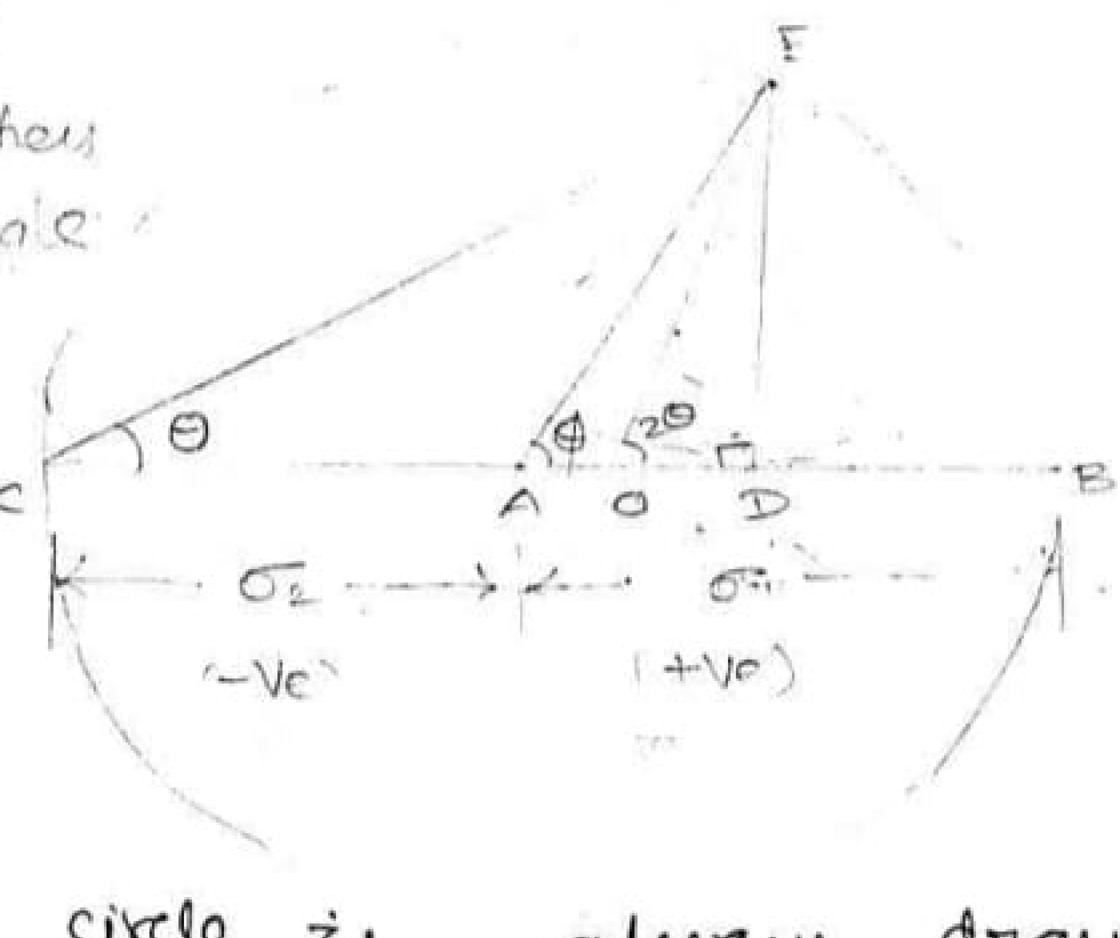
Both tensile (or) compressive.

1. Mohr's circle drawn with midpoint between C & B. The radius of mohrs circle is $(\sigma_t)_{max}$
2. From point 'C' draw a line with the angle 2θ. to find the point of intersection with the mohrs circle
3. From 'E' draw a ⊥ line to AB to find 'D' i.e AD = normal stress.



Tensile and compressive :

AD : normal stress
ED : Shear stress
AE : Resultant stress
ϕ : obliquity angle
σ_1 = major stress
σ_2 = minor stress



* The mohr's circle is always drawn with the midpoint between C and B. The radius of mohr's circle is OC or OB.
* Max. shear stress is radius of mohr's circle

① The stresses at a point in a bar are 200 N/mm^2 (tensile) and 100 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the maximum intensity of shear stress in the material at that point. The Determine using analytical & graphical method.

Solution :

BY USING ANALYTICAL METHOD :

$\sigma_1 = 200 \ N/mm^2$

$\sigma_2 = -100 \ N/mm^2$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$\cancel{\sigma_A = \sigma \cos 2\theta}$$

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{200 + (-100)}{2} + \frac{200 + 100}{2} \cdot \cos 2(30)$$

$$= 125 \ N/mm^2$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$= \frac{200 + 100}{2} \sin 2(30)$$

$$= 129.9 \ N/mm^2$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$\sigma_R = \sqrt{125^2 + 129.9^2}$$

$$\sigma_R = 180.2 \text{ N/mm}^2$$

$$\sigma_{t\,max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{200 + 100}{2} \Rightarrow 150 \text{ N/mm}^2.$$

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

$$\tan\phi = \frac{129.9}{125}$$

$$\phi = \tan^{-1}\left[\frac{129.9}{125}\right]$$

$$\phi = 46^\circ$$

GRAPHICAL

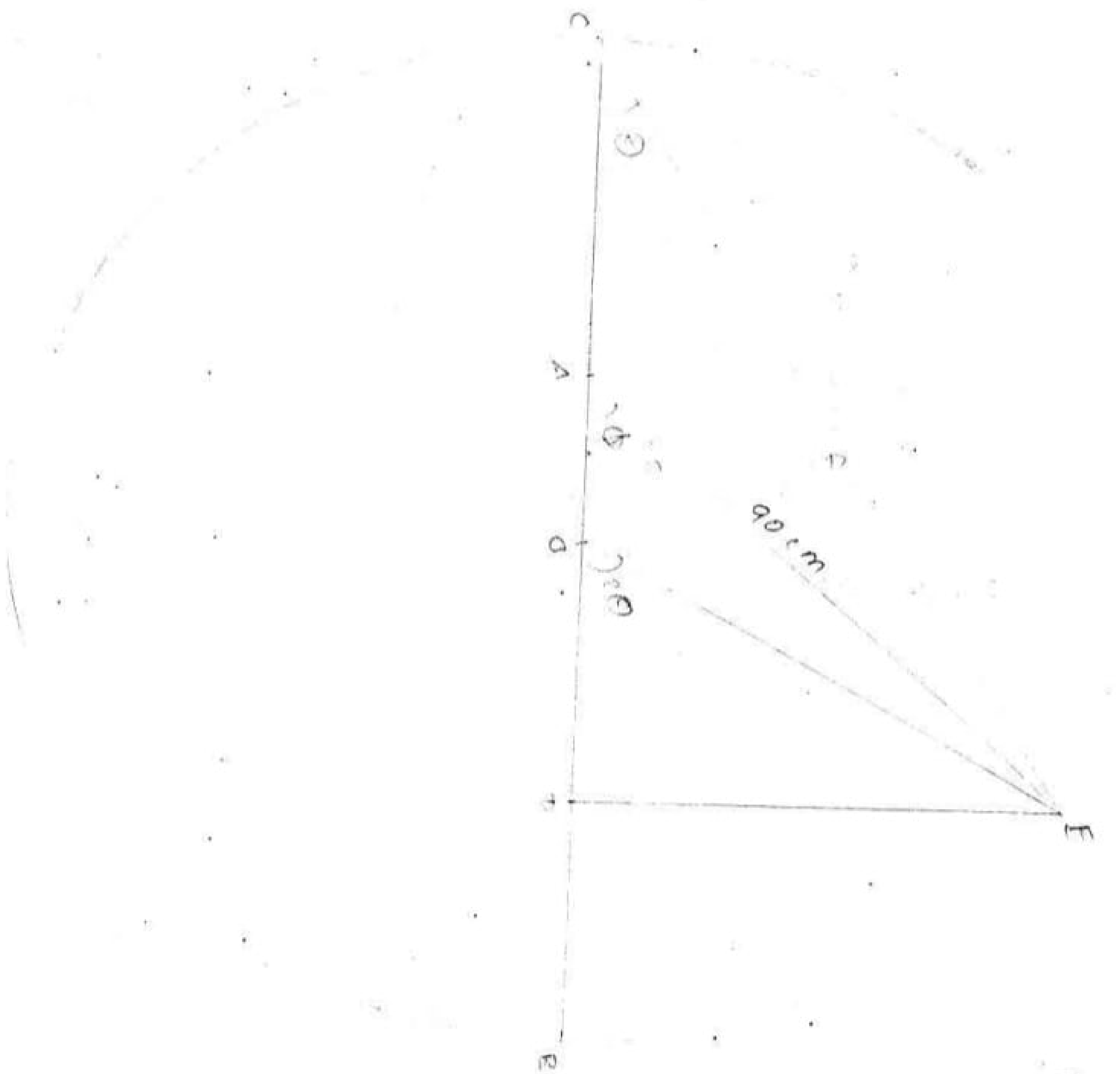
GRAPHICAL METHOD :

Scale: 1 cm = 20 N/mm²

$\sigma_1 = \frac{200}{20} = 10$ cm

$\sigma_2 = 100/20 = 5$cm

GRAPHICAL METHOD :



(2). At a point in a strained material the principle stresses are 100 N/mm² (tensile) & 40 N/mm² (tensile). Determine the resultant stress in magnitude & direction on a plane inclined at 60° to the axis of the major principle stress. What is the maximum intensity of shear stress in the material at that point. Determine using Analytical & Geometrical.

Solution:

ANALYTICAL METHOD :

$\sigma_1 = 100\ N/mm^2$

$\sigma_2 = 40\ N/mm^2$

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta$$

$$= \frac{100+40}{2} + \frac{100-40}{2}\cos 2(30)$$

$$\sigma_n = 70 + \cancel{30}\ 15$$

$$= 85\ N/mm^2$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta$$

$$= \cancel{200}\cdot \frac{100-40}{2}\sin 2(30)$$

$$\sigma_t = 25.98\ N/mm^2$$

$$\sigma_R = \sqrt{\sigma_n^2+\sigma_t^2}$$

$$= \sqrt{85^2+25.98^2}$$

$$= 88.88\ N/mm^2$$

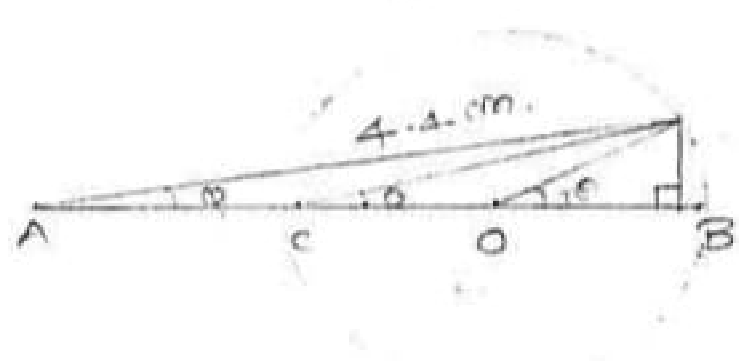
$$\sigma_{t\,max} = \frac{\sigma_1 - \sigma_2}{2} \Rightarrow \frac{100-40}{2} \Rightarrow 30 \ N/mm^2$$

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

$$\phi = \tan^{-1}\left(\frac{25.98}{85}\right)$$

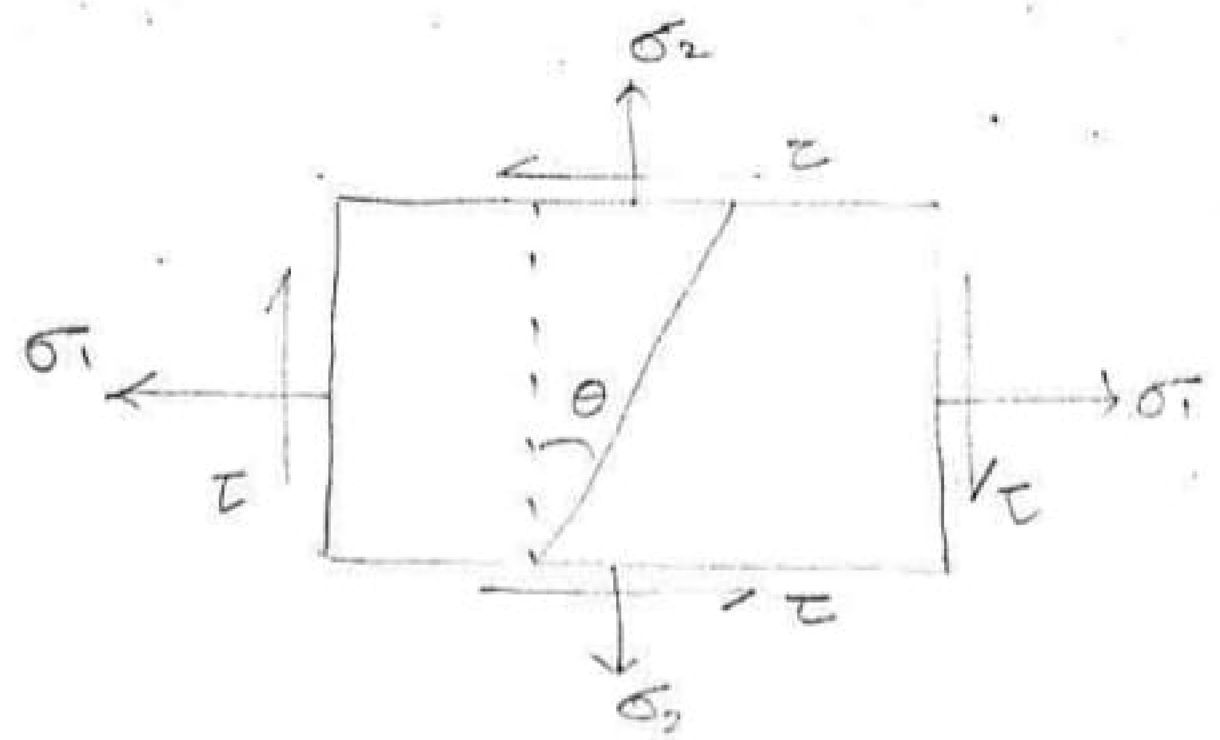
$$\phi = 16.9^\circ$$

GRAPHICAL METHOD :



Condition 3 :

Members subjected to direct stresses in two mutually perpendicular directions accompanied by simple shear stresses.



i). $\sigma_n = \dfrac{\sigma_1 + \sigma_2}{2} + \dfrac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$

ii) $\sigma_t = \dfrac{\sigma_1 - \sigma_2}{2} \cdot \sin 2\theta - \tau \cos 2\theta$

iii) $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

iv) Major principal stresses

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

v). Minor principal stresses

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

vi) Direction or position of principal plane

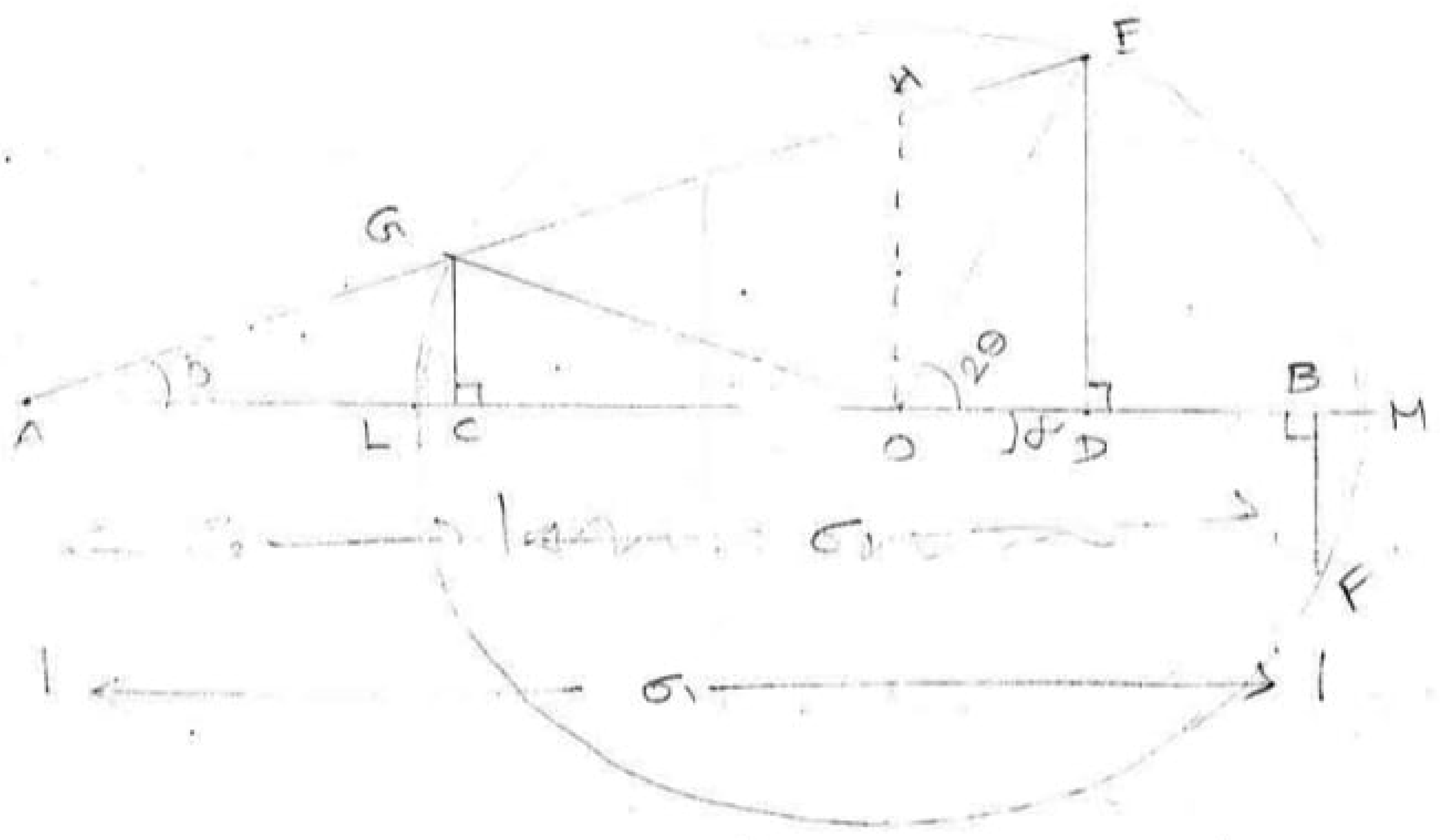
$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

vii) Max shear stress, $\sigma_{t\,max} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$

viii) Plane on which max. shear stress acts

$$\tan 2\theta = \frac{\sigma_1-\sigma_2}{2\tau}$$

GRAPHICAL METHOD:



Radius of Mohr's circle is drawn using the distance GO or FO, then GC and BF is equal to the value of shear stress.

Maximum shear stress is radius of Mohr's circle

AF represents resultant stress on oblique plane

AD represents normal

σ_{max} major principal stress

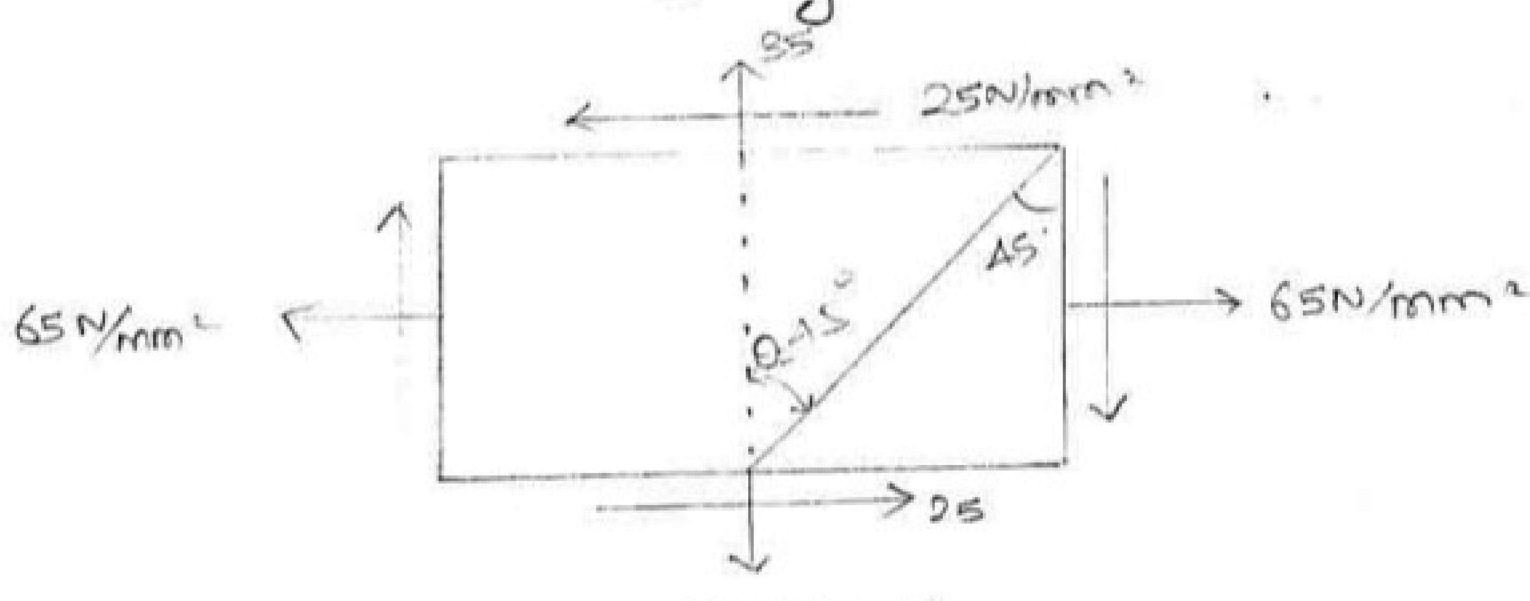
$$\sigma_{max} = AO + OM$$

$$\sigma_{min} = AO - LO$$

$$\tan 2\theta = \frac{BF}{OB}$$

PROBLEMS :

1. A point in a strained material is subjected to stress as shown in the fig. Determine the normal stress, tangential stress across the oblique plane using analytical method and check the answers using moh's circle.



GIVEN :

$$\sigma_1 = 65 \, N/mm^2$$

$$\sigma_2 = 35 \, N/mm^2$$

$$\tau = 25 \, N/mm^2$$

$$\theta = 45^\circ$$

Normal stress :

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{65+35}{2} + \frac{65-35}{2}\cos 2(45) + 25 \sin 2(45)$$

$$= 75 \text{ N/mm}^2$$

Tangential stress :

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$= \frac{65-35}{2}\sin 2(45) - 25\cos 2(45)$$

$$= 15 \text{ N/mm}^2$$

Using mohr's circle :

Scale 1 cm = 10 N/mm²

$\sigma_1 = 6.5$ cm (AB)

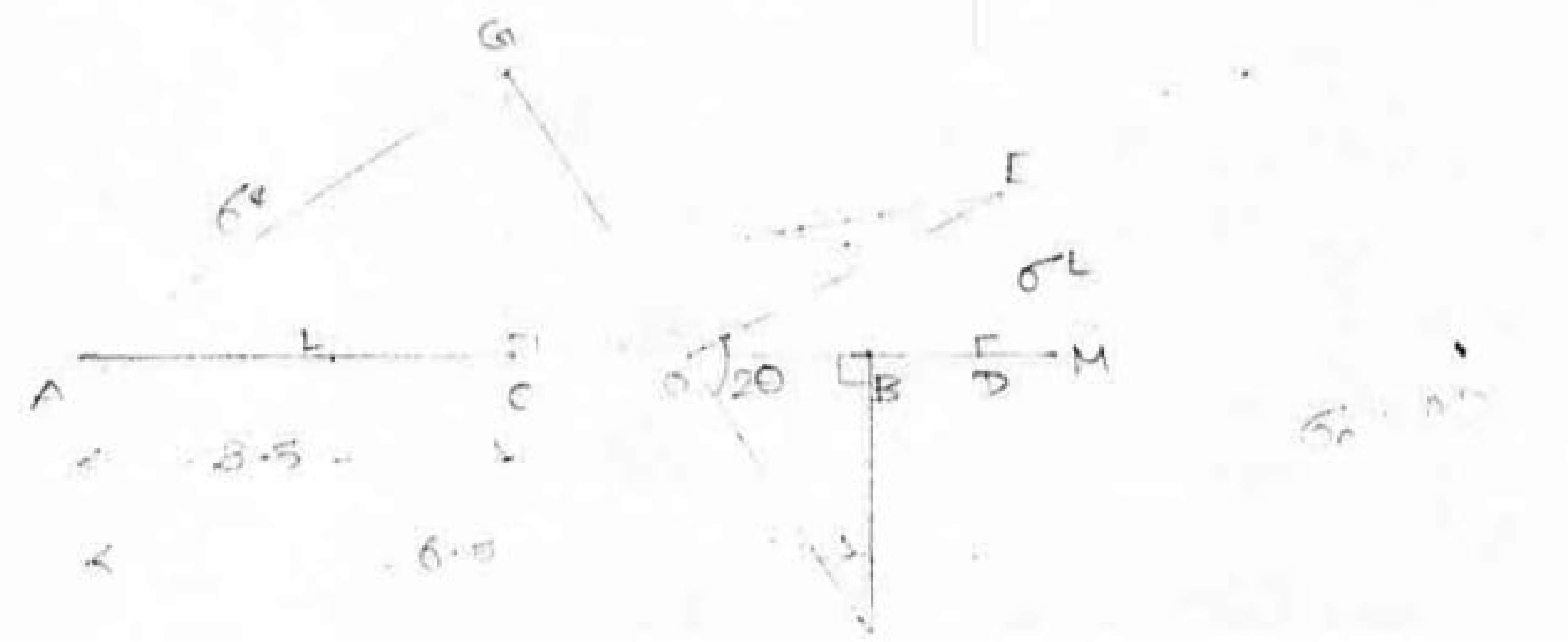
$\sigma_2 = 3.5$ cm

$\tau = 2.5$ cm.

AB : σ_1

AC : σ_2

Draw vertical lines from C, B with τ ...



Step 1 : Draw a line AB = σ_1 = 6.5 cm.

Step 2 : Draw a line AC = σ_2 = 3.5 cm from

3 : From points C and B draw vertical lines with the value of τ.

4 : Join the points G and F.

5 : The intersection pt is center for mohr's circle O.

6 : With GO (or) FO as radius draw the mohr's circle.

7 : Point O as center keep the protractor on the line GF mark angle 2θ

8 : Join line from pt: E perpendicular to AB. So ED gives σ_t (tangential stress)

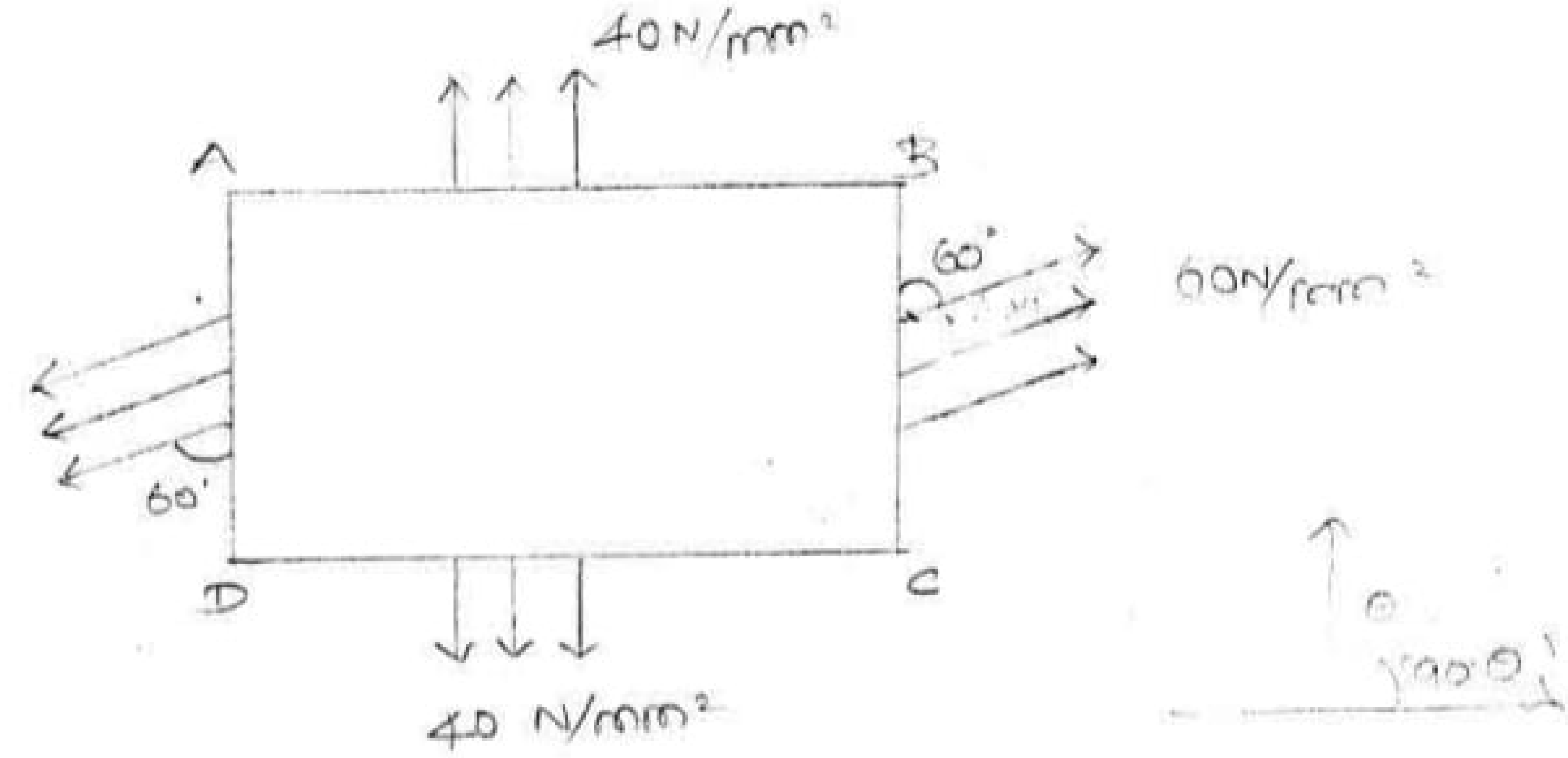
9 : Join the line AE = (Resultant stress)

10 : AD = Normal stress to oblique plane

σ_t = ED = 1.5 cm = 1.5×10 = 15 N/mm^2

σ_n = AD = 7.5 cm = 7.5×10 = 75 N/mm^2.

(2) A point in a strained material is subjected to stresses as shown in the figure. Locate the principle planes and evaluate the principle stresses. Find the maximum shear stress and the location of the plane which is acting.



SOLUTION :

Since the stress is inclined to the face BD (or) AD. Convert the inclined stresses in to normal and tangential to the face BC (or) AD.

Stress normal to the plane BC or AD

$= 60 \sin 60°$

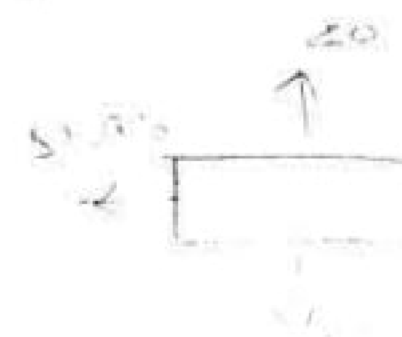
$= 51.96\ N/mm^2$

Tangential force BC

$= 60 \cos 60$

$= 30\ N/mm^2$

i. Location of principle plane :



$$\tan 2\theta = \frac{2\tau}{\sigma_1-\sigma_2}$$

$$\tan 2\theta = \frac{2 \times 30}{}$$

$$\theta = 39^\circ 36'$$

ii) Principle stresses :

$$\sigma_{max} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$\sigma_{min} = \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

iii) $$\sigma_{tmax} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\tau^2}$$

iv). Plane σ_{tmax} acts , $\tan 2\theta = \dfrac{\sigma_1-\sigma_2}{2\tau}$

- - - - - - - - - - - - - - - - - -

$$\sigma_{max} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{91.96}{2} + \sqrt{\left(\frac{11.96}{2}\right)^2 + 900}$$

$$= 45.98 + 30.59$$

$$= 76.57 \ N/mm^2$$

$$\sigma_{min} = 45.98 - 30.59$$

$$= 15.39 \ N/mm^2$$

$$\tan 2\theta = \frac{11.96}{60}$$

$$\theta = 5°38'$$

(3) Ans. $\sigma_2 = 600 \times 10^{-2}\ N/mm^2$

Resolving into stress normal to the face AB $= 800 \times 10^{-2} \cos 30°$

$= 6.92\ N/mm^2$

Stress tangential to face $= 4$

Location of principal plane.

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$= \frac{8}{\sigma_1 - \sigma_2} \Rightarrow \frac{8}{0.92}$$

$$\theta = 41°43'$$

Principal stress :

$$\sigma_{max} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= 6.46 + \sqrt{\left(\frac{0.92}{2}\right)^2 + 16}$$

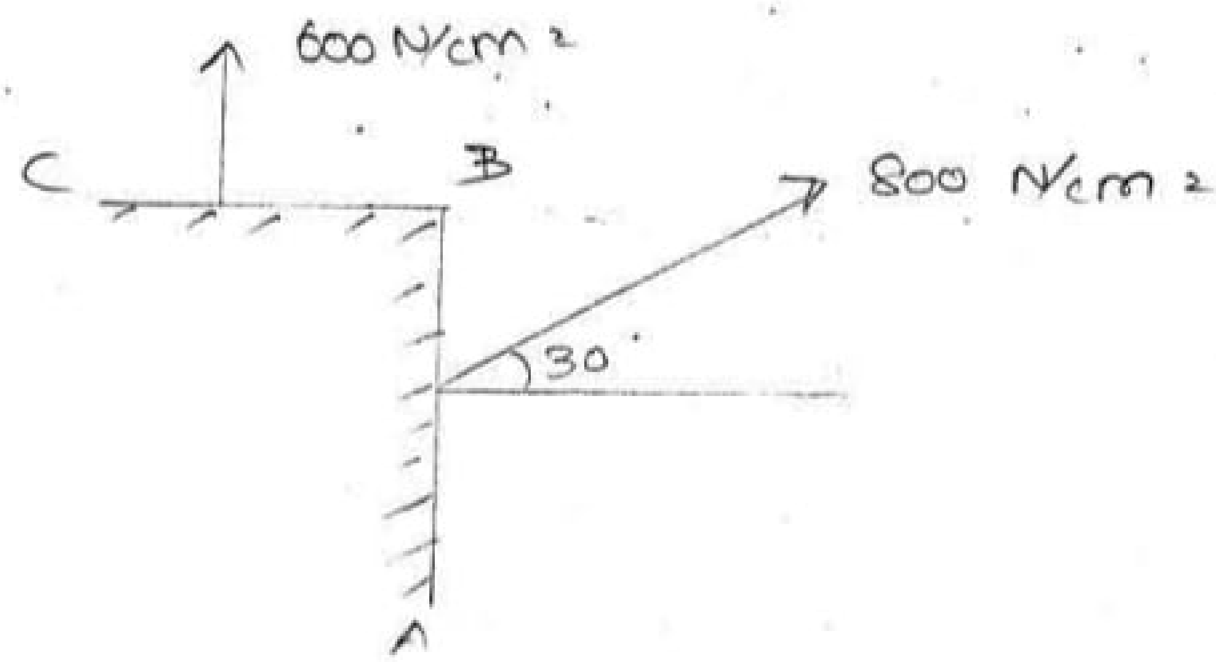
$= 4.02\ N/mm^2 + 6.46$

$= 10.48\ N/mm^2$ (Tensile)

σ_{min} $= 6.46 - 4.02$

$= 2.44\ N/mm^2$ (Tensile)

(3). The intensity of resultant stress on a plane AB at a point in a material under stress is 800 N/cm² and it is inclined at 30° to the normal to that plane as shown in the figure. The normal component of stress on other plane BC at right angle is 600 N/cm². Determine Resultant stress, Principal stress, Location of plane of principal stress and max shear stress.



$$\sigma_{tmax} = \frac{1}{2}\sqrt{(0.92)^2 + 64}$$

$$\sigma_{tmax} = 7.02 \text{ N/mm}^2$$

$$\tan 2\theta = \frac{0.92}{8}$$

$$= 3°\cancel{3}6'$$

$$\sigma_R = \sqrt{\sigma_t^2 + \sigma_n^2}$$

$$\sigma_R = 721.1$$

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UNIT - II

TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAMS

Beams :

A beam is a horizontal member which carries external load (super imposed load) and transmits or transfer load to the support.

TYPES OF BEAM :

1). Cantilever beam :

2). Fixed beam :

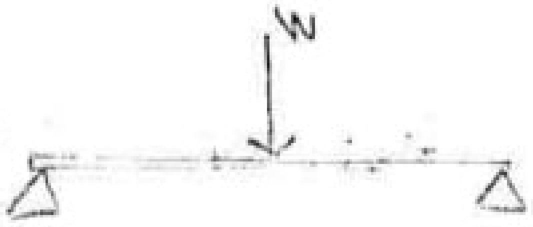
3). Simply supported beam:

4) Over hanging beam:

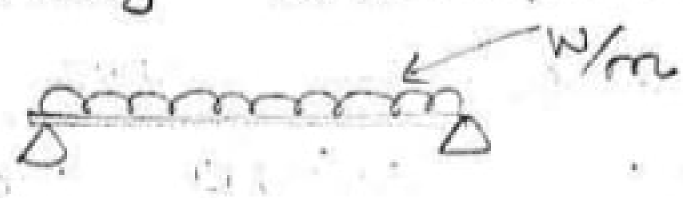
5). Continuous beam :

TYPES OF LOAD :

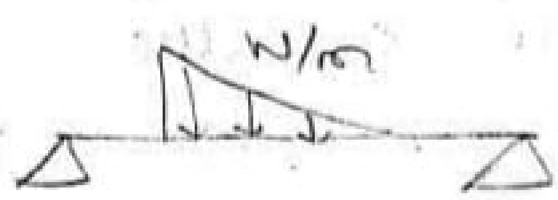
1) Point load (or) concentrated load :



2). Uniformly distributed load :



3). Uniformly varying load :



SHEAR FORCE & BENDING MOMENT DIAG :

Shear force is a force that is trying to shear off the section on which it acts and is obtained by algebric sum of all the forces including the reactions normal to the axis of the beam. The shear force diag shows the variation of shear force along the length of the beam.

Bending moment :

The moment that is trying to bend the beam and is obtained by the algebric sum of moments about the section of all forces including reaction acting on the beam.

(84)

Conditions for drawing shear force & Bending moment diag:

Condition I :

i) consider the left or right portion of the section.

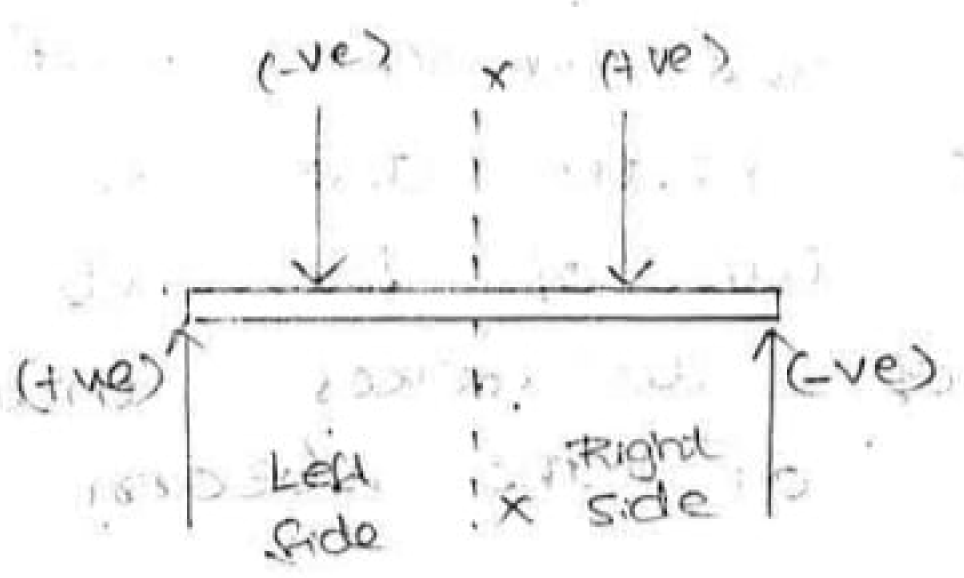
ii) Add the forces normal to the beam. The force on the right side of the section acting downward is positive. Force acting upward is -ve.

on the left side of the section force acting upward is +ve & downward is -ve.

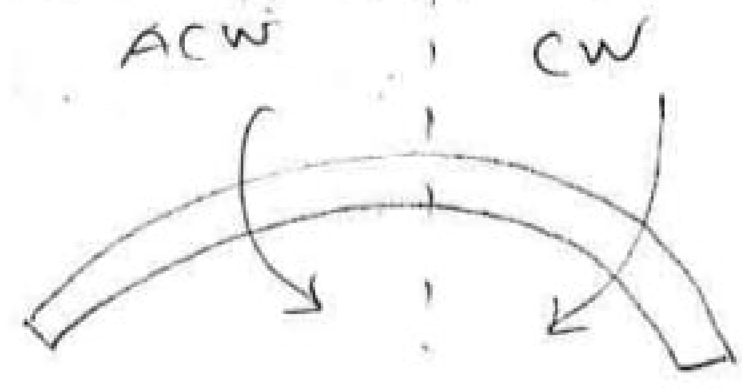
iii) The +ve values of shear force and bending moment are plotted above the base line and -ve value is plotted below the base line.

iv) The shear force between two vertical pt. load is constant and hence the shear force diag. between two vertical load will be horizontal.

v) The bending moment at the two supports of a simply supported beam and the end of the cantilever will be zero.

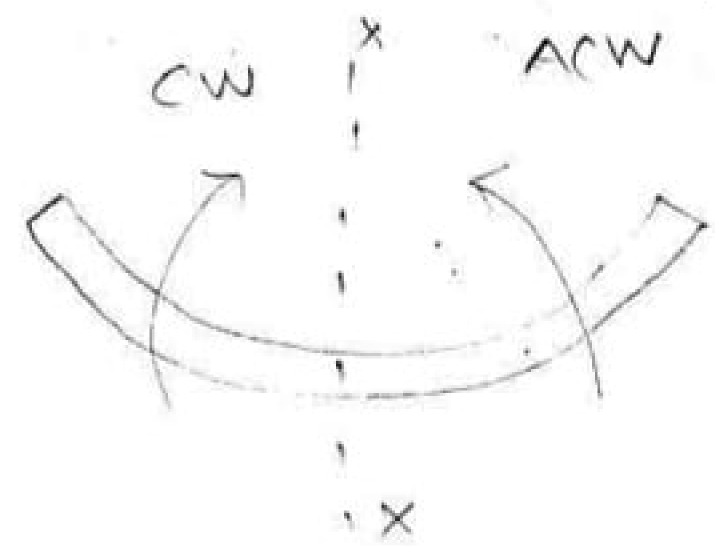


BM :



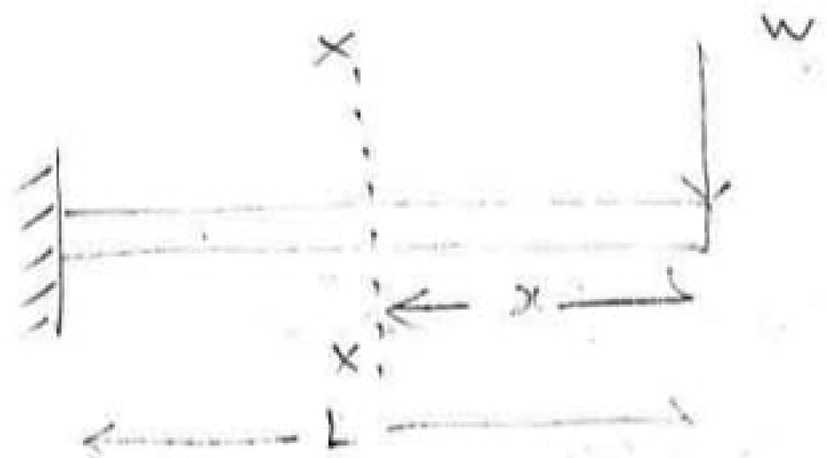
BM (-ve)

Hogging



Sagging

SHEAR FORCE AND BENDING MOMENT DIAG FOR CANTILEVER BEAM WITH POINT LOAD AT THE FREE END :



Shear Force at RHS :

S.F at x (↓(+ve))

$F_x = +W$

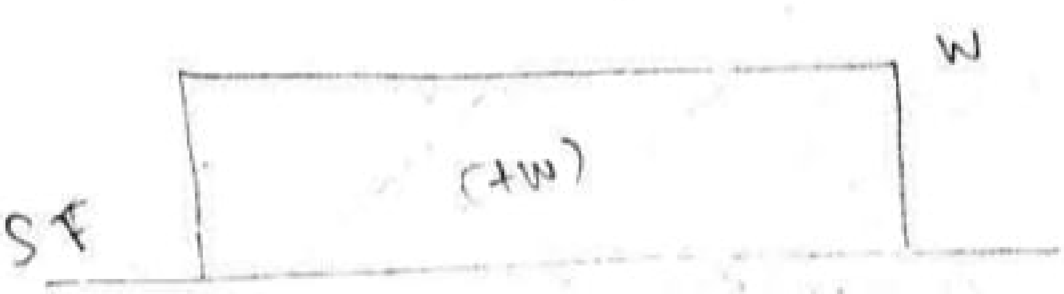
$BM_x = -W \times x$

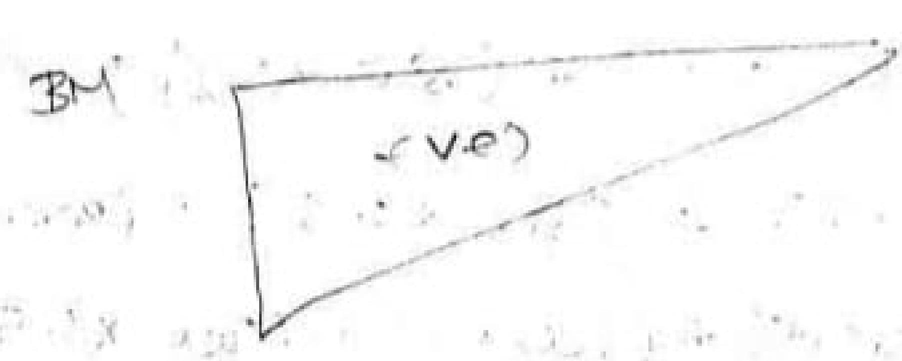
At $x = 0$

$BM = -W \times 0$

At $x = L$

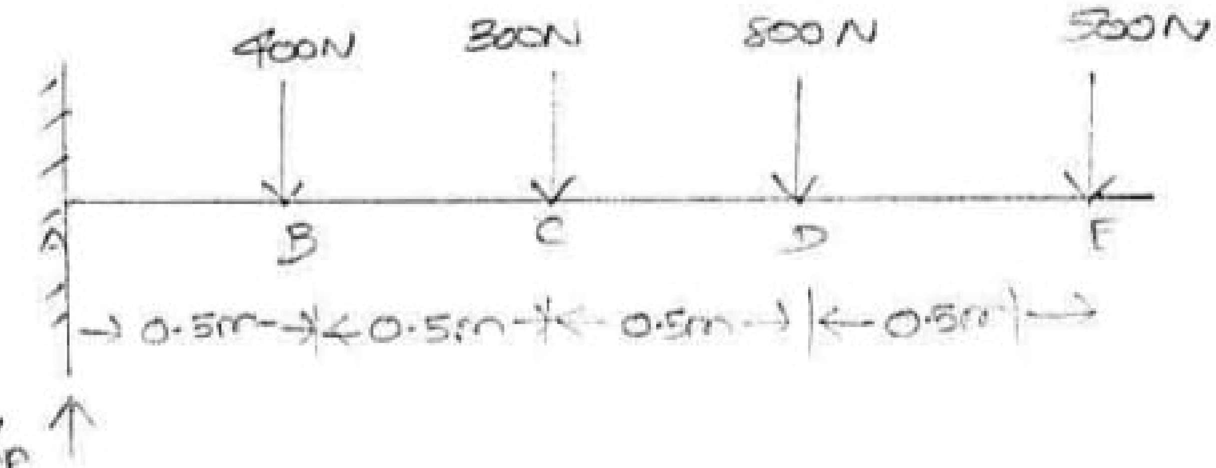
$BM : W \times L$





3/7/24 (88)

(1) Draw the shear force & bending moment diag for and cantilever beam 2m long subjected to forces as shown in the fig.



S.F
RHS
all downward forces are +ve
all upward be are -ve

$-R_A + 400 + 300 + 800 + 500 = 0$

$R_A = 2000\,N$

Shear force calculation :

SF at E = 500N

S.F at D = 500 + 800 = 1300N

S.F at C = 1300 + 300 = 1600 N

S.F at B = 1600 + 400 = 2000N

S.F at A = -2000 + 2000 = 0 N

Bending moment calculation :

BM at E = - 500 × 0 = 0

BM at D = $-[(500 \times 0.5) + (800 \times 0)] = -250$ N.m

BM at C = $-[(500 \times 1) + (800 \times 0.5) + (300 \times 0)] = -900$ Nm

BM at B = $-[(500 \times 1.5) + (800 \times 1) + (300 \times 0.5) + (400 \times 0)]$

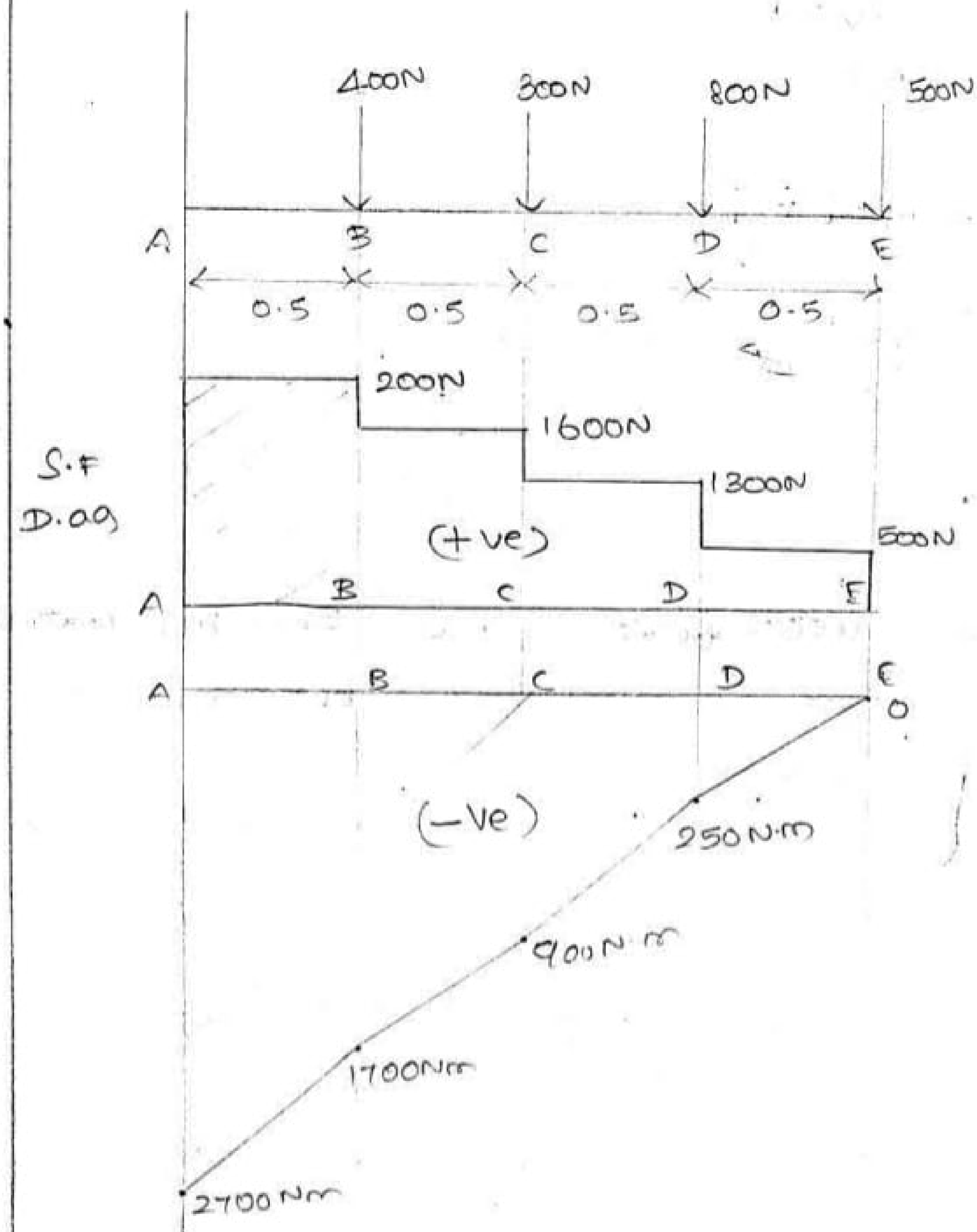
= -1700 Nm

BM at A = $-\{+(2000 \times 2) - [($

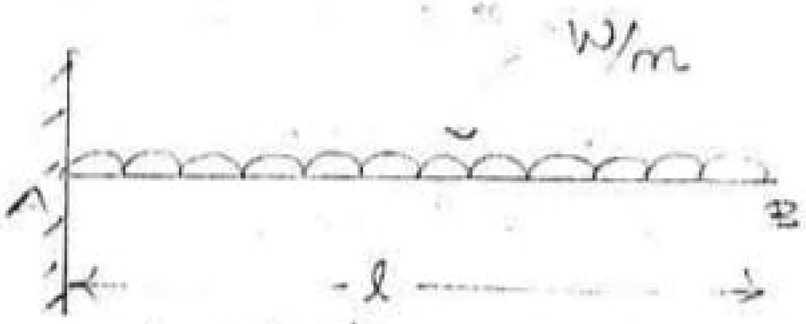
$-[(500 \times 2) + (800 \times 1.5) + (300 \times 1) + (400 \times 0.5)]$

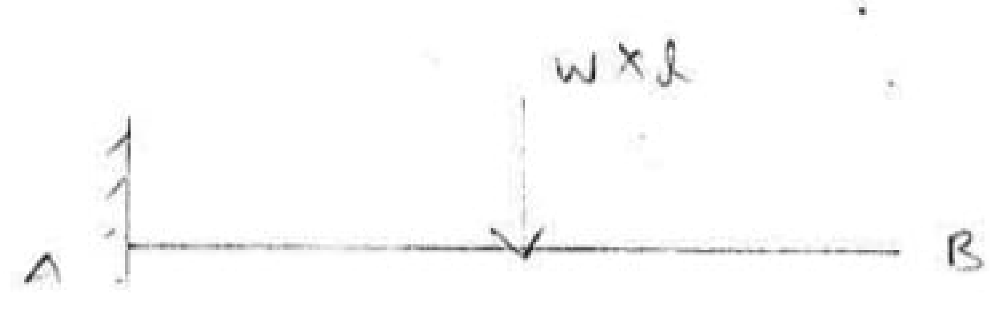
$+ 2000 \times 0$

= -2700 N-m



CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTED LOAD (UDL):



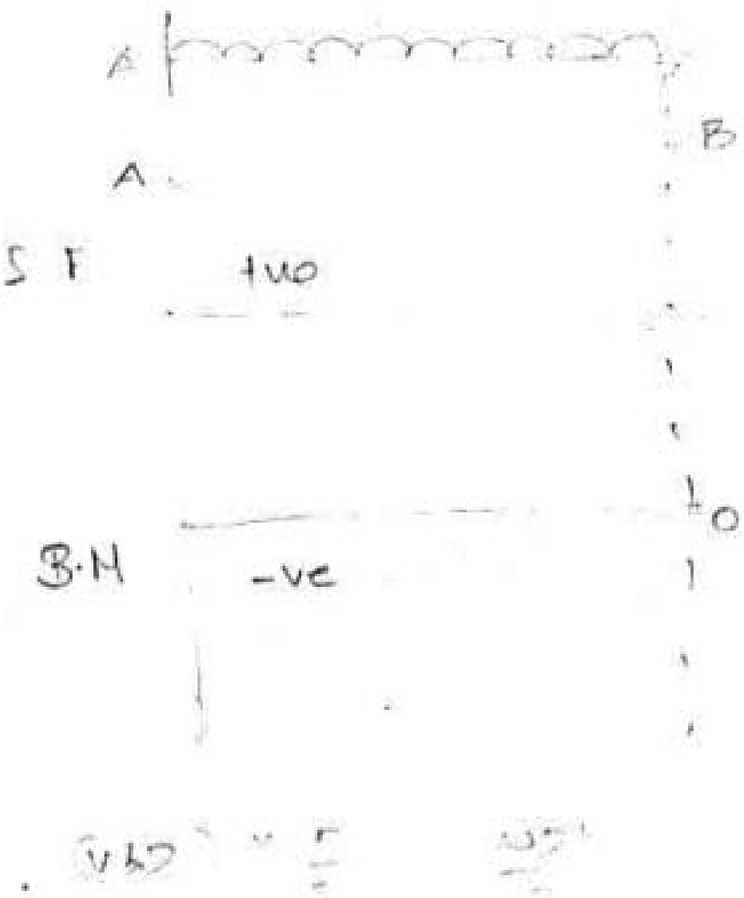


$S.F_A = w \times l$

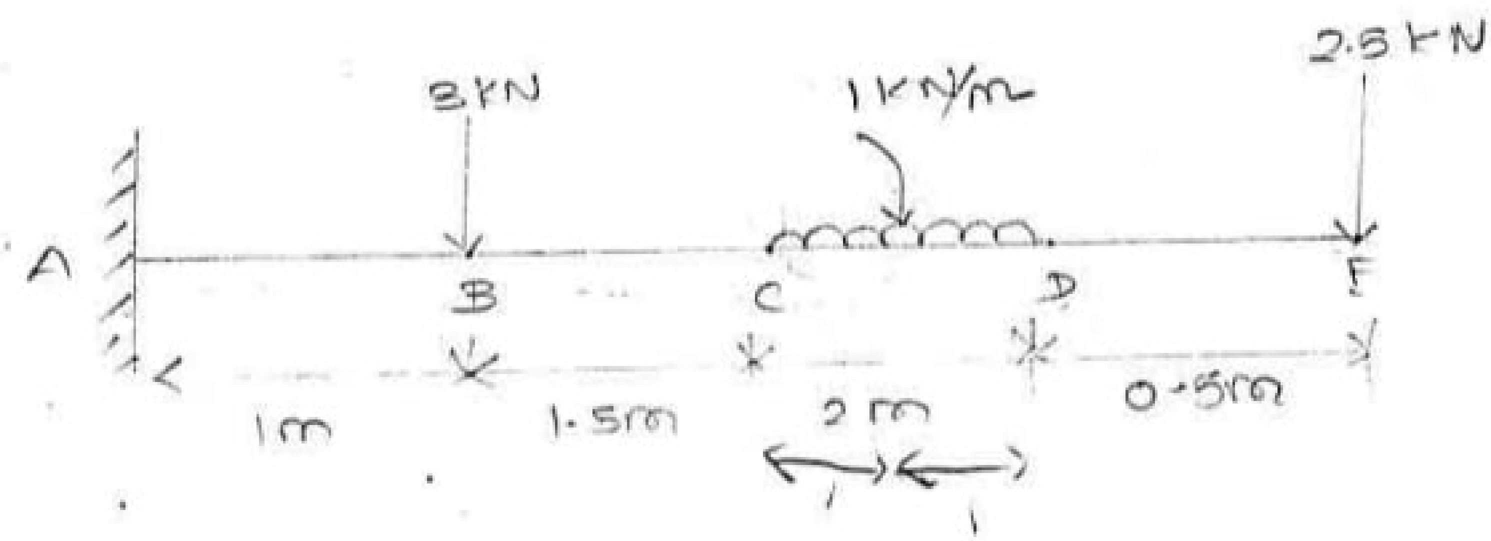
$= wl$

$B.M_A = w \times l \times \frac{l}{2}$

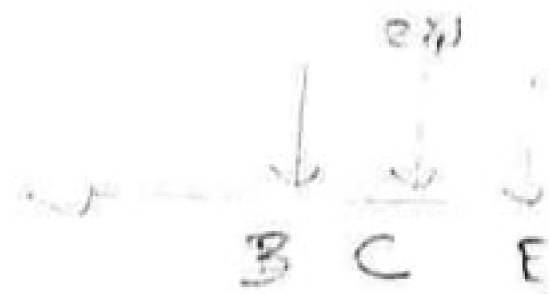
$= \frac{wl^2}{2}$



1. Draw the shear force and Bending moment diag for the cantilever beam as shown in fig.



Solution :



$R_A = 2.5 + (1 \times 2) + 3$

$= \cancel{6}\ 7.5\ KN$

S.F Diagram calc :

S.F at E = $2.5 \cancel{\times 0} = \cancel{0}$ 2.5 KN

S.F at D. = $0 + 2.5$ KN =

S.F at C = $2.5 + (1 \times 2)$ =

S.F at B = $2.5 + (1 \times 2) + 3$ =

S.F at A = 0

BM calculation:

BM at E $= 2.5 \times 0 = 0$

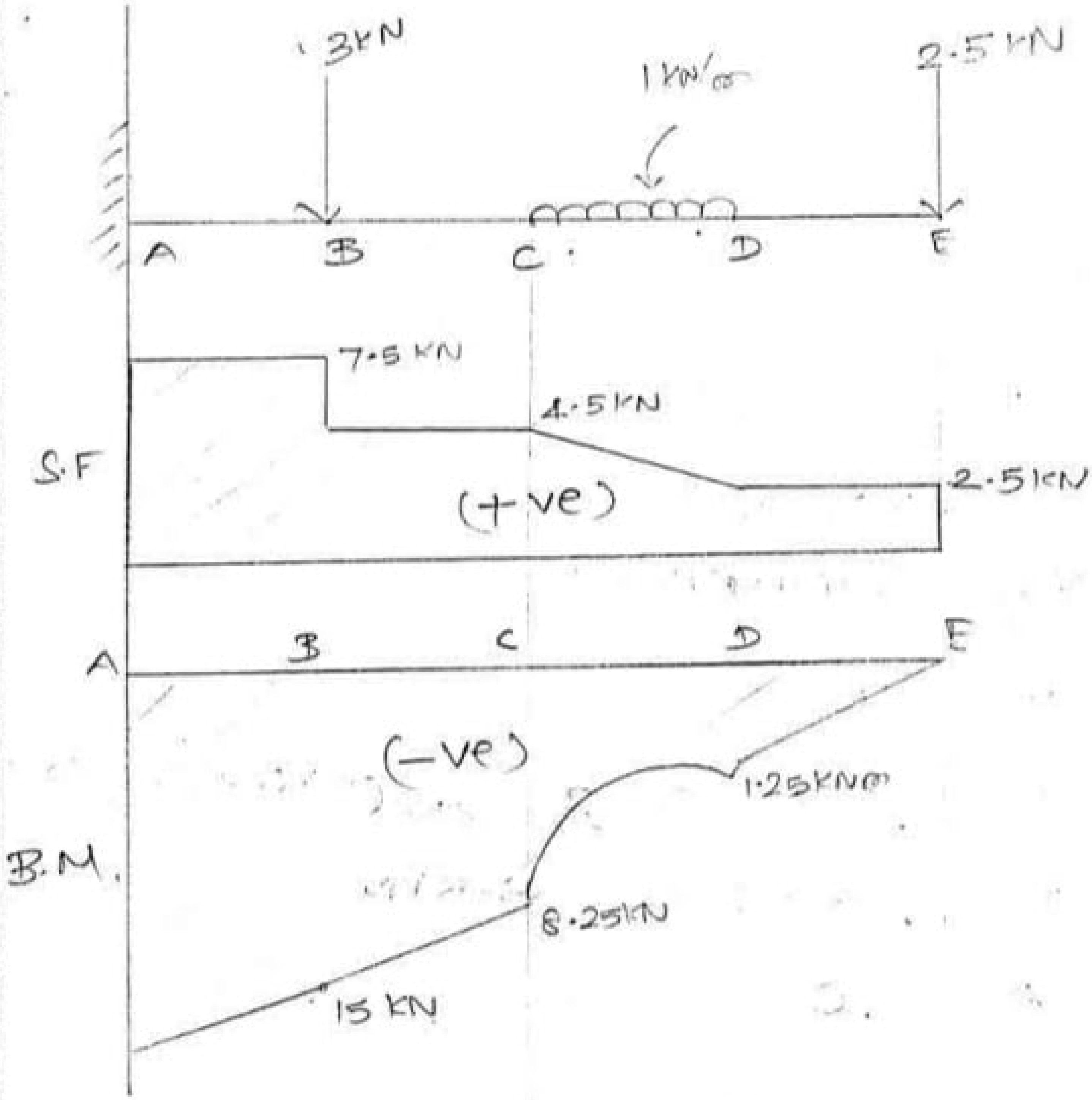
BM at D $= -[(2.5 \times 0.5) + (0)] = -1.25$

BM at C $= -[(2.5 \times 2.5) + (2 \times 1)] = -8.25$

BM at B $= -[(2.5 \times 4) + (2 \times 2.5)] = 15$

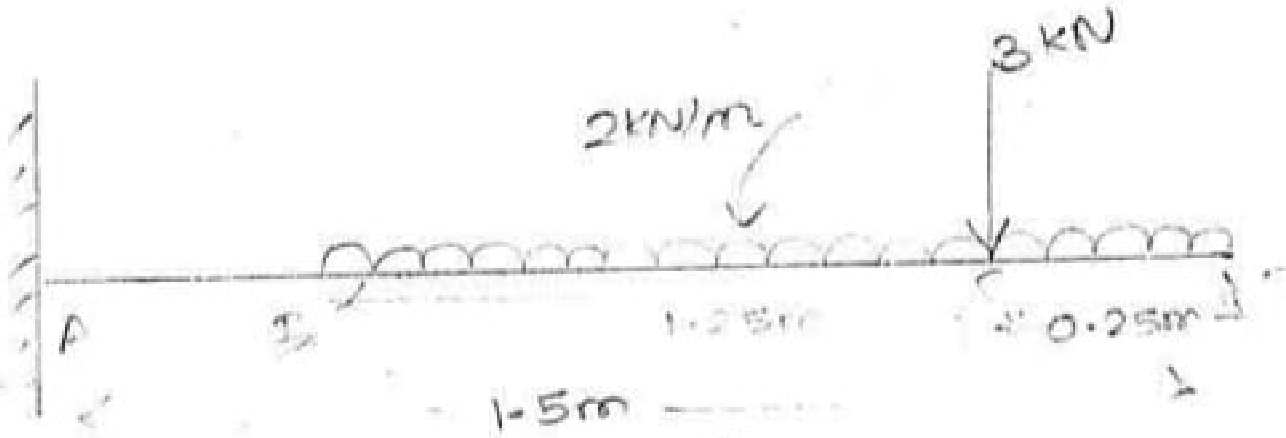
BM at A $= -[(2.5 \times 5) + (2 \times 3.5) + (3 \times 1)] + 7.5$

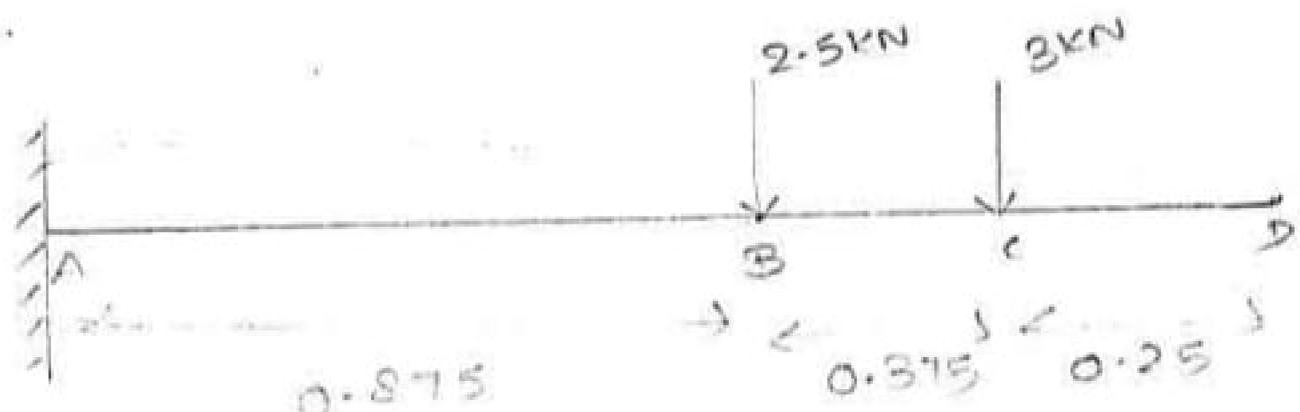
$= \cancel{-15}\ -22$



(90)

(2). A cantilever beam 1.5m long is loaded with uniformly distributed load of 2KN/m run over a length of 1.25m from the free end. it also carries a point load of 3KN at a distance of 0.25m. from the free end. Draw the shear force and B.M diag. for the beam.





Shear forc diag calculation:

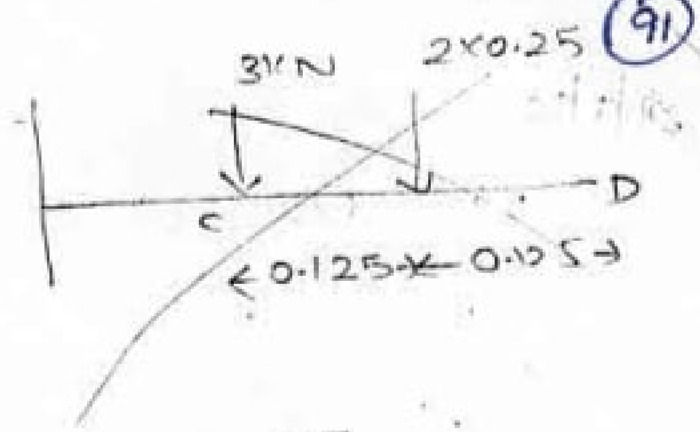
S.F at D = 0

S.F at C = 3KN + ~~2 = 3 KN~~ (2×0.25) = 3.5

S.F at B = 3 + 2.5 = <u>5.5KN</u>

S.F at A = 0

B.M at A: $(2\times1.25)\left(\frac{1.25}{2} + 0.25\right) + (3\times1.25)$



BM calc :

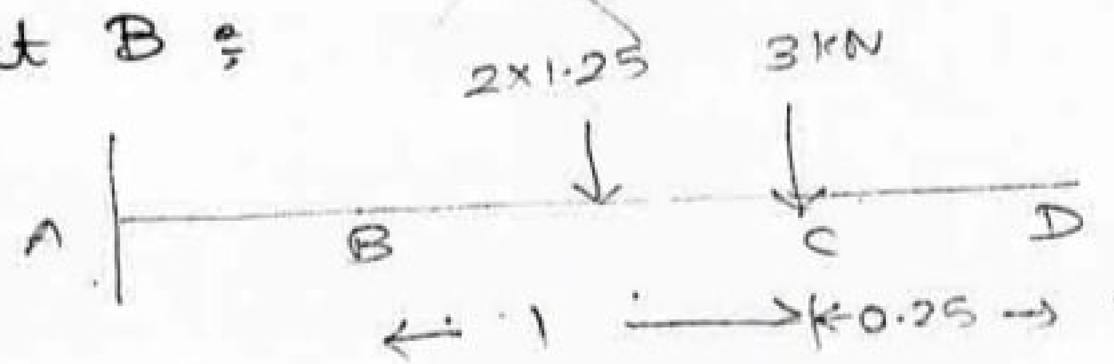
BM at D = 0

BM at C $= -[(3\times0) + (2\times0.25\times0.125)]$

$= -0.0625$ KN-m

$\left(2\times0.25\times\frac{0.25}{2}\right)$

BM at B :



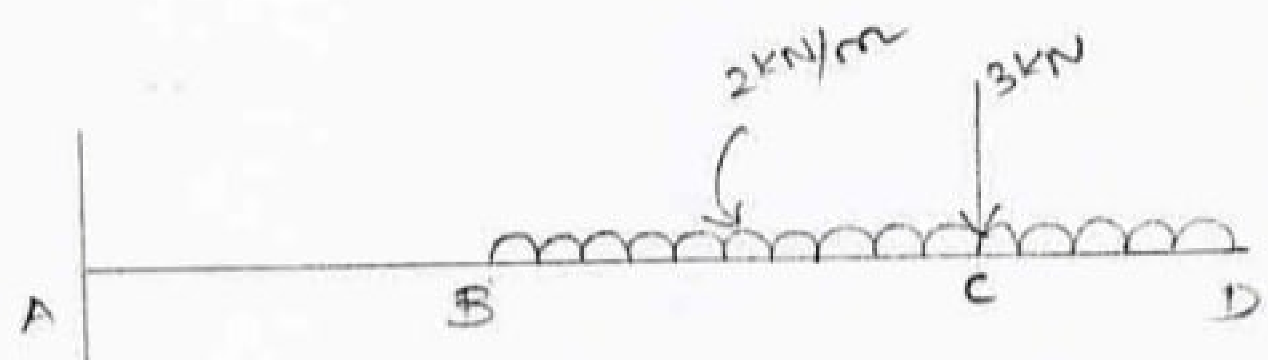
$= -[(3\times1) + (2\times1.25\times0.625)]$

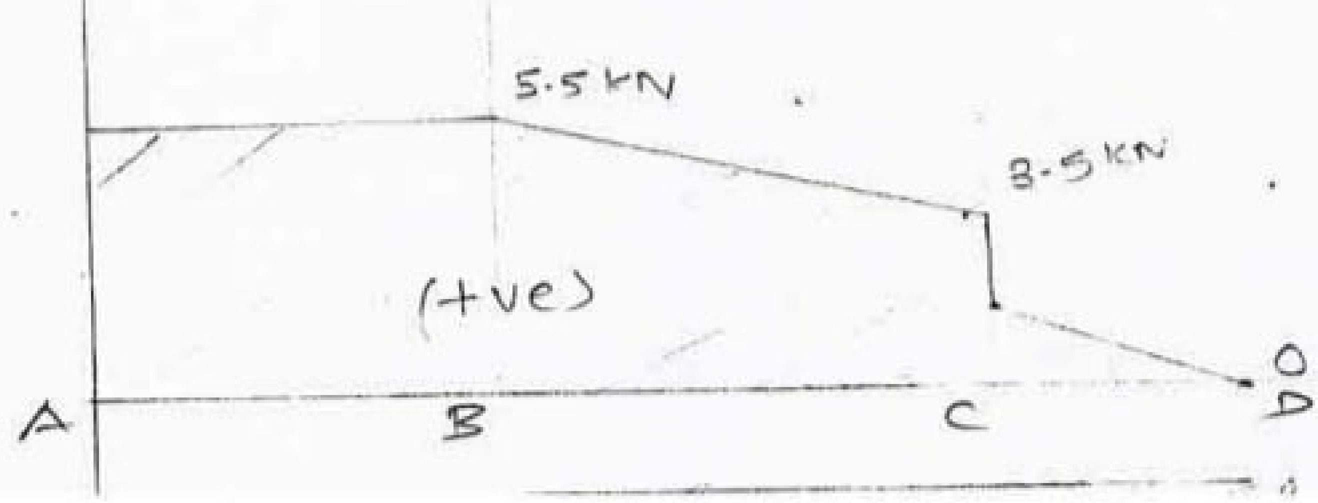
$= -4.56$ KN-m.

BM at A :

$= -[(3\times1.25) + (2\times1.25\times0.875)]$

$= -5.91$ KN.m.





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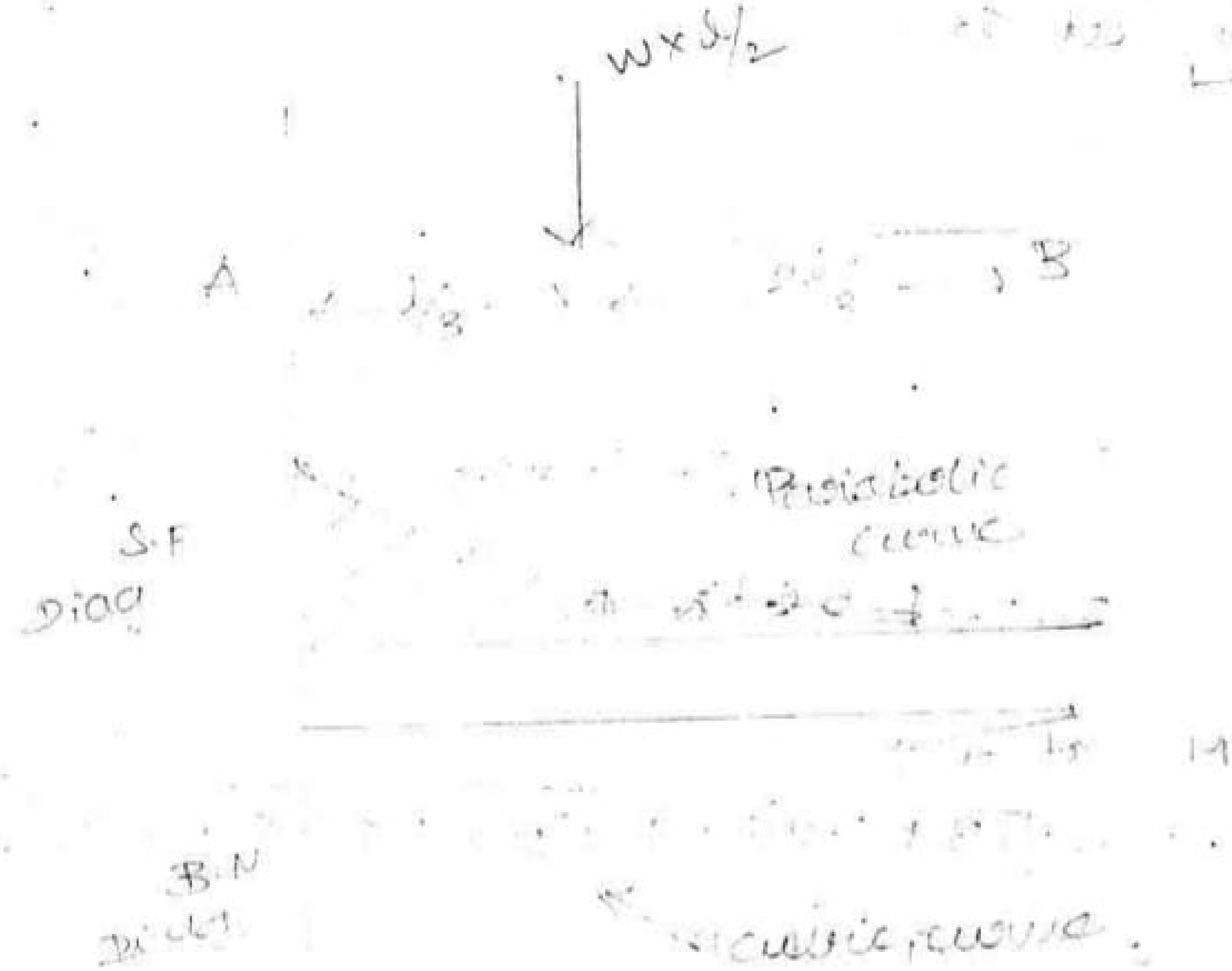
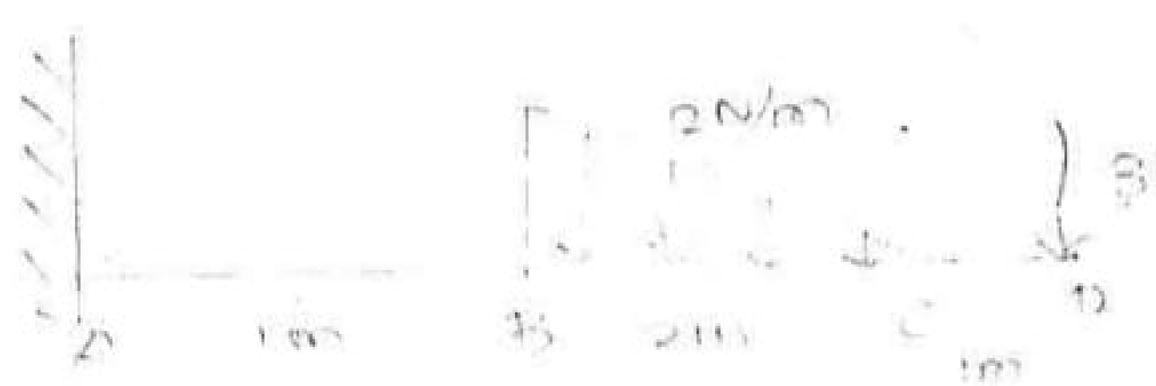
CANTILEVER BEAM WITH UNIFORMLY VARYING LOAD :

$\frac{1}{2} \times (w \times l)$



① Draw the shear Force & B.M diag for the given cantilever beam

$(2 \times 2)/2$

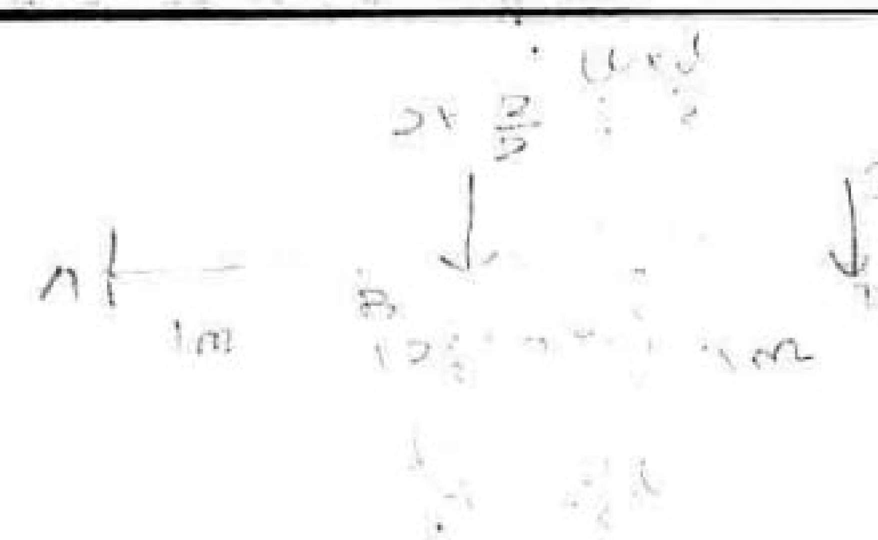


S.F calculation :

S.F at D = 3N

S.F at C = 3N

S.F at B = 3N + $2 \times \frac{2}{2}$ → $w \times \frac{l}{2}$



B.M :

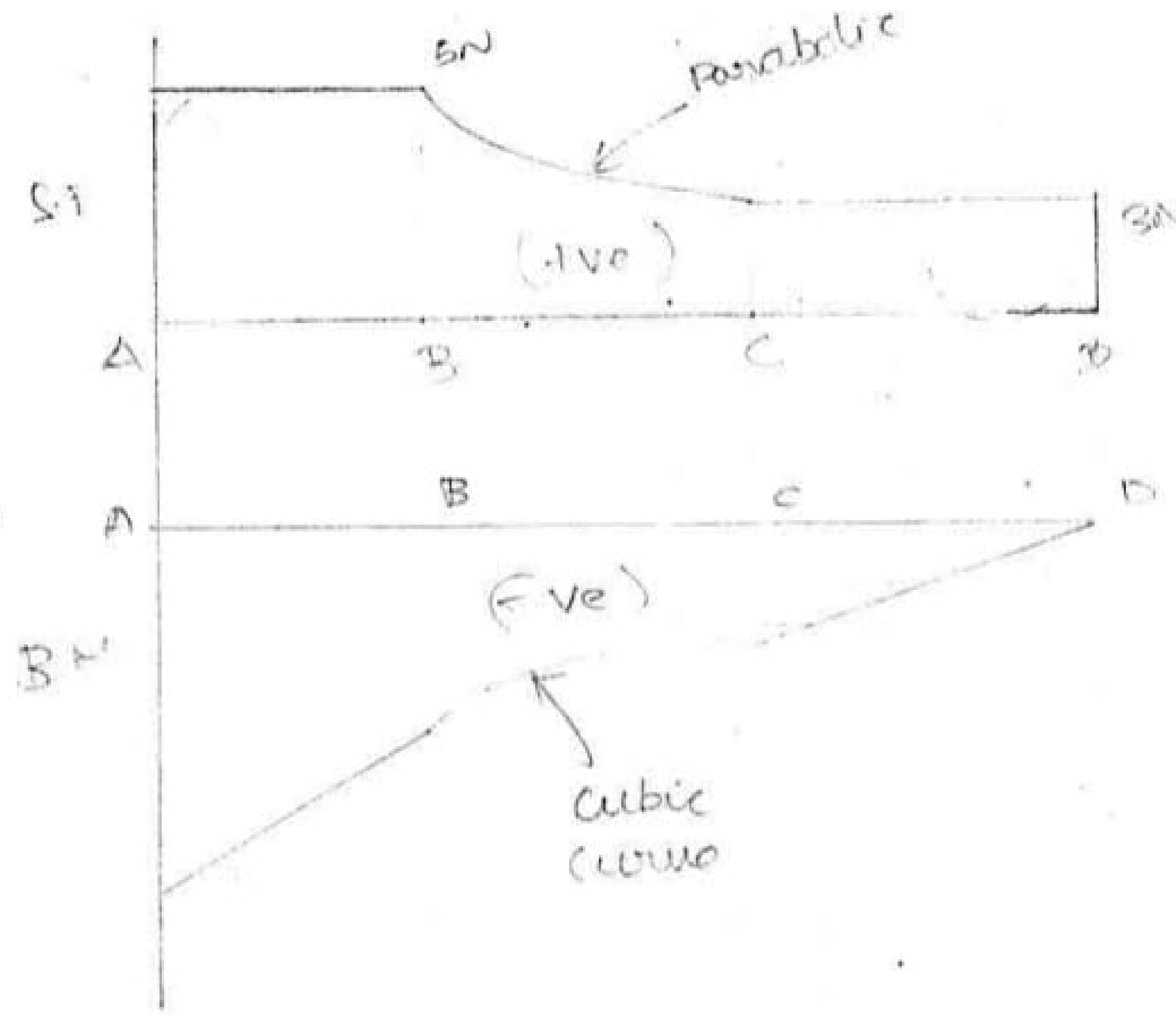
B.M at D = 0

B.M at C = $-1 \times 3 = 3$ N-m

B.M at B $= -3 \times 3 + \left(\left(2 \times \frac{2}{2}\right) \times \frac{2}{3}\right) = -10.33$ N-m

BM at A $= -(4 \times 3) + \left[2 \times \frac{2}{2} \times \left(1 + \frac{2}{3}\right)\right]$

$-(5 \times 3) - \left(\frac{1}{2} \times 2 \times 2\right)\left(\frac{1}{3} \times 2\right)$ $= -15$ N-m



(2). A simply supported beam carrying point load at the midpoint

(9/4)

Sol:

For equilibrium condition:

i) Sum of upward force = Sum of downward force

$$R_a + R_b = W$$

ii) Taking moment about one support.

Taking moment about A.

$$R_b \times l - W \times \frac{l}{2} = 0$$

$$R_b l = \frac{Wl}{2}$$

$$R_b = \frac{W}{2}$$

$$R_a + R_b = W$$

$$R_a + R_b = W$$

$$R_a = \frac{W}{2}$$

S.F at B $= -W/2$

S.F at C $= W - W/2 = W/2$

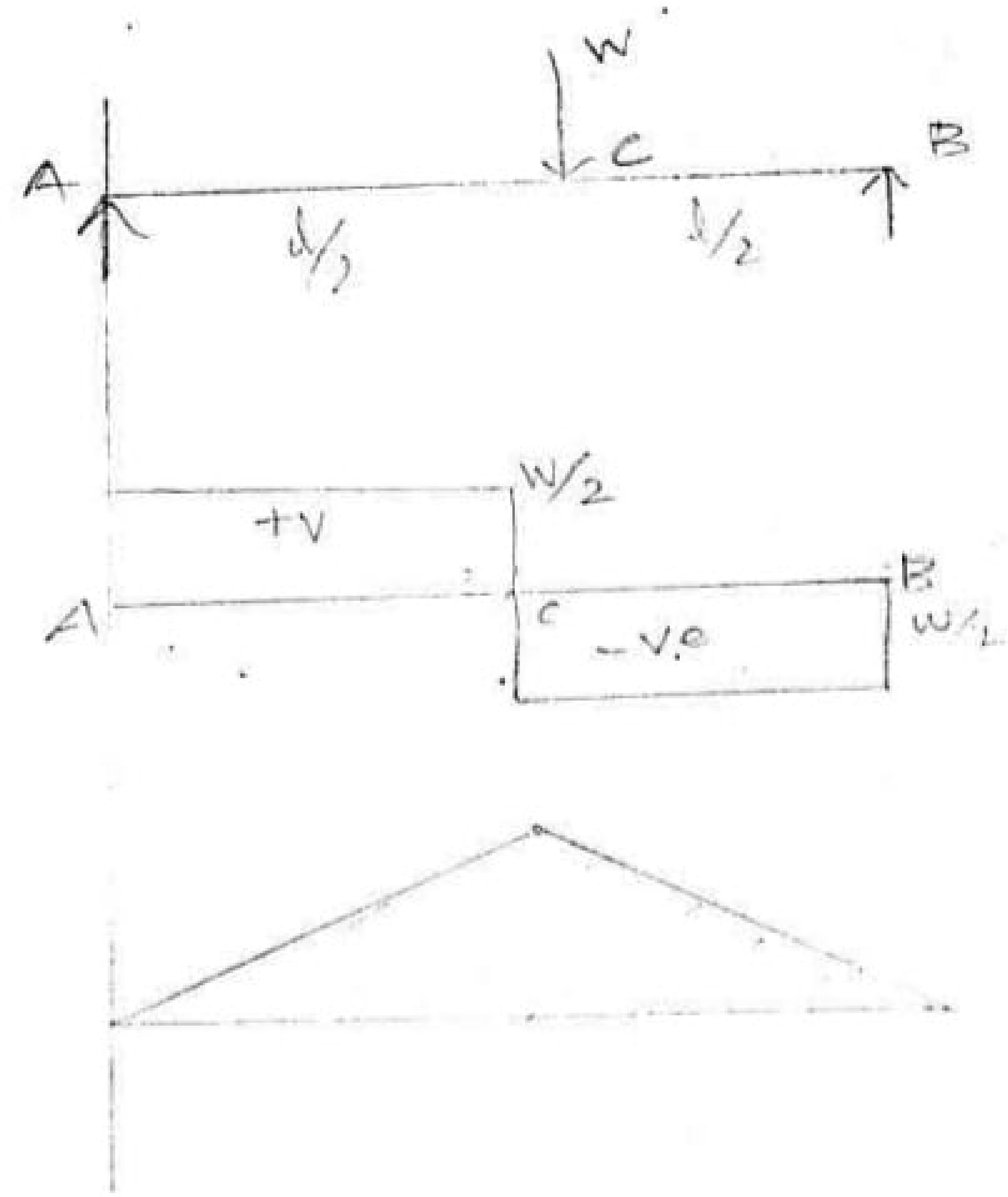
S.F at A $= W/2 - W/2 = 0$

B.M at B $= 0$

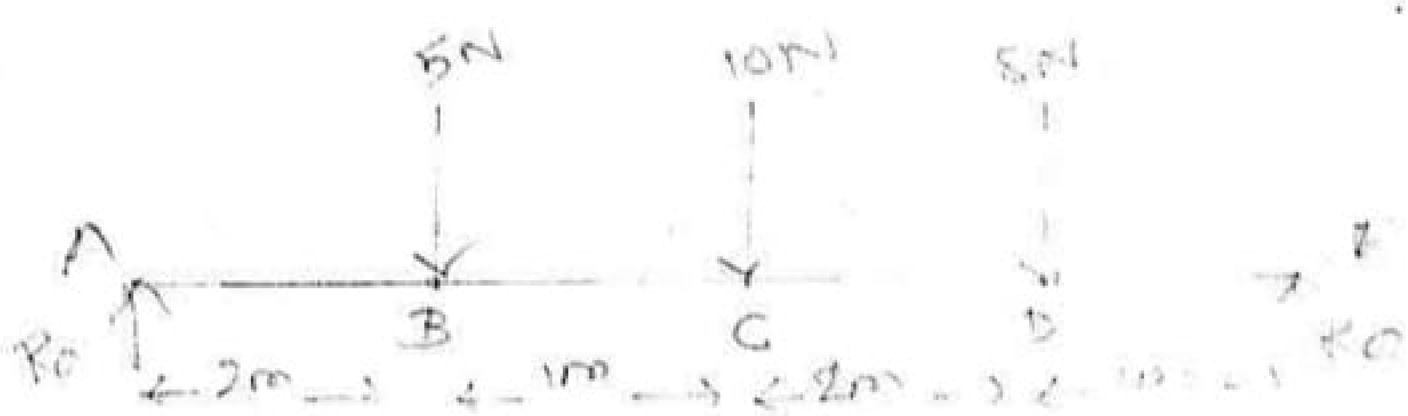
B.M at C $= \frac{W}{2} \cdot \times \frac{l}{2}$

B.M at A $= \frac{W}{2} \times l - W \times \frac{l}{2}$

$= 0$



(3) Draw the S.F & B.M diag for the simply supported beam as shown in the figure.



Solution:

To find Ra, Re:

under equilibrium condition.

$$R_a + R_e = 5 + 10 + 8$$

$$R_a + R_e = 23$$

Taking moment about one support.

Taking moment about A.

R. $\quad + R_e \times 6 = 0$

$R_a + 13.3 = 23$

$R_a = 9.6$ N

S.F at E $= -13.3$N

S.F at D $= -5.3$N

S.F at C $= 10 + 8 - 13.3 = -3.3$

S.F at B $= 10 + 8 + 5 - 13.3 = 5.2$

S.F at A $= 0$

B.M at E $= 0$

B.M at D $= (13.3 \times 1) - (8 \times 0)$

$= \cancel{5.3}\ 13.3$

B.M at C $= (13.3 \times 3) - (8 \times 2) - (10 \times 0)$

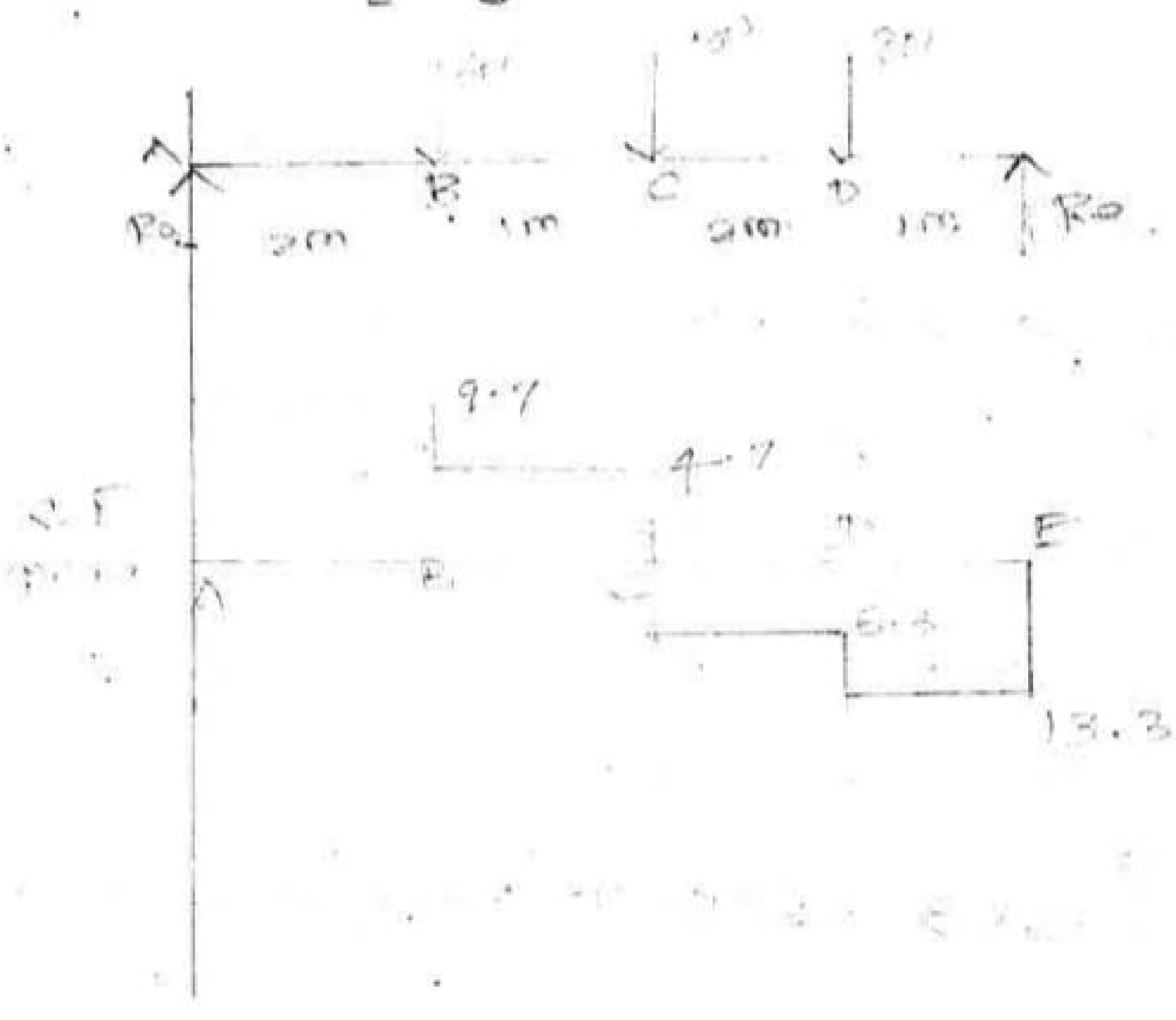
$= 23.9$

B.M at B $= (13.3 \times 4) - (8 \times 3) - (10 \times 1) - (5 \times 0) =$

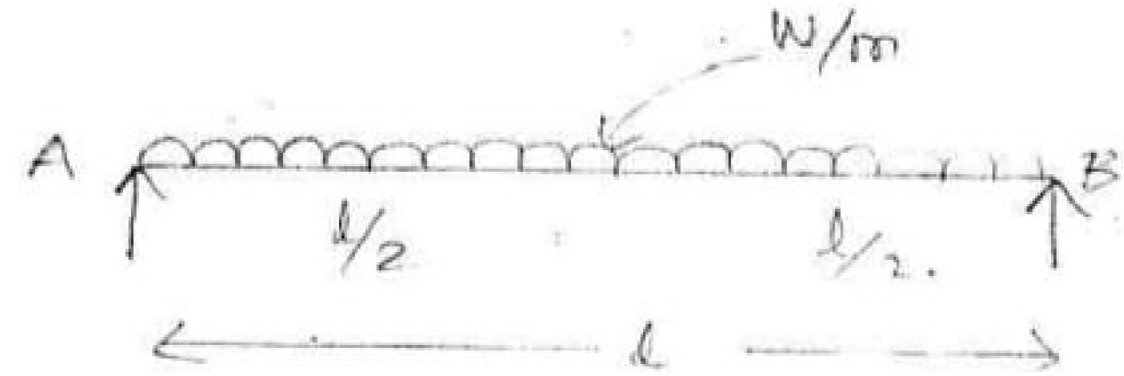
$= 19.32$

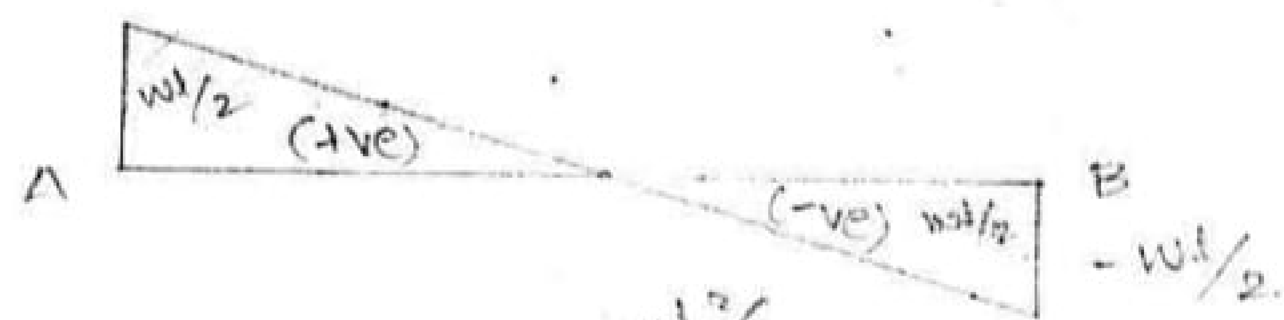
B.M at A $= (13.3 \times 6) - (8 \times 5) - (10 \times 3) - (5 \times 2)$

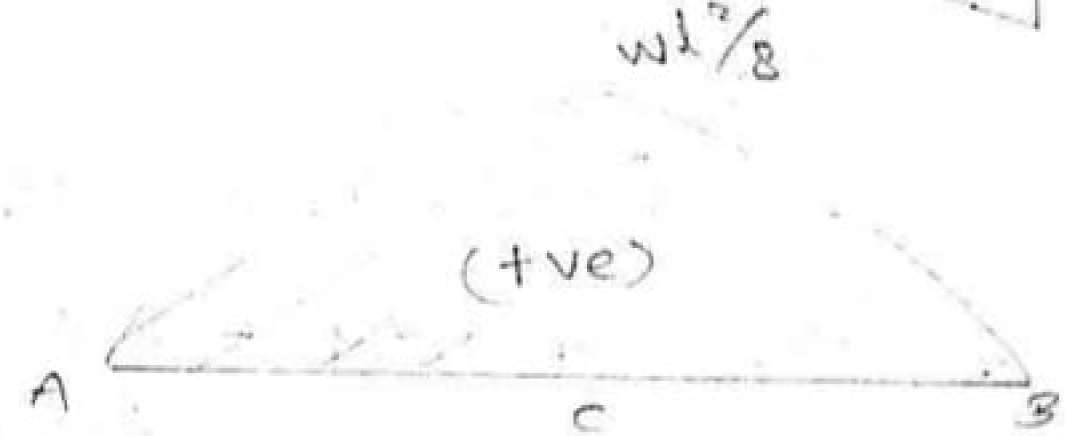
$= 0$



4. SIMPLY SUPPORTED BEAM CARRYING UNIFORMLY DISTRIBUTED LOAD :







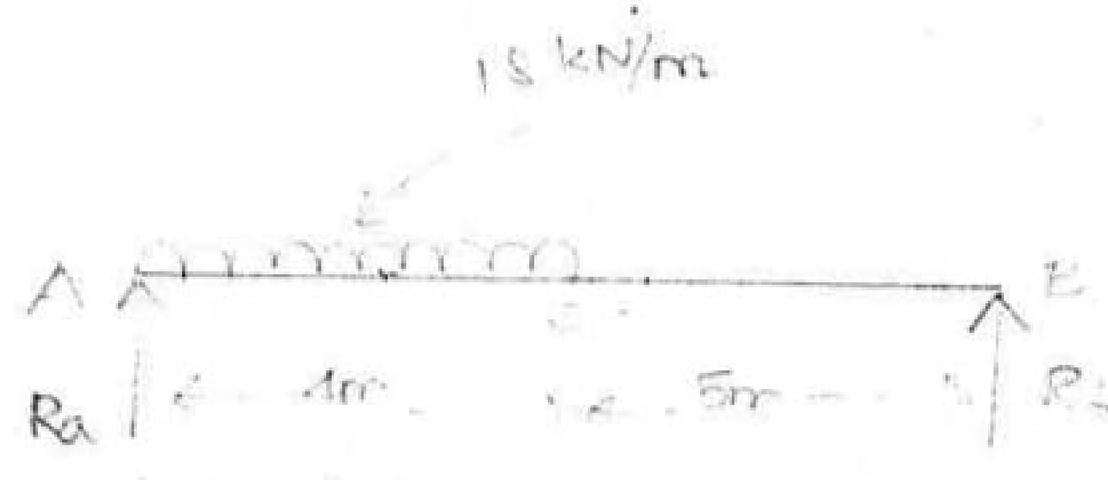
$$R_a + R_b = wl$$

$$R_a = wl/2$$

$$R_b = wl/2$$

$$\text{Max. B.M} = wl^2/8$$

4



Find Ra, Rb.

$$R_a + R_b = 18 \times 4.$$

Moment about A,

$$R_b \times 9 - (18 \times 4) \times \frac{4}{2} = 0$$

$$\boxed{R_b = 16\ KN}$$

$$\therefore R_a + 16 = 72$$

$$\boxed{R_a = 56\ KN}$$

S.F calculation:

S.F at B := -16 KN

S.F at C = $-16 + (18 \times 0) = -16$ KN

S.F at A without pt load $= -16 + (18 \times 4)$
$= 52$ KN

S.F at A with pt load $= -16 \times (18 \times 4) - 56$
$= 0.$

Bending Moment calc:

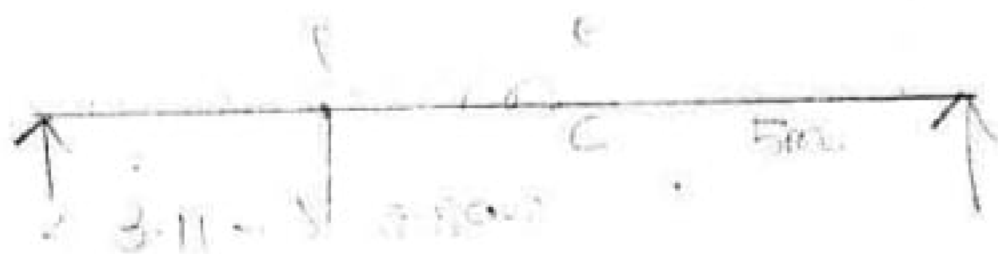
Since because of UDL, the shearforce value from -16 to $+56$ changes and at some point the shear force is zero because of the UVL. The distance x at which the shearforce is zero need to be found.

To find x where S.F is zero

$$-56 + 18 \times x = 0$$
$$18x = 56$$
$$x = 56/18$$

$$\boxed{x = 3.11\ m}$$

B.M :



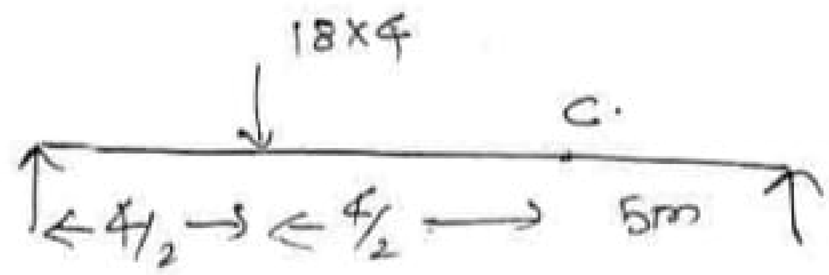
BM at B = 0

BM at C = $16 \times 5 = 80$ N m

BM at E =

$= 16 \times 5.89 - (18 \times 0.89 \times \frac{0.89}{2}) \Rightarrow 87.11$

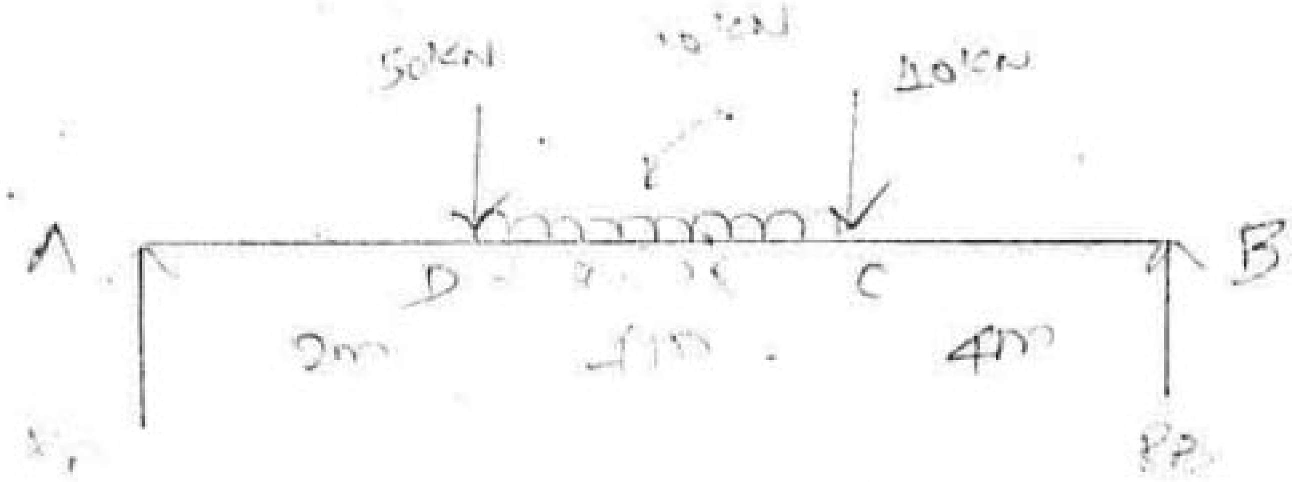
BM at A =



$= (16 \times 9) - (18 \times 4 \times \frac{4}{2})$

$= 0$

(5) (100)



Solution :

To find R_a, R_b :

$R_A + R_B = 50 + 40 + (10 \times 4)$

$R_A + R_B = 130$

Moment about A,

$R_B \cancel{\times} - (18 \times A) \times \frac{\cancel{4}}{\cancel{2}} = 0$

$10 R_B - (40 \times 6) - (50 \times 2) - ((10 \times 4) \times 4) = 0$

$10 R_B - (240) - (100) - (160) = 0$

$10 R_B = \cancel{460 \text{ KN}}\ 500$; $\boxed{R_B = 50 \text{ KN}}$

$R_A + \overset{50}{\cancel{460}} = 130$

$\boxed{R_A = 80 \text{ KN}}$

S.F calculation :

S.F at B $= -50$ KN

S.F at C $= -50 + 40 = -10$ KN

S.F at D $= -50 + 40 + 50 + (10 \times 4) =$
with pt load. $= 80$ KN

S.F at $\cancel{A}$ $= \cancel{0}\ -50 + 40 + (10 \times 4) = 30$ KN
without pt load

S.F at A $= 0$

~~Bending moment~~ :

TO find x :

$$-80 + 50 + 10 \times x = 0$$

$$10x = 30$$

$$\boxed{x = 3m}$$ from left.

Bending moment :

Bending moment at B = 0

B.M at C $= 50 \times 4 = 200$

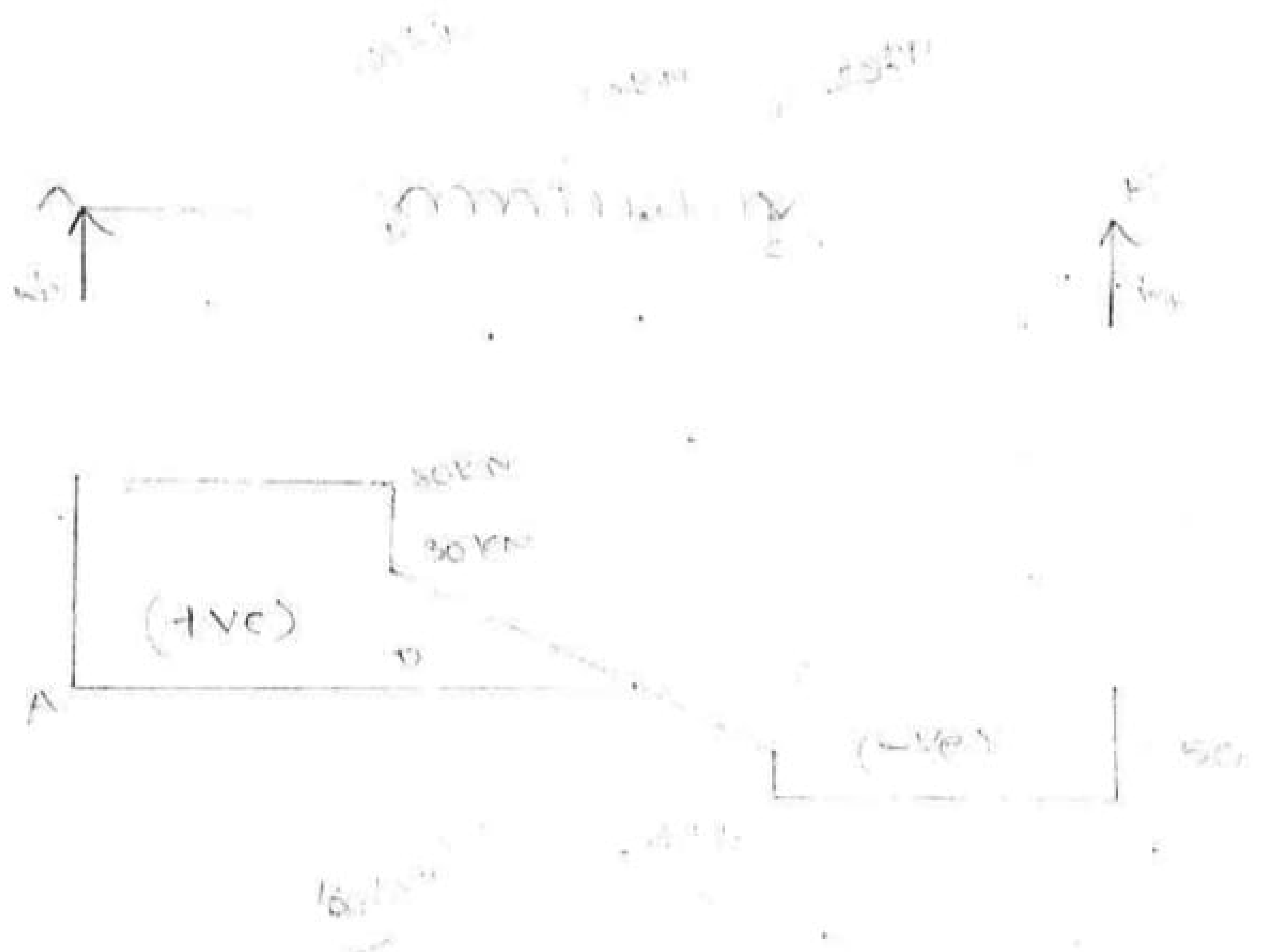
B.M at E $= (50 \times 5) - (40 \times 1) + (10 \times 1 \times 0.5)$

$= \cancel{210}\ 205$

B.M at D $= (50 \times 8) - (40 \times 4) - (10 \times 4 \times 2) - (50 \times 0)$

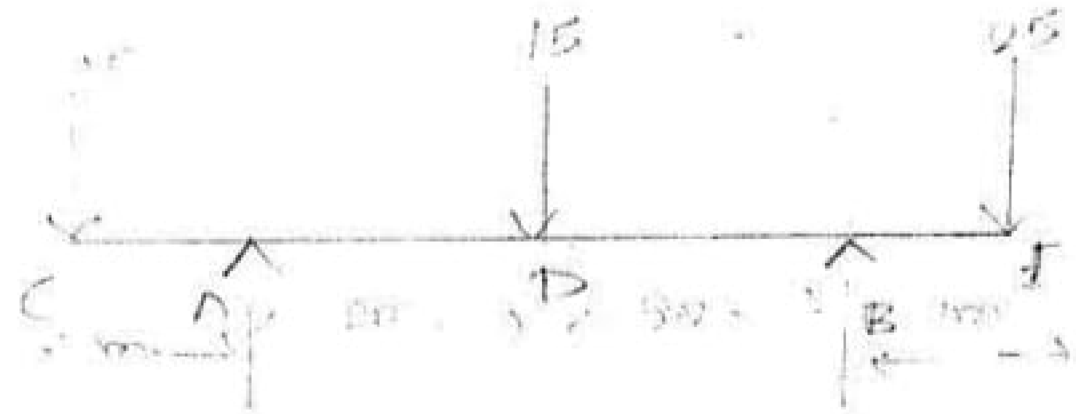
$= 160$

B.M at A $= 0$



OVER HANGING BEAM :

(1)



TO find Resultant,

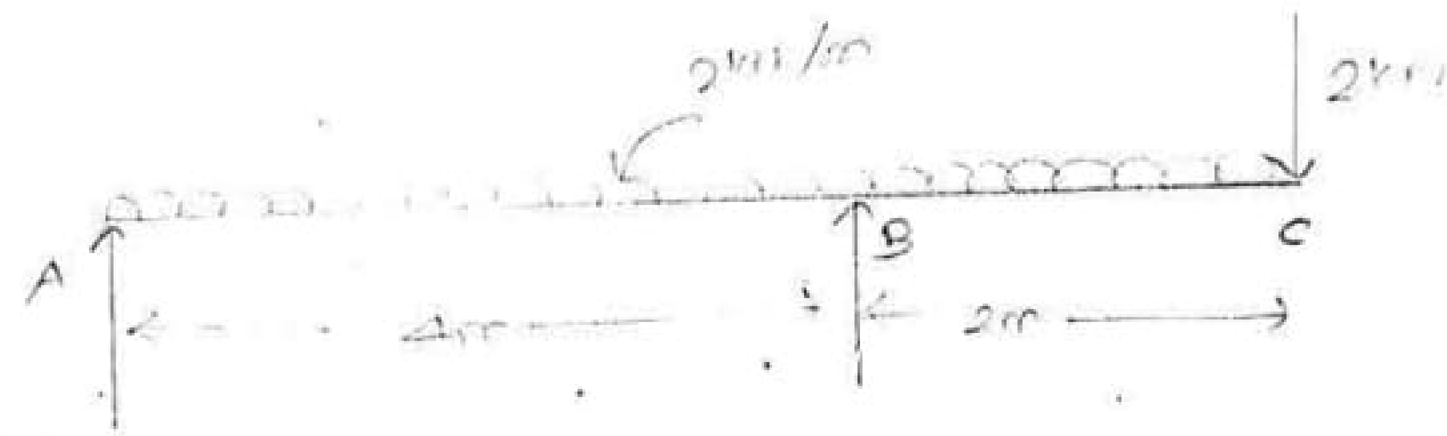
$$R_a + R_b = 10 + 15 + 25$$

The moment about C,

$$\Sigma (-25 \times 7) + (R_B \times 6) - (15 \times 3) + (R_A \times 1) = 0$$

(104)

1)



$R_a + R_b = (2\times6)+2$

$= 14\,kN$

Moment about A:

$(-2\times6) - (2\times6\times\frac{6}{2}) + R_b\times4 = 0$

$4R_b = 12+36$

$4R_b = 48\,kN$

$\boxed{R_b = 12\,kN}$

$\boxed{R_a = 2\,kN}$

S.F calculation:

S.F at C = 2 KN

S.F at B without pt load = 6 kN

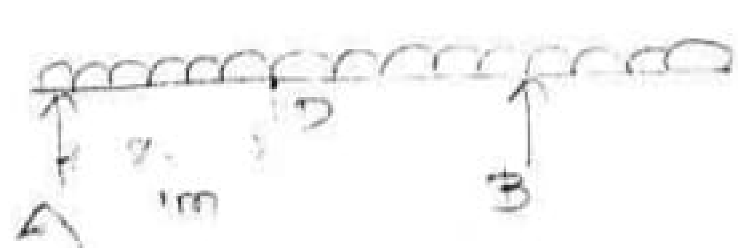
S.F at B with pt load = -6 kN

S.F at A without p.t load = 2 KN

S.F at A with p.t load = 0

To find x:

S.F changes from -6 kN to 2 kN b/w B to A.



$(2\times x) - 2 = 0$

$2x = 2$

Bending moment is zero : Point of contraflexure

B.M calculation :

B.M at C = 0

B.M at A. = 0

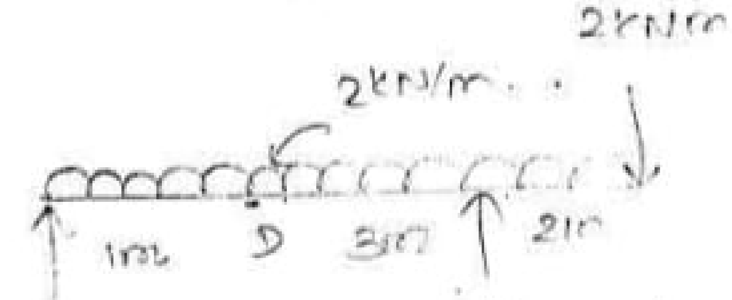
B.M at B $= -(2\times2)-(2\times2\times\frac{2}{2}) = -8$ KN-m.

B.M at A $= -(2\times6)+(12\times4)-(2\times6\times\frac{6}{2}) = 0$

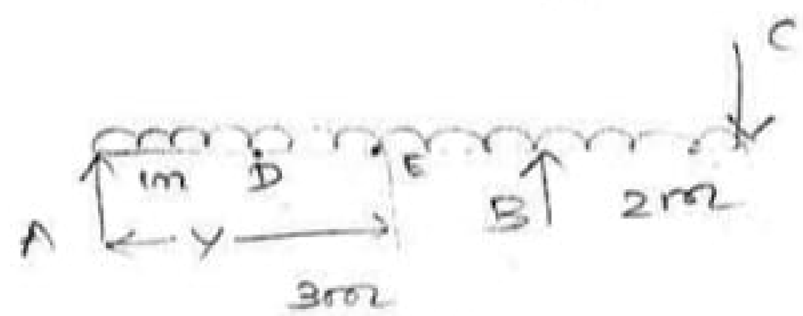
B.M at point where S.F is zero i.e D.

B.M at D $= -(2\times5)+(12\times3)-(2\times5\times\frac{5}{2})$

$= 1$ KN-m .



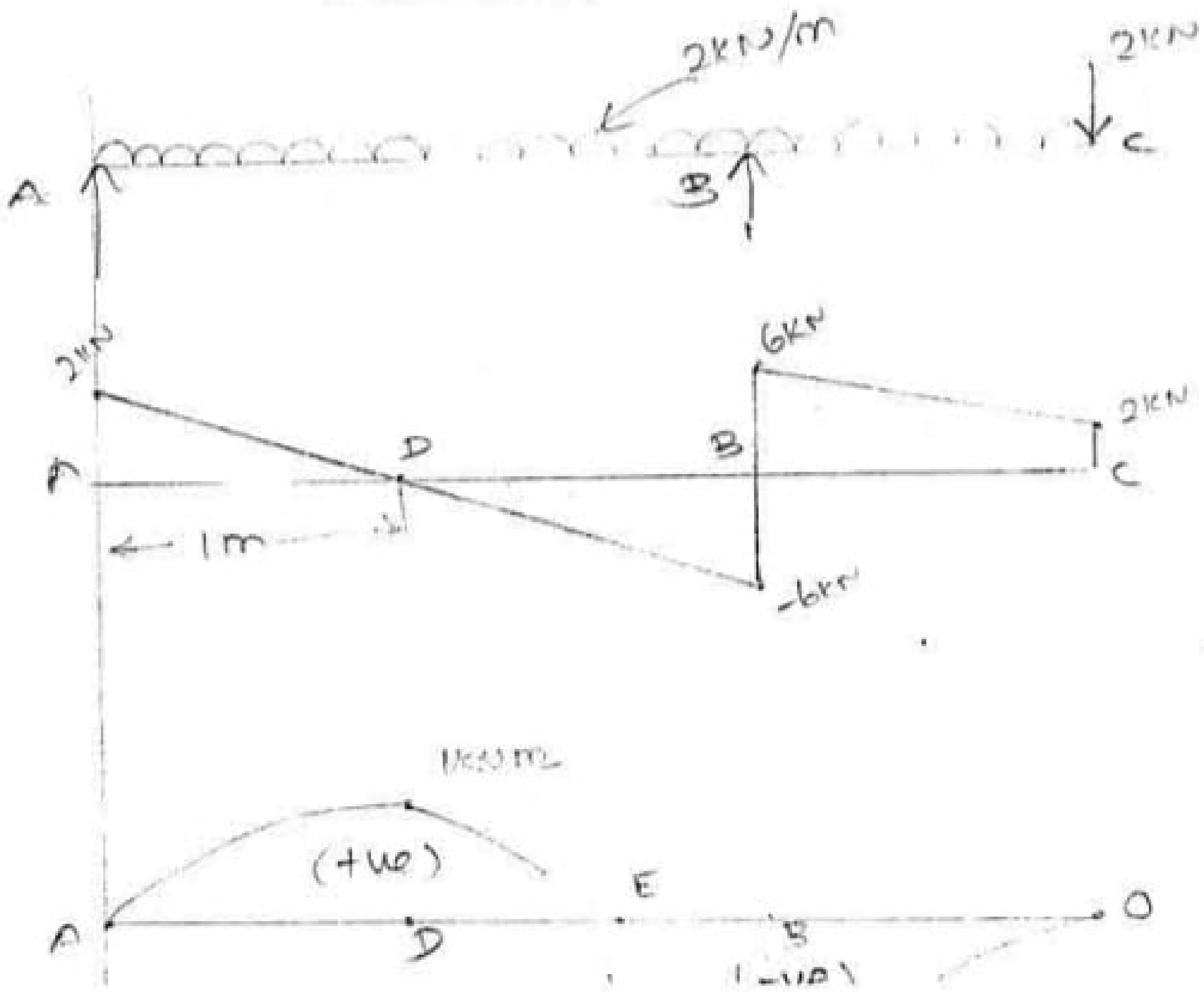
From B to D B.M changes from (-ve) to (+ve)



$(-2\times Y)+(2\times Y\times Y/2) = 0$

$-2Y+Y^2 = 0$

$\boxed{Y = 2m}$



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BEAMS CARRYING INCLINED LOAD AND SUBJECTED TO COUPLE :

* If inclined load is given resolve it into vertical and horizontal load. So vertical loads only caused S.F and B.M, Horizontal load will cause thrust or axial force.

* If one end is hinged other has roller support, roller support cannot provide any horizontal reaction. only hinged end provides horizontal rxns.

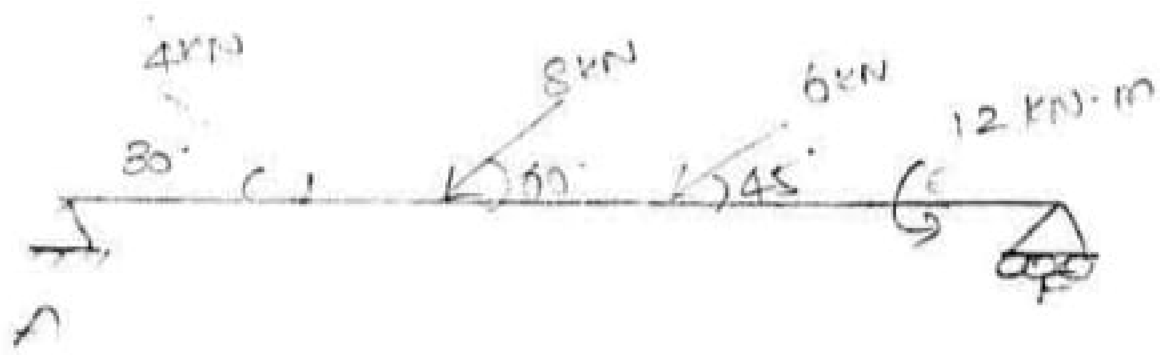
* If couple is acting the bending moment at that point increases and not the S.F but when finding the rxns the couple is considered by while taking bending moment.

* Where the couple acts the rxn will be opposite to couple direction

* If couple is anticlockwise it is +ve. If the couple acts clockwise it is -ve.

PROBLEMS :

(1)



Reactions at A & F:

R_A $H_A + V_A$ $- 4\sin 30 + 4\cos 30 - 8\sin 60$

$- 8\cos 60 - 6\sin 45 - 6\cos 45 + 12$

$+ V_F = 0$

$H_A + V_A - 2$

Horiz. force at $B = 4\cos 30 = 3.46$ kN

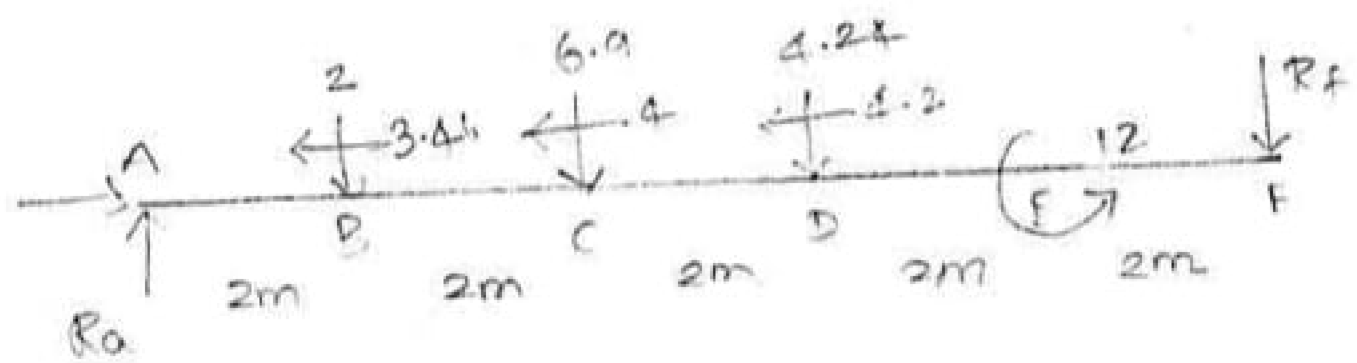
V.F at $B = -4\sin 30 = 2$ kN

H.F at $C = -8\cos 30 = 4$ kN

V.F at $C = 8\sin 30 = 6.9$ kN

Ho.F at $D = 6\cos 45 = 4.24$ kN

V.F at $D = 6\sin 45 = 4.24$ kN



$R_a + R_f = 2 + 6.9 + 4.24 \Rightarrow 13.14$

B.M about A:

$(-R_f \times 10) + 12 - (4.24 \times 6) - (6.9 \times 4) - 2 \times 2 = 0$

$\boxed{R_f = -4.48 \text{ kN}}$

$\boxed{R_a = 8.66 \text{ kN}}$

S.F calculation :

S.F at F $= -4.48$ kN

S.F at D $= -4.48 + 4.24 = ~~-8.72~~ -0.24$

S.F at C $= ~~8.72+~~ 6.9 - 0.24 = ~~15.62~~ 6.66$

S.F at B $= ~~15.62~~$ $6.66 + 2$ $= ~~19.08~~$ 8.66

S.F at A $= 0$

Bending Moment Calculation :

B.M at F $= 0$

B.M at E without couple $= -(-4.48) \times 2 = 8.96$

B.M at E with couple $= 8.96 + 12 = 20.96$ kN-m

Bending mom at D $= -(-4.48 \times 4) + 12$

$= 29.92$

B.M at C $= -(-4.48 \times \overset{6}{8}) + 12 - (4.2 \times 2)$

$= 30.2$

B.M at B $= -(-4.48 \times 8) + 12 - (4.2 \times 4) - (6.9 \times 2)$

$= 17.08$

B.M at A $= 0$

Axial force calculation : (-ve) $\longrightarrow$ (+ve) $\longleftarrow$

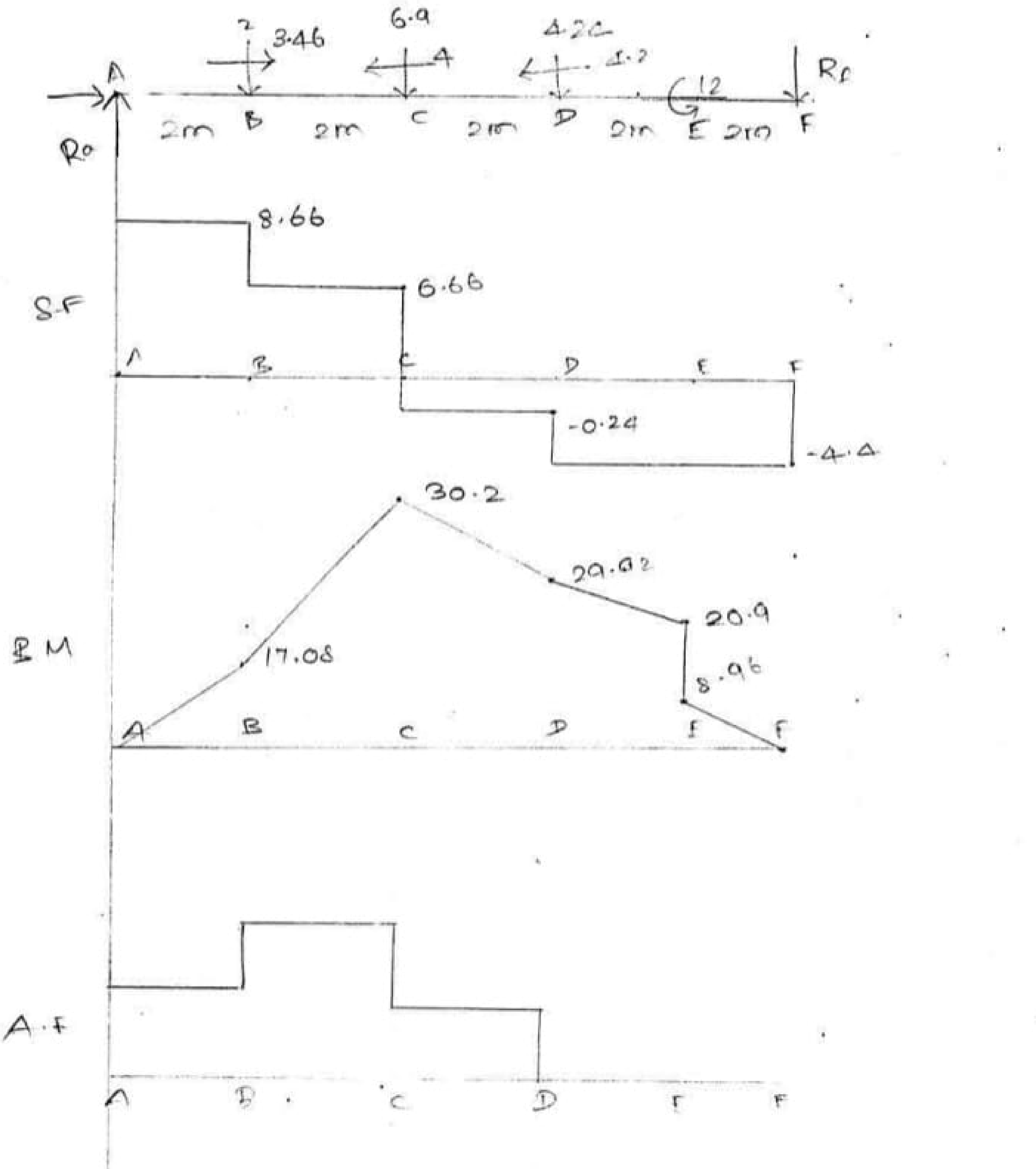
A.F at F = 0

A.F at D = 4.2 kN

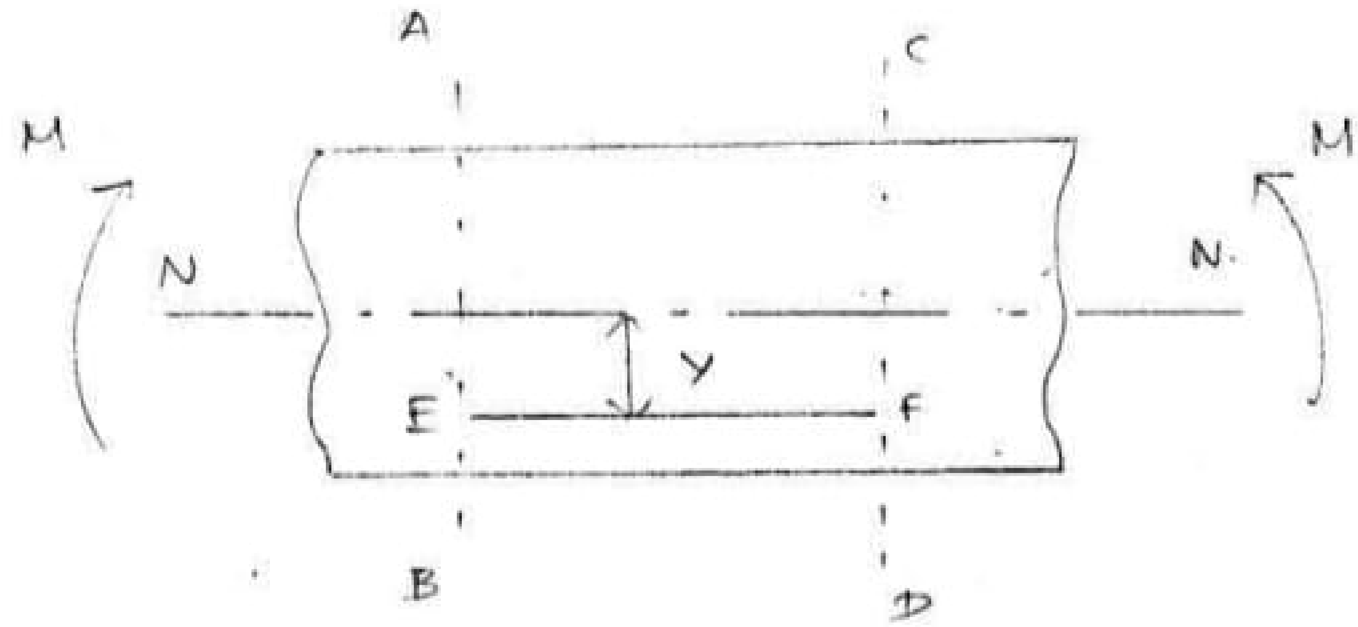
A.F at C = 4.2 + 6.9 = 11.1

A.F at B = $4.2 + \overset{4}{\cancel{6.9}} - 3.46 = \cancel{7.64}\ 4.74$

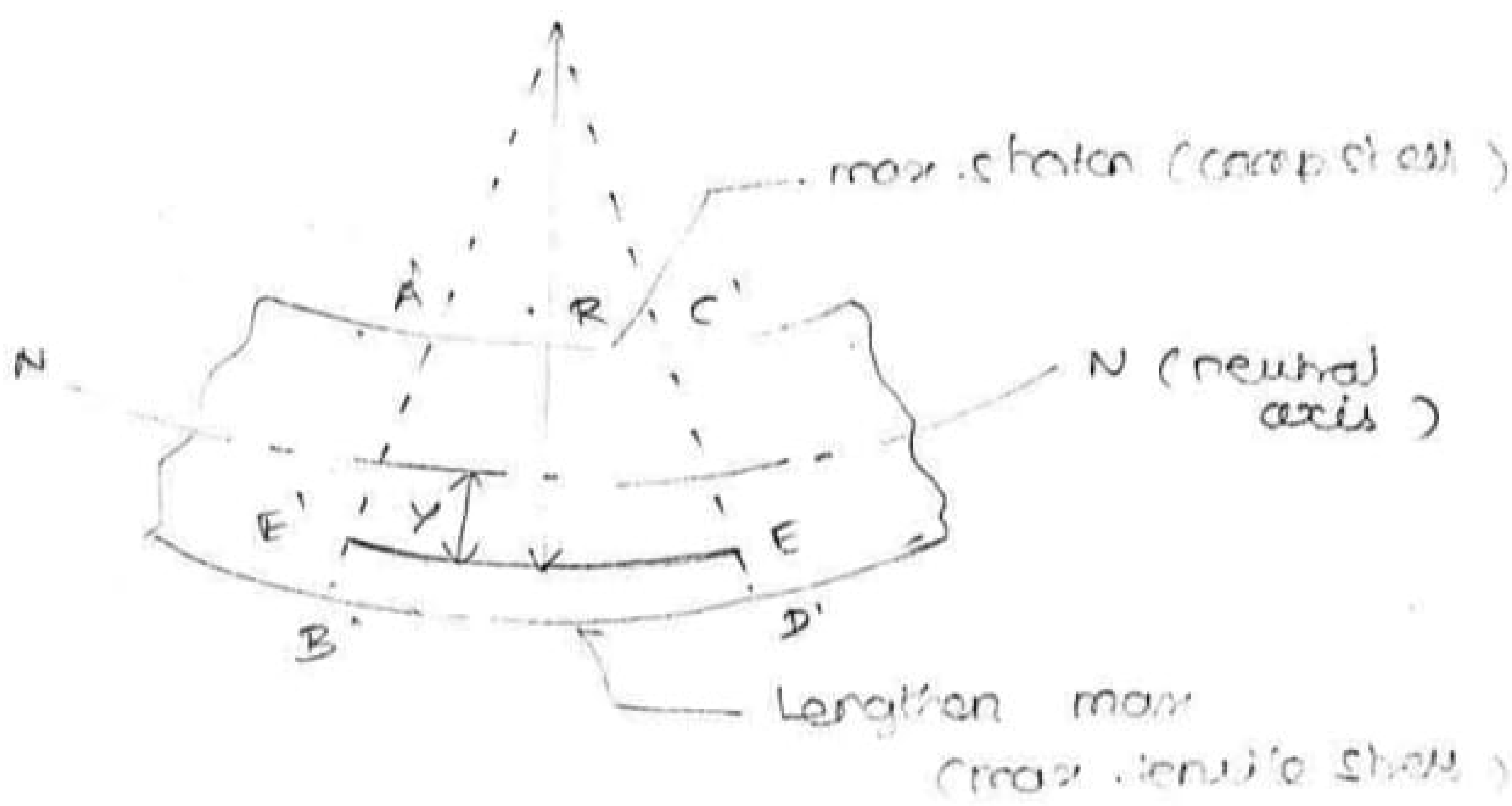
A.F at A = 0



BENDING STRESSES IN BEAMS :



Before Bending



AFTER BENDING

PURE BENDING (OR) SIMPLE BENDING :

If a length of the beam is subjected to constant bending moment and no shear force acts and that length of the beam is said to be pure bending.

The stresses are setup in the beam due to bending moment and it is called as bending stresses.

① STRAIN Along the depth of the beam:

$$\epsilon = \frac{Y}{R}$$

So, strain in a layer is proportional to its distance from the neutral axis.

② Stress along the depth of the beam:

Stress in a layer is directly proportional to the distance of the layer from the neutral axis.

$$\sigma = \frac{E}{R} \times y$$

where, y is distance of layer from neutral axis.

R = Radius of curvature of beam.

Moment of resistance :

Due to the compressive & tensile forces acting above & below the neutral layer will have moment about neutral axis. Total moment of these forces is about neutral axis for a section is known as moment of resistance of that section.

$$\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$$

where,

M = Bending moment (or) moment of resistance

I = Moment of Inertia

σ = Stress.

Y = Distance from neutral axis.

Bending stresses in symmetrical sections :

Section modulus for various beam sections :

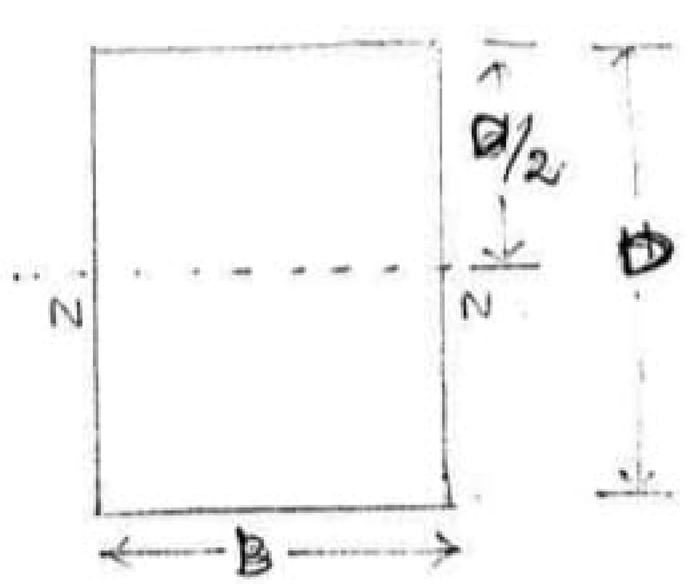
Section modulus :

It is the ratio of moment of Inertia about neutral axis to the distance of outermost layer from the neutral axis.

$$Z = \frac{I}{Y_{max}}$$

① Rectangular section :

$$I = \frac{BD^3}{12} \text{ mm}^4$$



$$Y_{max} = D/2 \text{ mm.}$$

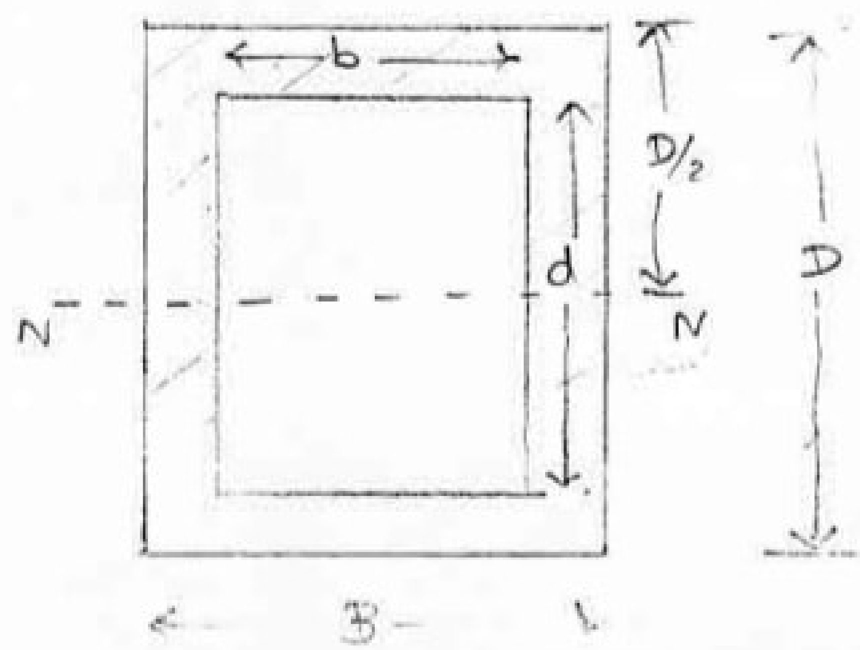
$$Z = \frac{I}{Y_{max}} = \frac{BD^2}{6} \text{ mm}^3.$$

② Hollow rectangular section :

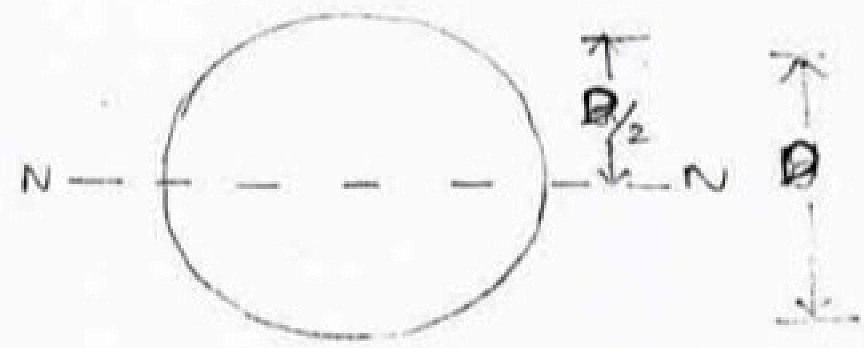
$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$Y_{max} = D/2$$

$$Z = \frac{1}{6D}\left(BD^3 - bd^3\right)$$



③ circular section :

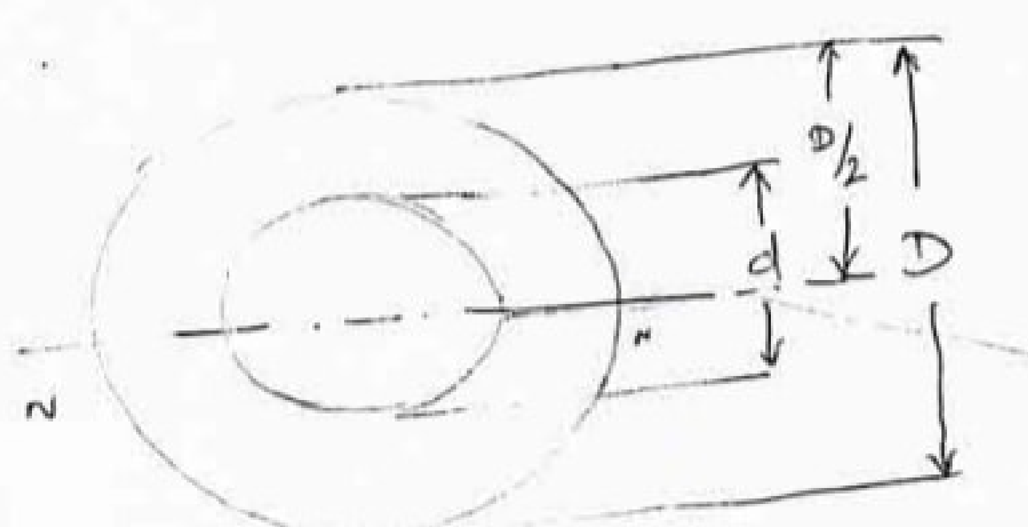


$$I = \frac{\pi}{64} D^4$$

$$Y_{max} = D/2$$

$$Z = \frac{\pi}{32} D^3$$

④. Hollow circular section :



$$I = \frac{\pi}{4}(D^4 - d^4) \qquad Y_{max} = D/2$$

$$Z = \frac{\pi}{\ldots}\left(D^4 - d^4\right)$$

1. Calc. the maximum stress induced in a cast iron pipe of external diameter 40 mm and internal diameter 20 mm & length is 4m when the pipe is supported at its end and carries a point load of 80 N at its center.

GIVEN :

$D = 40\,mm$

$d = 20\,mm$

$L = 4m = 4 \times 10^3\,mm$

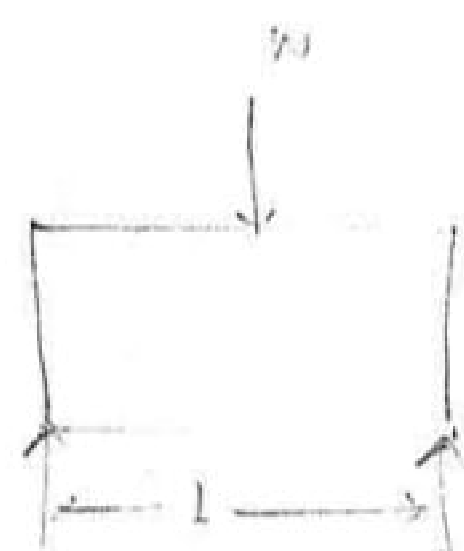
$P = 80N$

Solution :

$$\frac{M}{I} = \frac{\sigma}{y}$$

TO find σ_{max} :

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$



Bending moment,

$$M = \frac{WL}{4} = \frac{80 \times 4 \times 10^3}{4}$$

$$= 80{,}000$$

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(40^4 - 20^4)$$

$$= \cancel{1.884 \times 10^3}\ 11.7 \times 10^4$$

$$y_{max} = D/_2 \Rightarrow 20\,mm$$

$$\sigma_{max} = \frac{M \times Y_{max}}{I}$$

$$= \frac{80{,}000 \times 20}{\cancel{1884 \times 10^3}}$$

$$= 13.58 \ N/mm^2$$

②. A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of the section is to be twice the breadth and the stress in the timber is not to exceed 7N/mm2. Find the
i) dimension of the cross section.
ii) How would you modify the c/s of the beam if it carries a concentrated load of 20 kN placed at the center with the same ratio of breadth to depth.

GIVEN :

i) UDL $P = 20 kN = 20 \times 10^3 N$

$L = 3.6 m = 3.6 \times 10^3 mm$

$d = 2b$

$\sigma_{max} = 7 N/mm^2$

ii) Point load

Point Load= $\overset{20}{\cancel{3.6}} \times 10^3$ ~~mm~~ N

$L = 3.6 \times 10^3 mm$

$d = 2b$

$\sigma_{max} = 7 N/mm^2$

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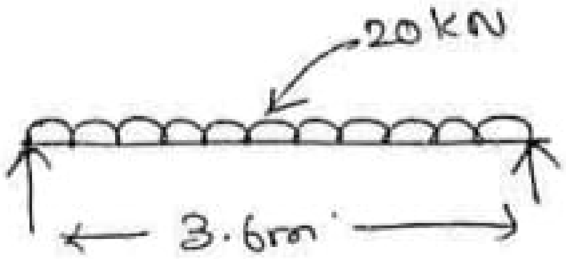


Sol :

To find b & d :

W.K.T $\frac{M}{I} = \frac{\sigma_{max}}{Y_{max}}$ —— ①

$M = \frac{WL^2}{8}$ (or) $\frac{WL}{\cancel{8}\,\cancel{4}\,8}$

when load is in N/m   Load is is 'N'

$M = \frac{WL}{8} = \frac{20 \times 3.6 \times 10^3}{8} = 9 \times 10^3$

$I = \frac{\pi}{64}(D^4 - d^4)$ $I = \frac{bd^3}{12}$

$I = \frac{b \times (2b)^3}{12}$

$I = \frac{8b^4}{12}$

$I = 0.66\, b^4$

$Y_{max} = \frac{d}{2} \Rightarrow \frac{2b}{2} \Rightarrow b$

$\sigma_{max} = 7$

$\frac{9}{0.66\, b^4} = \frac{7}{b}$

$\frac{9 \times 10^3}{0.66 \times 7} = b^3$

$\boxed{b = 124\, mm}$

$\therefore$ $d = 2b$

$$\boxed{d = 248 \text{ mm}}$$

Case ii) point load:

Bending moment,

$$BM,\ M = \frac{WL}{4}$$

$$= \frac{3.6\times10^{3} \times 10^{3} \times 20}{4}$$

$$M = 180\times10^{5}$$

$$\frac{180\times10^{5}}{0.66b^{4}} = \frac{7}{b}$$

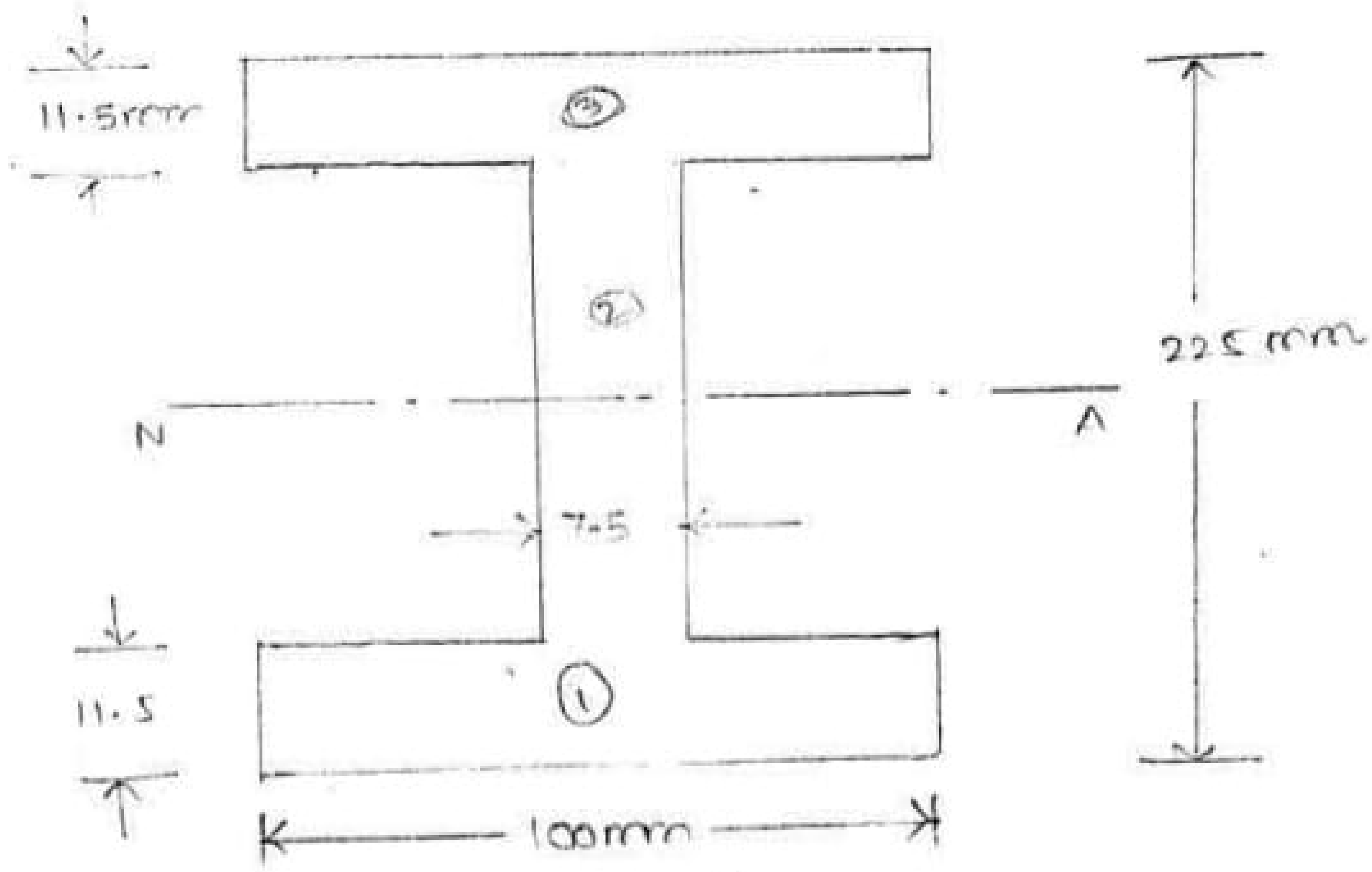
$$b^{3} = 3896103.8$$

$$\boxed{b = 157.3 \text{ mm}}$$

$d = 2b$

$$\boxed{d = 314.70 \text{ mm}}$$

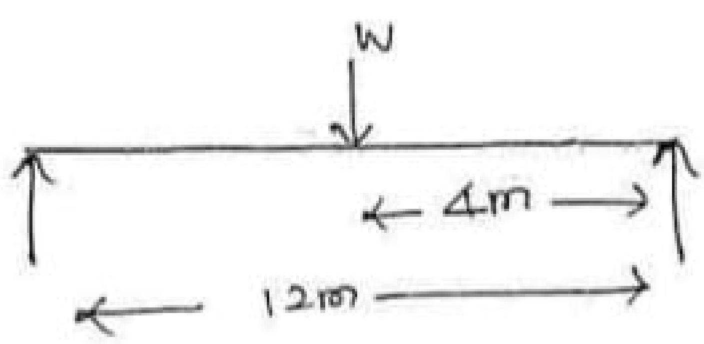
(3). An I section shown in the figure is simply supported over a span of 12 m if the maximum permissible bending stress is 80 N/mm2 what of concentrated load can be carried at a distance of 4 m from 1 support.



GIVEN :

$L = 12m = 12 \times 10^3 \; mm$

$\sigma_{max} = 80 \; N/mm^2$



Solution :

To find load 'W' :

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

Moment of Inertia,

$$I_1 = \frac{bd^3}{12} = \frac{100 \times 11.5^3}{12} = 126.739 \times 10^2 .$$

$$I_2 = \frac{bd^3}{12} =$$

Moment of Inertia

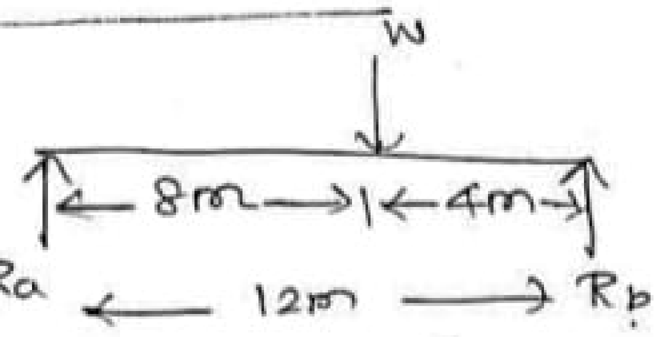
$$I = \left(\frac{100 \times 225^3}{12}\right) - \left[\frac{(100-7.5) \times (225 - 2 \times 11.5)^3}{12}\right]$$

$$= 949.21 \times 10^5 - 635.35 \times 10^5$$

$$I = 313.65 \times 10^5 \text{ mm}^4$$

$$Y_{max} = \frac{d}{2} = \frac{225}{2} = 112.5$$

Bending Moment M :



$$R_a + R_b = W$$

Taking moment about A :

$$(R_b \times 12) - (W \times 8) = 0$$

$$12 R_b = 8W$$

$$R_b = \frac{2W}{3}$$

$$R_a + R_b = W$$

$$\frac{2W}{3} + R_a = W \quad = \frac{3W - 2W}{3}$$

$$R_a = W/3$$

Bending moment :

B.M about A = 0

B.M about B = 0

$$B.M \text{ at } C = R_b \times 4$$

$$= \frac{2W}{3} \times 4$$

$$C = \frac{8W}{3} \quad (\text{max at } 'C')$$

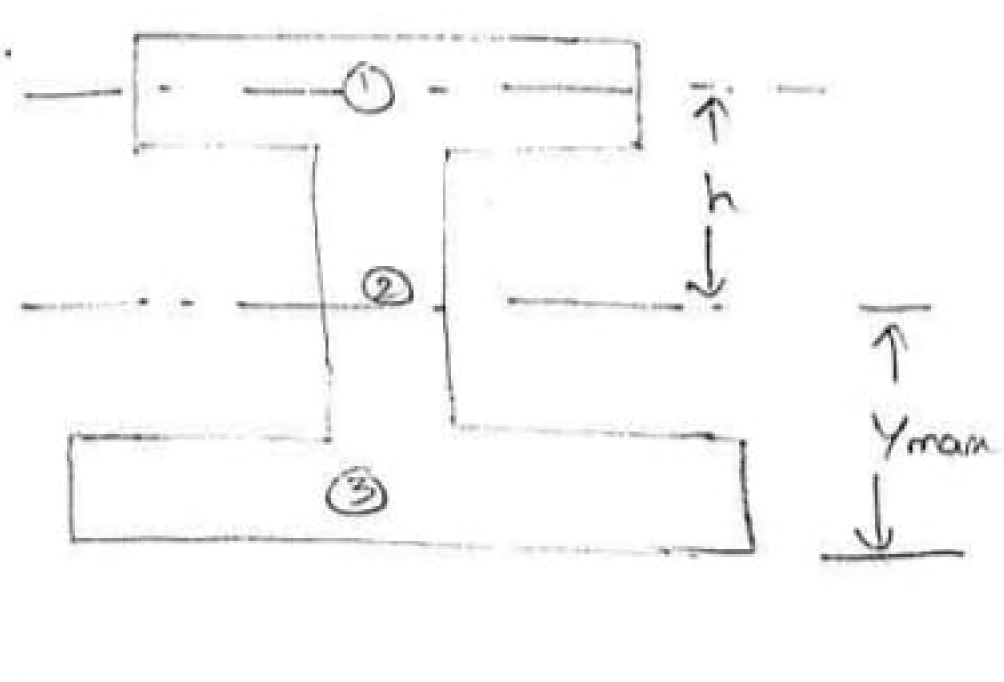
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\frac{8W/3}{31 \times 10^6} = \frac{80}{112.5}$$

$$\boxed{W = 8.26 \times 10^{3} N}$$

BENDING STRESS IN UNSYMMETRICAL SECTIONS :

In case of symmetrical section the neutral axis passes thorough the geometric section of the center but in unsymmetrical section it will not be same and hence the bigger value of y has to be used



$$Y_{max} = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3}{(A_1 + A_2 + A_3)}$$

MOI, I = MOI about C.G + (Area of h^2).

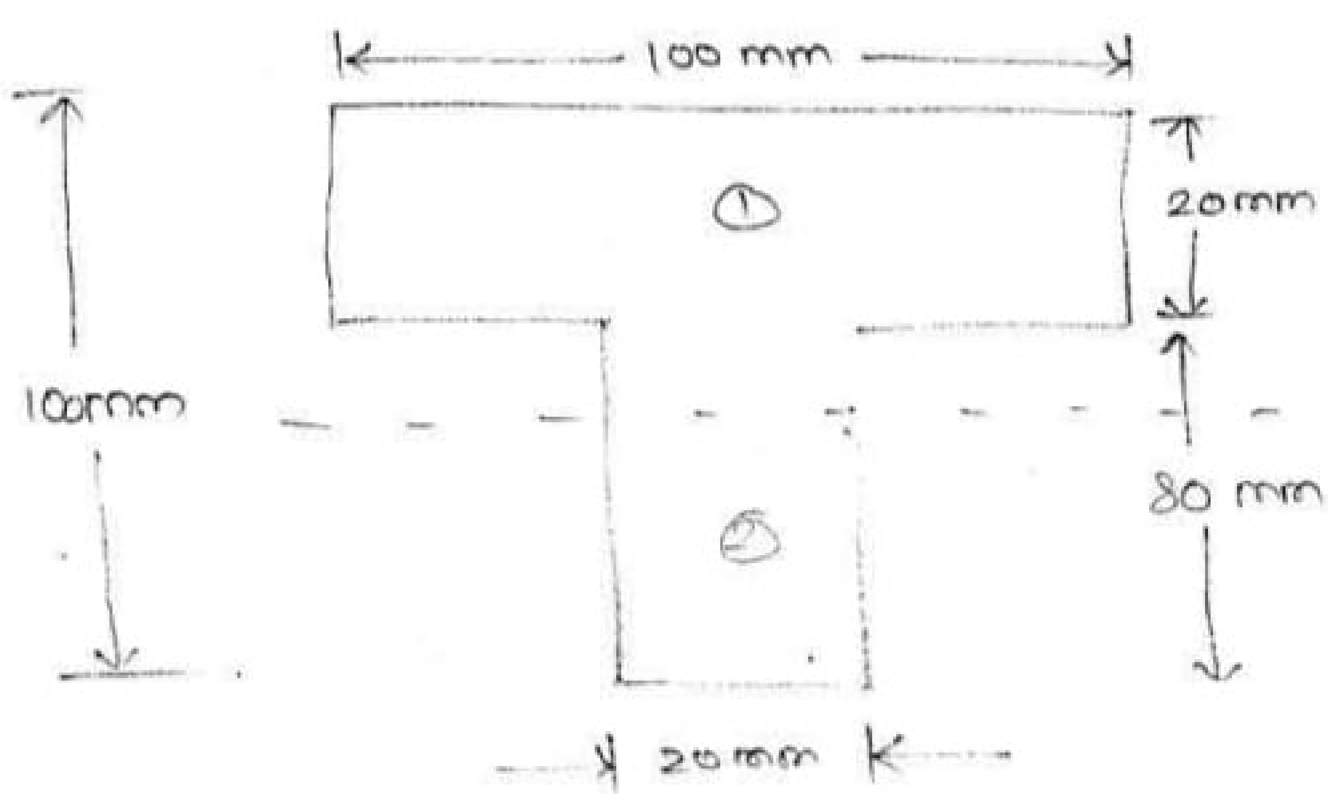
where,

A is area

y is distance of centre of gravity from the bottom.

h. distance of axis from centre of gravity.

1. A cast iron beam of T section is shown in the figure. The beam is simply supported on a span of 8 m. The beam carries a UDL of 1.5 kN/m for the entire span. Determine maximum tensile and compressive stresses



(122)

GIVEN :

$$L = 8m = 8 \times 10^3 mm$$

$$UDL = 1.5 KN/m \Rightarrow 1.5 \frac{\times 10^3}{10^3} \Rightarrow 1.5 N/mm$$

SOLUTION :

To find σ_c max & σ_t max.

W.K.T,

$$\frac{M}{I} = \frac{\sigma_{max}}{Y_{max}}$$

To find Y_{max},

$$Y_{max} = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3}{(A_1 + A_2 + A_3)}$$

$$A_1 = 100 \times 20 = 2000 mm$$

$$A_2 = 80 \times 20 = 1600 mm$$

$$Y_1 = 80 + \frac{20}{2} = 90 mm$$

$$Y_2 = \frac{80}{2} = 40 mm$$

$$Y_{max} = \frac{(2000)(90) + (1600)(40)}{2000 + 1600}$$

$$Y_{max} = 67.77 mm.$$

Neutral axis lies at a distance of 67.77 mm from bottom and 32.23 mm from top.

$$M = \frac{WL^2}{8} \Rightarrow \frac{1.5 \times (8 \times 10^3)^2}{8}$$

$$= 12 \times 10^6 \quad Nmm$$

MOI, I = Mom about C.G + (Area of h^2)

I = $I_1 + I_2$

I_1 = MOI of top section about C.G + ($A_1 \times$ dis. of its C.G from neutral axis)

$$= \frac{b_1 d_1^3}{12} + (A_1 \times h_1^2)$$

$$= \frac{100 \times 20^3}{12} + ((100 \times 20) \times 22.23^2)$$

$$= 66666.6 + 988345.8$$

$$I_1 = 10.55 \times 10^5$$

I_2 = M.O.I of bottom sec + ($A_2 \times h_2^2$)

$$= \frac{b_2 d_2^3}{12} + (A_2 \times h_2^2)$$

$$= \frac{20 \times 80^3}{12} + ((80 \times 20) \times (22.77)^2)$$

$$I_2 = 16.82 \times 10^5$$

$$I = I_1 + I_2$$

$$I = 27.37 \times 10^5$$

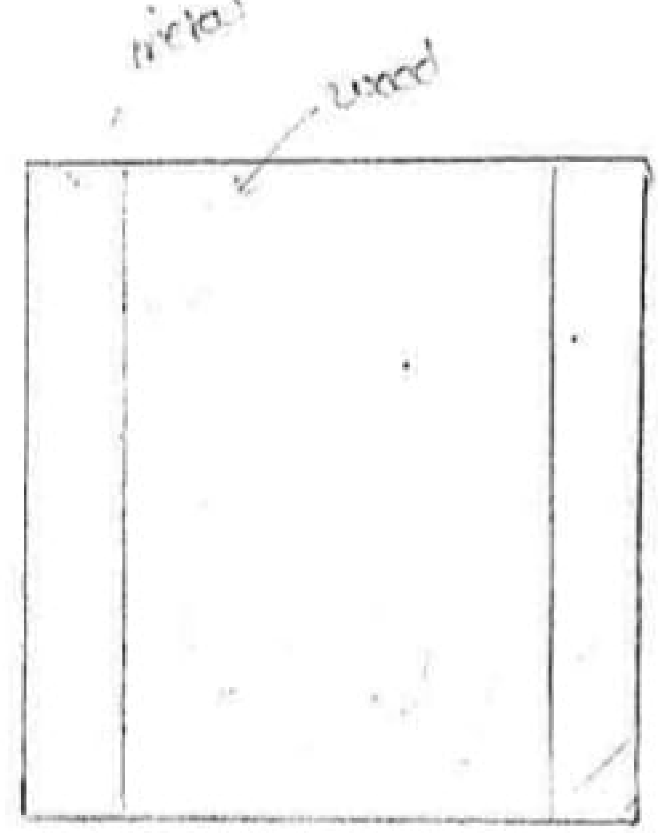
$\sigma_{t\,max}$ =

Max. comp. stress.

$$\frac{M}{I} = \frac{\sigma_{c\,max}}{y_{max}}$$

FLITCHED BEAMS: (or) composite beams:

A beam made up of two or more different materials assumed to be rigidly connected together and acts like a single piece known as composite beam or a wooden flitched beam.



i). strain constant, $\varepsilon_1 = \varepsilon_2 = \dfrac{\sigma_1}{E_1} = \dfrac{\sigma_2}{E_2}$

$$\sigma_1 = \frac{E_1}{E_2} \cdot \sigma_2$$

$$\boxed{\sigma_1 = m\sigma_2}$$

ii) Moment of resistance

$$M = M_1 + M_2$$

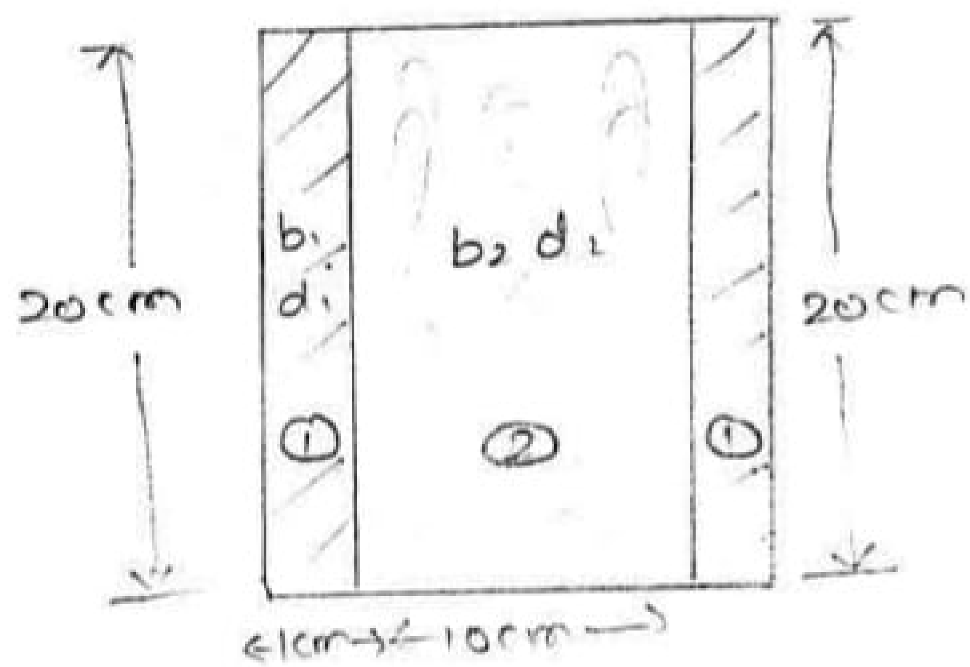
$$M_1 = \frac{\sigma_1}{y_1} I_1 \quad , \quad M_2 = \sigma_2 \, I_2 / y_2$$

iii) MOI

$$M = m I_1 + I_2$$

$m \rightarrow$ modular ratio $= \dfrac{E_1}{E_2}$

(4). A fixed beam consist of wooden joist 10cm wide and 20 cm deep strengthned by two steel plates 10mm thick and 20 cm deep. The max. stress in the wooden joist is 17 N/mm2. Find the corresponding max. stress attained in steel. Find the moment of resistance of the composite section. Take $E_s = 2 \times 10^5$ N/mm^2 and $E_w = 1 \times 10^4$ N/mm^2.



GIVEN :

$E_s = 2 \times 10^5$ N/mm^2 $\quad\quad$ $\sigma_w = 17$ N/mm^2

$E_w = 1 \times 10^4$ mm^2

$b_1 = 10$ mm

$d_1 = 200$ mm

$b_2 = 100$ mm

$d_2 = 200$ mm.

Solution:

Max. stress on steel: (σ_s)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_w}{E_w}$$

$$\frac{\sigma_s}{2 \times 10^5} = \frac{\sigma_w}{1 \times 10^4}$$

$$\sigma_s = 20\,\sigma_w$$

$$\sigma_s = 340\ N/mm^2$$

$$M = M_1 + M_2$$

$$M_s = \frac{\sigma_s}{y_s} \times I_s$$

$$M_w = \frac{\sigma_w}{y_w} \times I_w$$

$$y_s = \frac{200}{2} = 100\ mm$$

$$I_s = 2 \times \frac{b_1 d_1^3}{12}$$ (since 2 steel plates)

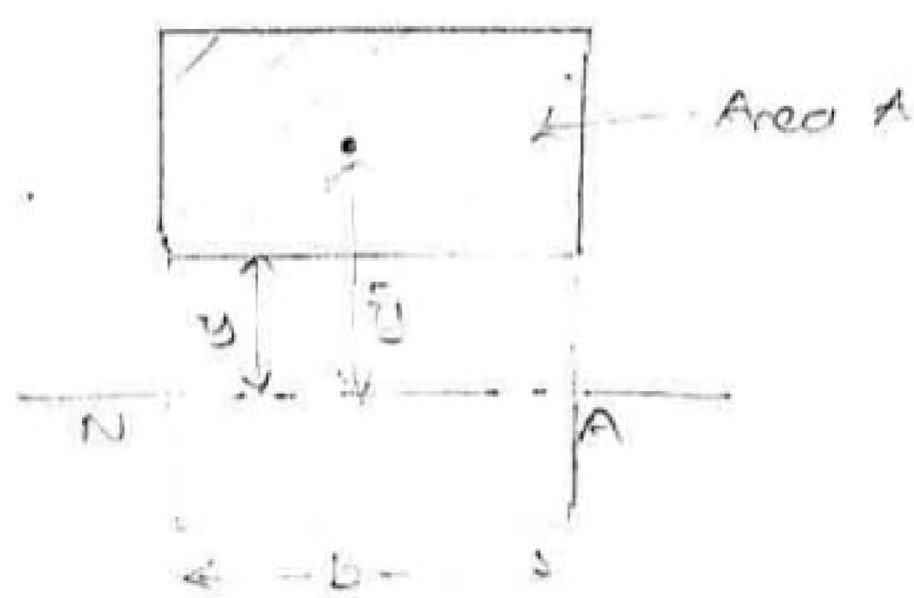
$$= 2 \times \frac{10 \times 200^3}{12}$$

$$= 13.3 \times 10^6$$

$$I_w = \frac{b_2 d_2^3}{12} = \frac{100 \times 200^3}{12} = 80 \times 10^7$$

18/8/14. **SHEAR STRESSES IN BEAMS :**

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

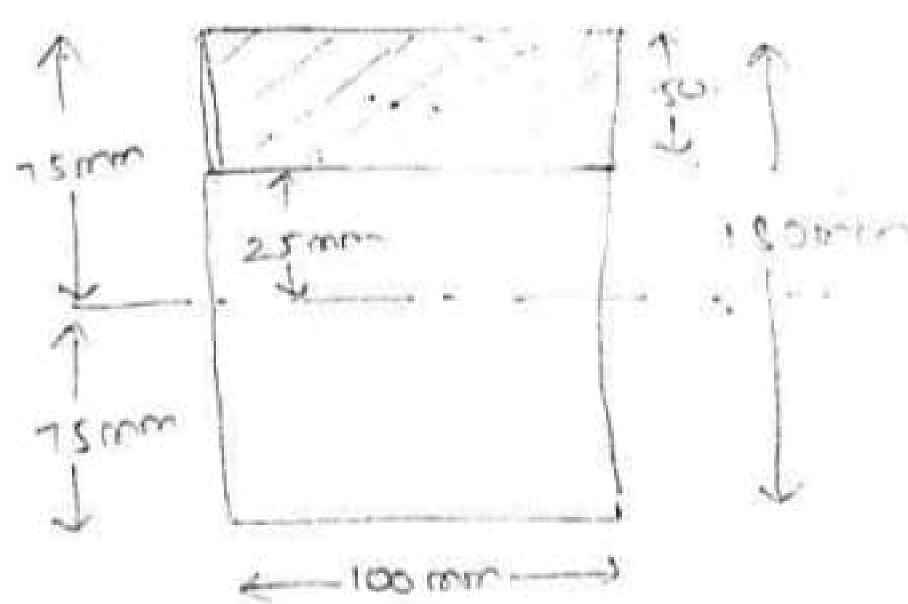


A - area of section

F - shear force

① A wooden beam 100 mm wide and 150 mm deep is simply supported over a span of 4 m. If shear force at a section of the beam is 4500 N. Find shear stress at a distance of 25 mm above neutral axis.

<u>GIVEN</u> :



$b = 100\,mm$

$d = 150\,mm$

$L = 4m$

$F = 4500\ N$

$y = 25mm.$

<u>Solution</u> :

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

$\bar{y} = 50\,mm$

$$I_{xx} = \frac{bd^3}{12} = \frac{100 \times \cancel{75}^3\ 150^3}{12} = \cancel{351.56 \times 10^4}\ 281.25 \times 10^5$$

$$\tau = 4500 \times \frac{5000 \times 50}{\cancel{351.56 \times 10^4}\ 281.25 \times 10^5 \times 100}$$

$\tau = 0.4003\ N/mm^2$

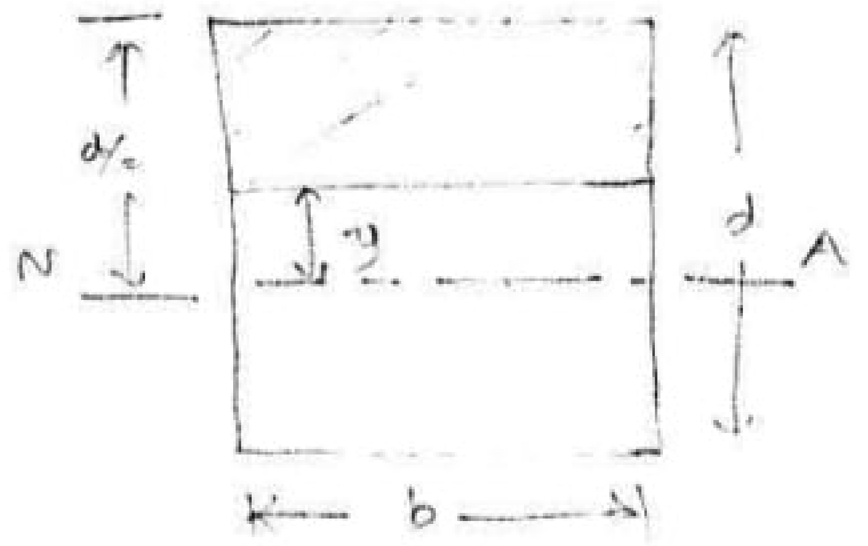
Shear stress distribution for different sections:

1. Rectangular section:

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau_{avg} = \frac{F}{b \times d}$$

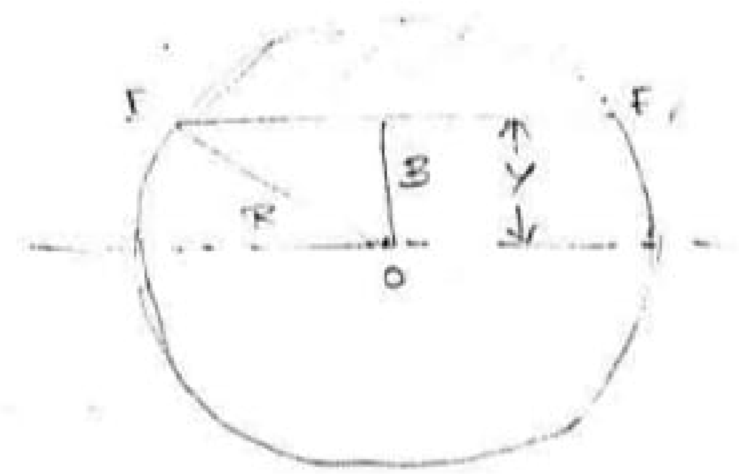
$$\tau_{max} = 1.5\,\tau_{avg}$$



2. Circular section:

$$\tau = F/3I\,(R^2 - y^2)$$

$$\tau_{avg} = F/\pi r^2, \quad \tau_{max} = \frac{4}{3}\tau_{avg}$$

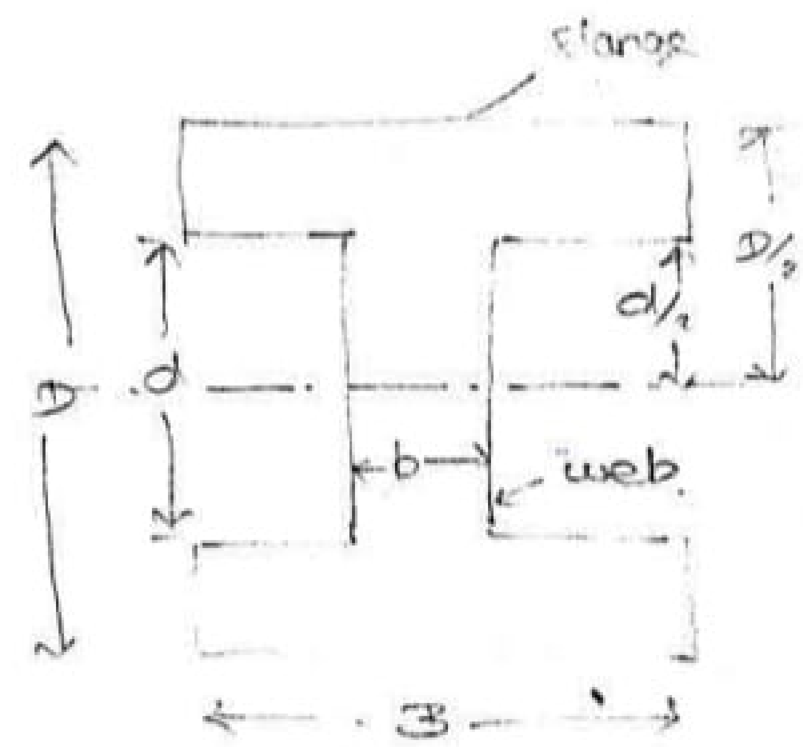


3. I section:

τ in upper flange $= 0$

τ in lower flang $= \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$

$$\boxed{y = \frac{d}{2}}$$



shear stress in web =

$$\tau = \frac{F}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

$$\tau_{max} = \frac{F}{Ib}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

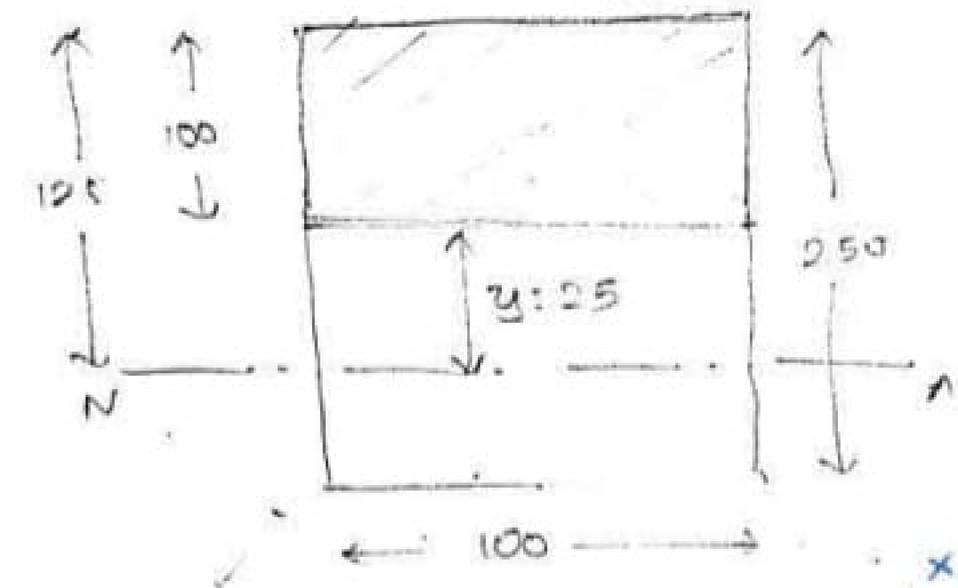
① A rectangular beam 100 mm wide and 250 mm deep is subjected to a ~~max. shear~~ force of 50 KN. Determine i) avg. shear stress ii) Max. shear stress. iii) shear stress at a distance of 25mm from neutral axis.

Given :

$b = 100\,mm$

$d = 250\,mm$

$F = 50 \times 10^3 N$.



Solution :

iii) $\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$

$\frac{F\,A\bar{y}}{Ib} = \frac{50\times10^3 \times (\cancel{125}\ 100 \times 1000 \times \frac{250}{2})}{\frac{100\times250^3}{12} \times 1000}$

$I = \frac{bd^3}{12} = \frac{100 \times \cancel{100^3}\ 250^3}{12} = \cancel{83.33\times10^5}\ \ 130.208 \times 10^6$

$\tau = \frac{50\times10^3}{2\times 83.33\times10^5}\left(\frac{250^2}{4} - 25^2\right)$

$\tau = \cancel{3\times10^{-3}}\left(\cancel{1875}\ 15000\right) \Rightarrow 1.92\times10^{-4} \times 15000$

$\tau = \cancel{5.62} \Rightarrow 2.88\ N/mm^2$

i). $\tau_{avg} = \frac{F}{b\times d} = \frac{50\times10^3}{100\times250} = 2\ N/mm^2$

ii) $\tau_{max} = 1.5 \times \tau_{avg}$

$= 3\ N/mm^2$

(2). A circular beam of diameter 150 mm is subjected to a max. shear stress of 15 N/mm² . Find the i) Avg shear force acting at a distance of ii) shear stress acting at a distance of 20 mm from the neutral axis.
: y.

GIVEN :

$\tau_{max} = 15 \ N/mm^2$

Sol :

i) $\tau_{max} = \frac{4}{3} \times \cancel{15} \ \tau_{avg}$

$15 = \frac{4}{3} \times \tau_{avg}$

$\tau_{avg} = 11.25$

ii) $\tau_{avg} = \frac{F}{\pi R^2}$

$11.25 = \frac{F}{(3.14) \times (75^2)}$

$F = 198.703 \times 10^3 \ N.$

ii). $\tau = \frac{F}{3I} (R^2 - y^2)$

$I = \frac{\pi d^4}{64} = \frac{(3.14)(150)^4}{64} = 248.37 \times 10^5$

$\tau = \frac{198.703 \times 10^3}{3 \times 248.37 \times 10^5} (75^2 - 20^2)$

$\tau = 13.93 \ N/mm^2$

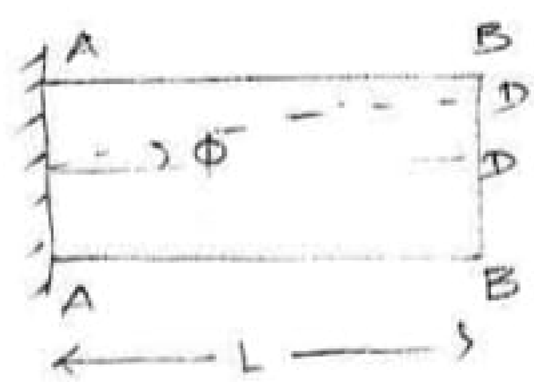
19/8/14

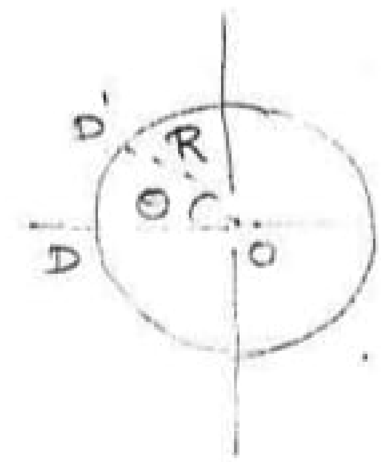
UNIT-III

TORSION OF SHAFTS AND SPRINGS

A shaft is said to be in torsion when equal & opposite torques are applied at the two ends of the shaft.

Shear stress produced in a shaft subjected to torsion :





Distortion due to torque, T = DD'

Shear strain $= \frac{DD'}{L}$

Also $\tan\phi = \frac{DD'}{L}$

If ϕ value is very small $\tan\phi = \phi$

$$\phi = \text{shear strain} = \frac{DD'}{L}$$

From fig. 2 $DD' = OD' \times \theta$
$= R \times \theta$

$$\therefore \phi = \frac{R\theta}{L}$$

R - radius of shaft
l - length
T = Torque applied
τ = shear stress
C = modulus of rigidity
θ = angle of twist
φ = shear strain.

∴ Modulus of rigidity, $C = \frac{\text{shear stress}}{\text{shear strain}}$

$$C = \frac{\tau}{\frac{R\theta}{L}}$$

$C = \tau l$ $\boxed{\tau = CR\theta}$

If q is shear stress induced at a radius r from the centre then

$$\frac{\tau}{R} = \frac{q}{r}$$

Maximum torque transmitted by circular solid shaft : (study the derivation) X.

$$T = \frac{\pi}{16} \tau D^2$$

where,

T = max. torque

τ = shear stress

D = Diameter of the shaft.

Maximum torque transmitted by hollow circular shaft : (Study the derivation) X.

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

D_o = outer diameter

D_i = inner diameter

Power transmitted by the shaft :

$$P = \frac{2 \pi N T}{60} \text{ watt}$$

(or)

$$= \omega \times T$$

where $\omega = \frac{2 \pi N}{60}$

where, N = Speed in R.P.M.

T = torque.

Note :

If maximum torque is not given consider T as T_{max}.

Problems

① Find the maximum shear stress induced in a solid circular shaft of dia 15 cm when the shaft transmits 150 kW power at 180 rpm.

Sol :

$\tau_{max} = ?$

$D = 15\ cm \Rightarrow 15 \times 10^{-2}\ m$

$P = 150 \times 10^{3}\ W$

$N = 180$

$$P = \frac{2\pi N T}{60}$$

$$150 \times 10^{3} = \frac{2 \times \pi \times 180 \times T}{60}$$

$$T = 7961.78\ N \cdot m$$

$$T = \frac{\pi}{16} \tau D^{3}$$

$$7961.78 = \frac{\pi}{16} \times \tau \times (15 \times 10^{-2})^{3}$$

$$\tau = 12.02 \times 10^{6}\ N/m^{2}$$

(134)

(2). A solid steel shaft has to transmit 75 kW at 200 rpm taking allowable shear stress as 70 N/mm² . Find suitable diameter for the shaft with max torque transmitted at each revolution exceeds the mean by 30%.

GIVEN:

$P = 75 \times 10^3 \, W$

$N = 200 \, rpm.$

$\tau = 70 \, N/mm^2 = 70 \times 10^6 \, N/m^2$

$T_{max} = 100 + 30\% \, T$

$T_{max} = 1.3 T$

Sol: TO find Dia 'D':

$$P = \frac{2 \pi N T}{60}$$

$$75 \times 10^3 = \frac{2 \times \pi \times 200 \times T}{60}$$

$$T = 3582.80 \text{ N-m}$$

$$T_{max} = 4657.6 \text{ N-m}$$

$$T = \frac{\pi}{16} \tau D^3$$

$$4657.6 = \frac{\pi}{16} \times 70 \times 10^6 \times D^3$$

$$D^3 = 3.390 \times 10^{-4}$$

$$\boxed{D = 0.069 \, m}$$

(8) A hollow shaft is to transmit 300 kW power at 80 rpm. If the shear stress is not to exceed 60 N/mm² and the internal dia is 0.6 times of the ext. dia. Find the external & internal diameters assuming that the max torque is 1.4 times the mean.

Given :

$$P = 300 \times 10^3 \, W$$

$$N = 80 \, rpm$$

$$\tau = 60 \, N/mm^2$$

$$T_{max} = 1.4T$$

$$D_i = 0.6 D_o$$

Sol :

To find Dia D_o & D_i :

$$P = \frac{2\pi N T}{60} \, (W \quad N\text{-}m)$$

$$T_{max} = 1.4 T$$

$$T_{max} = \frac{\pi}{16} \tau \left(\frac{D_o^4 - D_i^4}{D_o} \right)$$

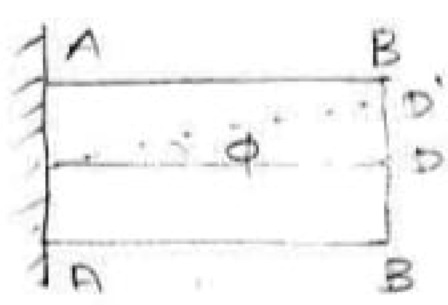
19/8/14.

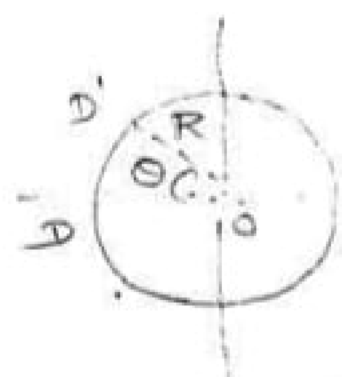
UNIT - III

TORSION OF SHAFTS AND SPRINGS

A shaft is said to be in torsion when equal and opposite torques are applied at two ends of the shaft.

Shear stress produced in a shaft subjected to torsion :





Distortion due to torque, $T = DD'$

Shear strain $= \frac{DD'}{L}$

Also $\tan\phi = \frac{DD'}{L}$

If ϕ value is very small $\tan\phi = \phi$

ϕ = shear strain $= \frac{DD'}{L}$

From fig. 2. $DD' = OD' \times \theta$

$= R \times \theta$

$\therefore \phi = \frac{R\theta}{L}$

$\therefore$ Modulus of rigidity, $C = \frac{\text{shear stress}}{\text{shear strain}}$

$$C = \frac{\tau}{\frac{R\theta}{L}}$$

$$C = \frac{\tau L}{R\theta} \quad ; \boxed{\tau = \frac{CR\theta}{L}}$$

If q is shear stress induced at a radius r from the centre then.

$$\frac{\tau}{R} = \frac{q}{r}$$

Maximum torque transmitted by circular solid shaft : (Study the derivation) X

$$\boxed{T = \frac{\pi}{16} \tau D^3}$$

where,

T = max. torque

τ = Shear stress.

D = Diameter of the shaft.

Maximum torque transmitted by hollow circular shaft : (Study the derivation) X

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

D_o = outer diameter.

D_i = inner diameter.

Power transmitted by the shaft :

$$P = \frac{2\pi N T}{60} \text{ watt}$$

(or)

$$P = \omega \times T$$

where,

$$\omega = \frac{2\pi N}{60}$$

where, N = speed in r.p.m

T = torque.

NOTE :

If maximum torque is not given consider T as T_{max}.

PROBLEMS :

① Find the maximum shear stress induced in a solid circular shaft of dia 15 cm when the shaft transmits 150 kW power at 180 rpm.

GIVEN :

$T_{max} = ?$

$\tau = ?$

$D = 15\,cm \Rightarrow 15 \times 10^{-2}\,m$

$P = 150 \times 10^{3}\,W$

$N = 180$

$$P = \frac{2\pi NT}{60}$$

$$150 \times 10^{3} = \frac{2 \times \pi \times 180 \times T}{60}$$

$$T = 7961.78\ N\cdot m$$

$$T = \frac{\pi}{16}\tau D^{3}$$

$$7961.78 = \frac{\pi}{16} \times \tau \times (15 \times 10^{-2})^{3}$$

$$\tau = 12.02 \times 10^{6}\ N/m^{2}$$

(38)

(2) A solid steel shaft has to transmit 75 kW at 200 rpm taking allowable shear stress as 70 N/mm². Find suitable diameter for the shaft with max. torque transmitted at each revolution exceeds the mean by 30 %.

GIVEN :

$P = 75 \times 10^3\ W$

$N = 200\ rpm$

$\tau = 70\ N/mm^2 \Rightarrow 70 \times 10^6\ N/m^2$

$T_{max} = 100 + 30\ \%\ T$

$= 1.3T$

Solution :

To find diameter 'D'

$$P = \frac{2\pi NT}{60}$$

$$75 \times 10^3 = \frac{2 \times \pi \times 200 \times T}{60}$$

$$T = 3582.80\ N\text{-}m$$

$$T_{max} = 4657.6\ N\text{-}m$$

$$T = \frac{\pi}{16} \tau D^3$$

$$4657.6 = \frac{\pi}{16} \times 70 \times 10^6 \times D^3$$

$$D^3 = 3.390 \times 10^{-4}$$

$$D = 0.069\ m.$$

(3) A hollow shaft is to transmit 300 kW power at 80 rpm. If the shear stress is not to exceed 60 N/mm² and the internal dia is 0.6 times of the ext. dia. Find the external & internal diameters assuming that the max. torque is 1.4 times the mean.

GIVEN :

$P = 300 \times 10^3 \, W$

$N = 80 \, rpm$

$\tau = 60 \, N/mm^2$

$T_{max} = 1.4T$

$D_i = 0.6 D_o$

Sol :

To find Diameters D_o & D_i :

$$P = \frac{2\pi N T}{60}$$

$$300 \times 10^3 = \frac{2 \times \pi \times 80 \times T_{max} \, \cancel{1.4T}}{60}$$

$$T = \cancel{25591.44} \quad 35828.02 \text{ N-m}$$

$$T_{max} = 1.4T$$

$$= 1.4 \times \cancel{25591.44} \quad 35828.02 \Rightarrow 50159.23$$

$$T_{max} = \cancel{35828.02} \quad 50159.23 \text{ N-m.}$$

$$T_{max} = \frac{\pi}{16} \tau \frac{(D_o^4 - D_i^4)}{D_o}$$

$$35828.02 = \frac{\pi}{16} \times 60 \frac{(D_o^4 - (0.6D_o)^4)}{D_o}$$

$$35828.02 = 11.775 \, (0.870 \, D_o^3)$$

$$D_o = \cancel{349.37 \text{ mm}} \quad 0.169 \text{ m} //$$

$D_i = 0.101 \text{ m} //$

(3) In a hollow circular shaft of outer and inner dia. 20 cm & 10 cm respectively. The shear stress not to exceed 40 N/mm². Find the max. torque which the shaft can safely transmit.

(4) Two shafts of the same material & of same length are subjected to same torque. If the first shaft is of solid circular section and the second shaft is of hollow circular section whose internal dia is 2/3 of outside dia and the max. shear stress developed in each shaft is same. compare the weights of the shafts.

GIVEN :

T = Torque transmitted by shaft.

τ = Max. shear stress.

D = Dia. of solid shaft.

D_o = outer dia of hollow shaft.

$D_i = \frac{2}{3} D_o$

L = Length of the shaft.

W_s = weight of solid shaft.

W_h = weight of hollow shaft.

Solution :

Find the relation between dia of solid shaft & hollow shaft.

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \left[\frac{D_o^4 - \left(\frac{2}{3} D_o\right)^4}{D_o} \right]$$

$D = 0.92 D_o$

Density $= \dfrac{\text{Mass}}{\text{volume}}$

$\rho_s = \dfrac{\text{weight}}{\frac{\pi}{4} D^2 L}$ (mass $\times$ 9.81) $= \dfrac{W_s \times 9.81}{\frac{\pi}{4} D^2 L}$

$\rho_H = \dfrac{W_h \cancel{\times} 9.81}{\frac{\pi}{4}(D_o^2 - D_i^2) L}$

$\dfrac{W_s}{W_h} = \dfrac{\dfrac{\rho_s \times \frac{\pi}{4} D^2 \times L}{9.81}}{\dfrac{\rho_h \times \frac{\pi}{4}(D_o^2 - D_i^2) \times L}{9.81}}$ $\quad \rho_s = \rho_h$ $\quad$ ~~m = W × 9.81~~

$\dfrac{W_s}{W_h} = \dfrac{D^2}{(D_o^2 - D_i^2)} = \dfrac{(0.92 D_o)^2}{D_o^2 - \left(\frac{2}{3} D_o\right)^2}$ $\quad$ mass × 9.81 = W. $\quad m = \frac{W}{9.81}$

$$\boxed{\frac{W_s}{W_h} = \frac{1.52}{1}}$$

(4) A solid circular shaft and a hollow circular shaft whose inside dia is 3/4 of the outside dia made of same material of equal length and are required to transmit a given torque. Compare the weights of two shafts if the max. shear stress developed in the two shaft are equal.

143 Solution :

Relation b/w solid shaft & hollow shaft

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \left[\frac{D_o^4 - (\frac{3}{4} D_o)^4}{D_o} \right]$$

$$D^3 = 0.68 D_o^3$$

$$D = 0.88 D_o$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\rho = \frac{\text{weight (mass} \times 9.81)}{\frac{\pi}{4} D^2 L}$$

$$\rho_s = \frac{W_s \times 9.81}{\frac{\pi}{4} D^2 L}$$

$$\rho_h = \frac{W_h \times 9.81}{\frac{\pi}{4} (D_o^2 - D_i^2) L}$$

$$\frac{W_s}{W_h} = \frac{\frac{\rho_s \times \frac{\pi}{4} D^2 \times \cancel{L}}{\cancel{9.81}}}{\frac{\rho_h \times \frac{\pi}{4} (D_o^2 - D_i^2) \times \cancel{L}}{\cancel{9.81}}}$$

$$\frac{W_s}{W_h} = \frac{D^2}{(D_o^2 - D_i^2)} = \frac{(0.88 D_o)^2}{(D_o^2 - (\frac{3}{4} D_o)^2)}$$

$$\frac{W_s}{W_h} = \frac{1.77}{1}$$

(5) A hollow shaft has to transmit 85 kWh at 180 rpm and allowable shear stress as 95 N/mm². If the inner dia of the shaft is 4/5 of outer dia. Find the diameters of next. also Find the power and torque by considering the outer dia of hollow shaft as solid shaft diameter.

Solution:

Polar moment of Inertia (J):

$$J = \frac{\pi}{32} D^4$$

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$$CJ = \frac{Tl}{\theta}$$

20/8/14

POLAR MOMENT OF INERTIA (J):

It is defined as moment of inertia of the area about the axis perpendicular to the plane of figure and passing through the center of gravity of the area.

$$J = \frac{\pi}{32} D^4 \quad \text{— solid shaft}$$

$$J = \frac{\pi}{32} (D_o^4 - D_i^4) \quad \text{— Hollow shaft}$$

TORQUE IN TERMS OF POLAR M.O.I:

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

θ = Angle of twist

POLAR MODULUS: (Z_p)

It is the ratio of the polar moment of inertia to the radius of the shaft. It is also called as torsion section modulus.

$$Z_p = \frac{J}{R}$$

R - radius of the shaft.

Solid shaft, $Z_p = \frac{\pi}{16} D^3$

Hollow shaft, $Z_p = \frac{\pi}{16} \left(\frac{D_o^4 - D_i^4}{D_o} \right)$

Strength of the Shaft:

... shaft means the max torque

TORSIONAL RIGIDITY (or) STIFFNESS :

It is defined as the product of modulus of rigidity and polar moment of inertia of the shaft.

$$\text{Torsional rigidity} = C \times J$$

$$= \frac{TL}{\theta} \qquad \left[\frac{T}{J} = \frac{C\theta}{L}\right]$$

If $\theta = 1$ & $L = 1$ then

Torsional rigidity = Torque.

θ in terms of radian.

It is also defined as torque required to produce a twist of one radian per unit length of the shaft.

Problems :

(1). Determine the diameter of solid shaft which will transmit 90 kW at 160 rpm. Also determine the length of the shaft if the twist must not exceed $1°$ over the entire length. The max. shear stress is $60\ N/mm^2$. Take C as $8 \times 10^4\ N/mm^2$

GIVEN :

$P = 90 \times 10^3\ W$

$N = 160$ rpm.

$\theta = 1° = 1 \times \frac{\pi}{180}$ rad

$\tau = 60\ N/mm^2 = 60 \times 10^6\ N/m^2$

$C = 8 \times 10^4\ N/mm^2$

$= 8 \times 10^{10}\ N/m^2$

Solution : Dia. of Shaft :

1) $P = \frac{2\pi N T}{60}$

$$90 \times 10^3 = \frac{2 \times \pi \times 160 \times T}{60}$$

$$T = 5374.20 \; N/m^2 \; N\text{-}m$$

$$T = \frac{\pi}{16} \tau D^3$$

$$5374.20 = \frac{\pi}{16} \times 60 \times 10^6 \; D^3$$

$$D^3 = 4.564 \times 10^{-4}$$

$$D = 0.076 \; m$$

$$\boxed{D = 76 \; mm}$$

ii) Length of the shaft :

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{R}$$

$$J = \frac{\pi}{32} \times D^4$$

$$J = \frac{\pi}{32} \times 0.076^4 = 3.27 \times 10^{-6}$$

$$\frac{5374.20}{3.27 \times 10^{-6}} = \frac{8 \times 10^{10} \times 0.0174}{L}$$

$$L = 0.88 \; m$$

$$L = 880 \; mm$$

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② A hollow shaft having inside diameter 60% of its outside diameter is to replace a solid shaft transmitting the same power at the same speed. Calculate % of saving in material if the material to be used is also same.

GIVEN:

Saving in material

$d_i = 60\% \; D_o$

$D_i = 0.6 D_o$

Solution:

Saving in material = Saving in area

$$= \frac{A_s - A_h}{A_s}$$

Find the relationship between dia of solid & hollow shaft.

$\therefore T_s = T_h$

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \frac{(D_o^4 - D_i^4)}{D_o}$$

$$D^3 = \frac{(D_o^4 - (0.6 D_o)^4)}{D_o}$$

$$D^3 = \cancel{0.64 D_o^3} \; \cancel{0.1296 D_o^3} \quad 0.8704 D_o^3$$

$$D = 0.95 D_o$$

$$A_s = \frac{\pi}{4} D_i^2$$

$$A_s = \frac{\pi}{4} (0.95 D_o)^2$$

$$= 0.708 D_o^2 m^2$$

$$A_h = \frac{\pi}{4} (D_o^2 - D_i^2)$$

$$= \frac{\pi}{4} (D_o^2 - (0.6 D_o)^2)$$

$$A_h = 0.502 D_o^2 m^2$$

Saving in material $= \frac{0.708 D_o^2 - 0.502 D_o^2}{0.708 D_o^2}$

$= 0.2909$

% of saving in material $= 0.29 \times 100$

$= 29\ \%$

0 3

(751)

③. A hollow shaft of diameter ratio $\frac{3}{8}$ (internal to outer dia) is to transmit 375 kW power at 100 rpm. The maximum torque beam being 20% greater than the mean. The shear stress not to exceed 60 N/mm² and twist in a length of 4 m not to exceed 2°. Calculate its internal and external diameter which would satisfy both the above conditions. Assume & $C = 0.85 \times 10^5$ N/mm².

GIVEN :

$$\frac{D_i}{D_o} = \frac{3}{8}$$

$$D_i = \frac{3}{8} D_o$$

$$P = 375 \times 10^3 \text{ W}$$

$$N = 100 \text{ rpm}$$

$$T_{max} = 1.2T \qquad = T + 0.2T$$

$$L = 4 \text{m}$$

$$\theta = 2^\circ = 2 \times \frac{\pi}{180}$$

$$C = 0.85 \times 10^{11} \text{ N/m}^2$$

Solution :

$$P = \frac{2\pi NT}{60}$$

$$375 \times 10^3 = \frac{2 \times \pi \times 100 \times T}{60}$$

$$T = 35.8 \times 10^3 \text{ N-m}$$

$$T_{max} = 1.2 \times 35.8 \times 10^3$$

$$T \quad = 42960 \text{ N-m}$$

(1) Dia. of shaft when τ not to exceed $60 N/mm^2$:

For Hollow shaft, $T_{max} = \frac{\pi}{16} \tau \left(\frac{D_o^4 - D_i^4}{D_o} \right)$

$$42960 = \frac{\pi}{16} \times 60 \times 10^6 \left(\frac{D_o^4 - (\frac{3}{8} D_o)^4}{D_o} \right)$$

$$D_o = 0.16 m \simeq 160 mm$$

$$\therefore \quad D_i = \frac{3}{8} D_o \Rightarrow \frac{3}{8} \times 0.16$$

$$D_i = 0.06 m \simeq 60 mm.$$

ii) Dia of shaft when twist not to exceed $2°$:

$$\frac{T}{J} = \frac{C\theta}{L} = \cancel{\frac{\tau}{R}} \quad \text{(should not use)}$$

$$J = \frac{\pi}{32} (D_o^4 - D_i^4) \qquad T = \frac{\pi}{16} \tau (\ldots$$

$$J = \frac{\pi}{32} (0.16^4 - 0.06^4) \qquad \frac{LT}{C\theta} = J$$

$$J = 6.30 \times 10^{-5}$$

$$= 0.0962 \, D_o^4 \, m^4$$

$$D_o = 0.158 m \simeq 158 mm$$

$$D_i = 0.059 \simeq 59 mm.$$

Always take larger value for safe design.

$$D_o = 160 mm$$

$$D_i = 60 mm$$

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④ Determine the dia. of solid shaft which will transmit 300 KW at 250 rpm. The max. shear stress should not exceed 30 N/mm^2 and twist should not be more than 1° in a shaft length of 2m. Take $C = 1 \times 10^5\ N/mm^2$.

H.W

⑤ A hollow shaft having an internal diameter 40% of its external diameter, transmits 562.5 KW power at 100 rpm. Determine external diameter of shaft. If the shear stress not to exceed 60 N/mm^2 and twist in a length of 2.5 m should not exceed 1.3°. Assume maximum torque = 1.25 mean torque and modulus of rigidity is $9 \times 10^4\ N/mm^2$

H.W

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④ ANS

Given:

$P = 300 \times 10^3$ W

$N = 250$ rpm

$\tau = 30\ N/mm^2$

$\theta = 1° = \pi/180$

$L = 2m$

$C = 1 \times 10^5\ N/mm^2$

Sol:

$$P = \frac{2\pi N T}{60} = \cancel{2 \times \pi \times}$$

$$300 \times 10^3 = \frac{2 \times \pi \times 250 \times T}{60}$$

$$T = 11464.96\ N\text{-}m$$

i) Diameter of the shaft when τ not to exceed $30\,N/mm^2$

for solid shaft,

$$T = \frac{\pi}{16}\tau D^3$$

$$11464.96 = \frac{\pi}{16} \times 80\times10^6 \times D^3$$

$$D^3 = 1.947\times10^{-3}$$

$$D = 0.124\ m$$

$$D = 124\ mm$$

ii) Diameter of shaft when twist should not be more than $1°$.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$ all in mm

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$J = \frac{\pi}{32}\times D^4$$

$$\cancel{\tau\frac{\pi}{32}}\ 1000\times\frac{11464.96}{J} = \frac{10^5\times\frac{\pi}{180}\cancel{\times10^6}}{2\times1000}$$

$$J = 13.144\times10^6$$

$$10^6\times13.144 = \frac{\pi}{32}\times D^4$$

$$D^4 = 133.95\times10^6$$

$\cancel{D = 3.40\ m}$ $D = 107.58\ mm$

$\cancel{D = 340\ mm.}$

If the diameter 107.58 is taken.

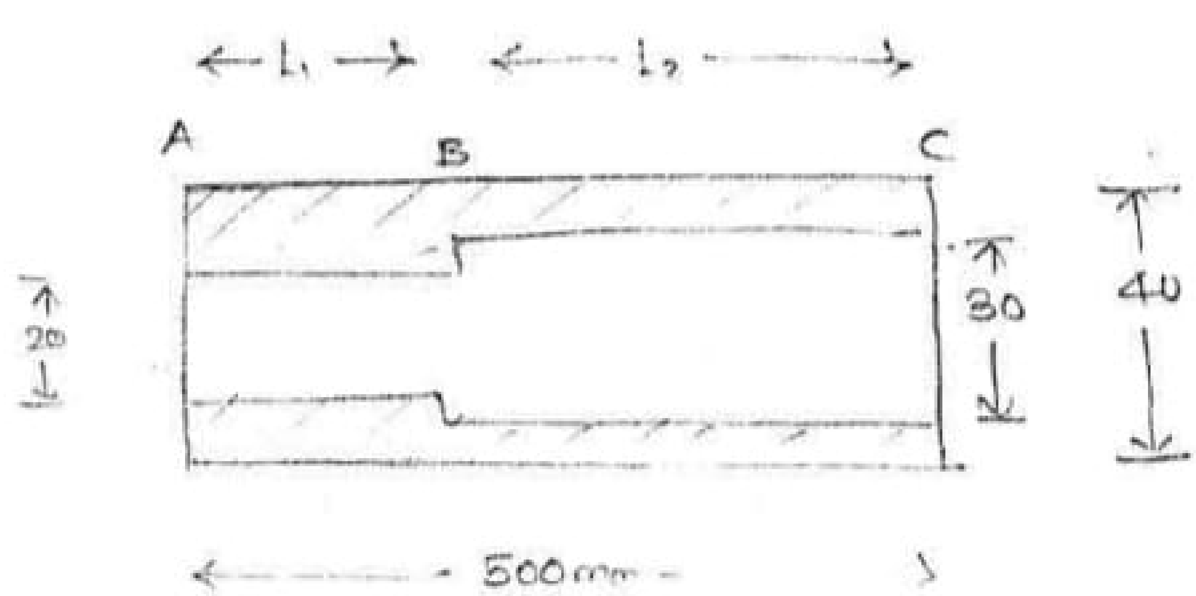
$$\therefore\ 11464.96\times10^3 = \frac{\pi}{16}\times\tau\times(107.58)^3$$

$$\boxed{\tau = 46.92\ N/mm^2}$$

STRENGTH OF SHAFT OF VARYING SECTIONS :

When a shaft is made up of different length and of different diameter. The torque transmitted by individual section should be calculated first. The strength of such a shaft is the minimum value of least there torque.

① A shaft ABC of 500 mm length and 40 mm external diameter is bored for a part of its length AB to a 20 mm diameter and for remaining length BC to a 30 mm diameter bore. If the shear stress not to exceed 80 N/mm² . Find the maximum power the shaft can transmit at a speed of 200 rpm. If the angle of twist in the length of 20 mm diameter bore is equal to the 30 mm diameter bore find the length of the shaft for 20 mm and 30 mm bore.



GIVEN :

$L = 500$ mm
$D_o = 40$ mm
$D_{i_2} = 30$ mm
$\tau = 80$ N/mm²

Solution :

TO find max. power :

$$T = \frac{\pi}{16} \tau \frac{(D_o^4 - D_i^4)}{D_o}$$

$$T_1 = \frac{\pi}{16} \tau \frac{(D_o^4 - D_{i_1}^4)}{D_o^{\cancel{4}}}$$

$$= \frac{\pi}{16} \times 80 \frac{(40^4 - 20^4)}{40^{\cancel{4}}}$$

$$= \cancel{235.5} \quad 942 \times 10^3 \text{ N-mm}$$

$$T_2 = \frac{\pi}{16} \tau \frac{(D_o^4 - D_{i_2}^4)}{D_o^{\cancel{4}}}$$

$$= \frac{3.14}{16} \times 80 \frac{(40^4 - 30^4)}{40}$$

$$T_2 = 686.87 \times 10^3 \text{ N-mm}$$

Safe power transmitted is minimum of two values $= 686.87 \times 10^3$

$= 687$ N-m.

$$P = \frac{2\pi N T}{60}$$

$$P = \frac{2 \times \pi \times 200 \times 687}{60}$$

$$P = 14381.2 \text{ W}$$

$$P = 14.38 \text{ KW}$$

ii) To find length of AB & BC L_1 & L_2 :

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\theta = \frac{TL}{JC}$$

(for safe T & C are same for the given shaft.)

$$\therefore \theta_{AB} = \frac{TL_1}{J_1 C} \qquad \theta_{BC} = \frac{TL_2}{J_2 C}$$

$$\therefore \frac{\cancel{T}L_1}{J_1 \cancel{C}} = \frac{\cancel{T}L_2}{J_2 \cancel{C}}$$

$$\frac{L_1}{J_1} = \frac{L_2}{J_2}$$

$$J_1 = \frac{\pi}{32}(Do^4 - Di_1^4)$$

$$= \frac{\pi}{32}(40^4 - 20^4)$$

$$= 235.50 \times 10^3 \text{ mm}^4$$

$$J_2 = \frac{\pi}{32}(Do^4 - Di_2^4)$$

$$= \frac{\pi}{32}(40^4 - 30^4)$$

$$= 171.71 \times 10^3 \text{ mm}^4$$

$$\therefore \frac{L_1}{235.5 \times 10^3} = \frac{L_2}{171.71 \times 10^3}$$

$$L_1 = 1.375 L_2$$

WKT, $L_1 + L_2 = 500$

$$1.375 L_2 + L_2 = 500$$

$$\boxed{\begin{aligned} L_2 &= 210.5 \text{ mm} \\ L_1 &= 289.5 \text{ mm} \end{aligned}}$$

② A steel shaft ABCD having a total length of 2.4m consist of three length having different section as follows. AB is hollow having inside & outside diameter of 80 mm and 50 mm, then CD diameter of 70 mm, BC diameter of 80 mm. If the angle of twist is same for each section: Determine the length of each section and the total angle of twist if the maximum shear stress in the hollow portion is 50 N/mm². Take $C = 8.2 \times 10^4$ N/mm².

GIVEN :

$D_o = 0.4 D_o$

$D = 562.5 \times 10^3$ watt

$N = 100$ rpm

$\tau = 60$ N/mm²

$L = 2.5$ m

$\theta = 1.3 \times \frac{\pi}{180}$

$T_{max} = 1.25\ T$

$C = 9 \times 10^4$ N/mm²

$$P = \frac{2\pi N T}{60}$$

$$T_{max} = \frac{P \times 60}{\ldots} \Rightarrow \frac{562.5 \times 10^3 \times 60}{\ldots 100}$$

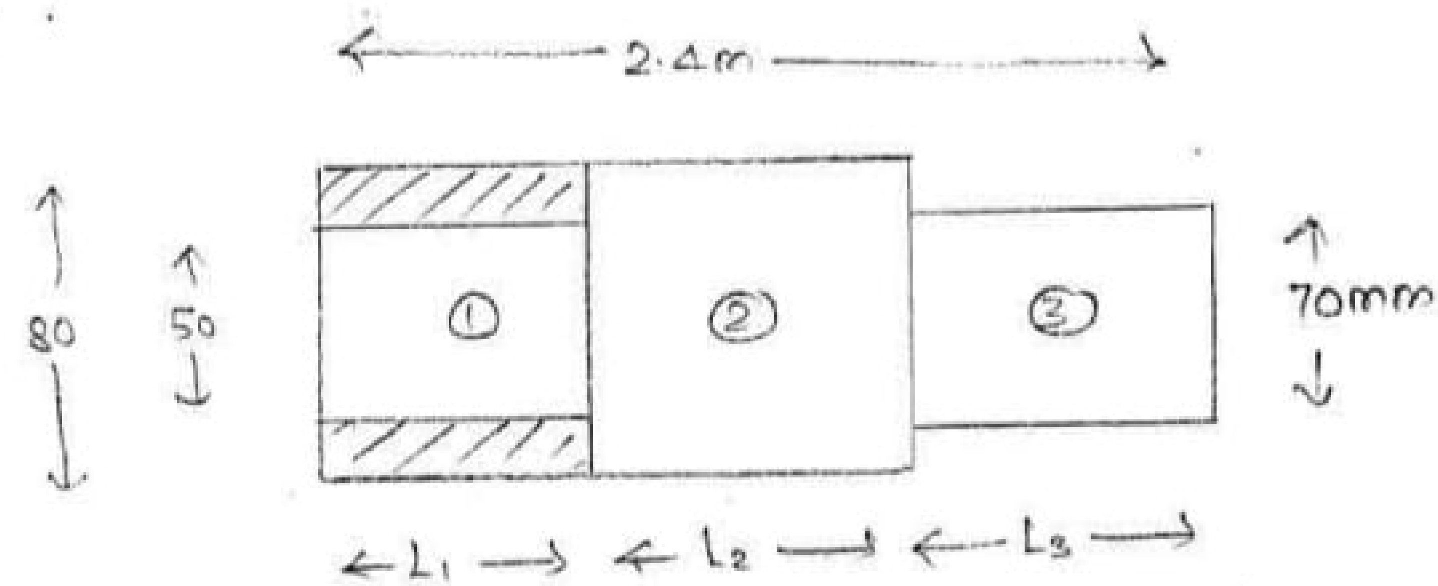
$T_{mean} = 53742.03822$ Nm

$= 53742.03 \times 10^3$ Nmm

$T_{max} = 1.25 \times 53742.03 \times 10^3$

$= 67177537.5$ N-mm.

i) Diameter of the hollow shaft when a



GIVEN :

$L = 2.4 = 2400$ mm

Shaft AB : Length L_1

$D_{o1} = 80$mm

$D_{i1} = 50$mm

Shaft BC. :

Length $= L_2$

$D_2 = 80$mm.

Shaft CD :

Length $= L_3$

$D_3 = 70$ mm

Angle of twist, $\theta_1 = \theta_2 = \theta_3$

max τ, in hollow portion $= 50$ N/mm²

Solution :

To find L_1, L_2, L_3 :

W.K.T.

$$\frac{T}{J} = \frac{C\theta}{L}$$

$\therefore \theta_1 = \theta_2 = \theta_3$

$$\theta_1 = \frac{T_1 L}{CJ_1} , \quad \theta_2 = \frac{T_2 L}{CJ_2} ; \quad \theta_3 = \frac{T_3 L}{CJ_3}$$

$$\therefore \frac{T_1 L_1}{\cancel{C} J_1} = \frac{T_2 L_2}{\cancel{C} J_2} = \frac{T_3 L_3}{C J_3}$$

$$J_1 = \frac{\pi}{32} (D_o^4 - D_i^4)$$

$$= \frac{\pi}{32} (80^4 - 50^4)$$

$$= 3405918.75$$

$$= 340.5918 \times 10^4 \ mm^4$$

$$J_2 = \frac{\pi}{32} D^4$$

$$= \frac{\pi}{32} \times 80^4$$

$$= 401.920 \times 10^4 \ mm^4$$

$$J_3 = \frac{\pi}{32} D^4$$

$$= \frac{\pi}{32} \times 70^4$$

$$= 235.59 \times 10^4 \ mm^4$$

$$\frac{L_1}{340.5\times10^4} = \frac{L_2}{401.92\times10^4} = \frac{L_3}{235.5\times10^4}$$

$L_1 = \cancel{0.847 L_2}\ 1.44\times L_3$

$L_2 = 1.71\times L_3$

$L_1 + L_2 + L_3 = L$

$1.44 L_3 + 1.71 L_3 + L_3 = 2.4\times10^{+3}$

$4.15 L_3 = 2.4\times10^3$

$\boxed{L_3 = \cancel{0.57}\ mm}$ $\boxed{L_3 = 578\ mm}$

$L_1 = 1.44\times0.57$

$\boxed{L_1 = \cancel{0.83}\ mm}$ $\boxed{L_1 = \cancel{988}\ 832\ m}$

$L_2 = 1.71\times0.57$

$\boxed{L_2 = \cancel{0.988}\ mm}$ $\boxed{L_2 = 988\ mm}$

On 1 2

To find total angle of twist:

$\theta = \theta_1 + \theta_2 + \theta_3$

$$\frac{C\theta_1}{L_1} = \frac{\tau_1}{R_1}$$

$$\frac{8.2\times10^4\times\theta_1}{2.4\times10^3} = \frac{50}{40}$$

$\theta_1 = 0.0126$ rad.

Since $\theta_1 = \theta_2 = \theta_3$

$$\theta = 0.7273 + 0.7273 + 0.7273$$

$$\boxed{\theta = 2.1819^\circ}$$

COMPOSITE SHAFT :

Composite shaft is made up of two or more different materials.

Conditions : ① :

Total torque = Sum of torques transmitted by individual shaft.

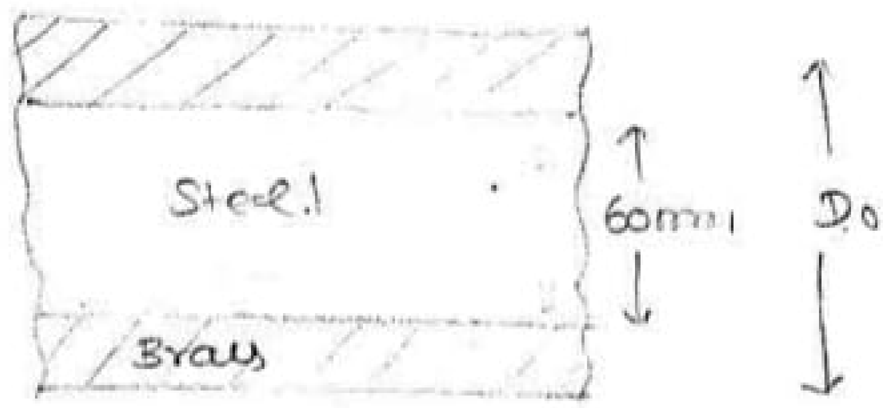
$$T = T_1 + T_2$$

Condition : ② :

Angle of twist is same. (i.e) $\theta_1 = \theta_2$

① A composite shaft consist of steel rod 60 mm diameter surrounded by a closely fitting tube of brass. Find the outside diameter of the brass tube so that when a torque of 1000 N-m is applied to the composite shaft, it will be shared equally by the two materials. Take C ~~F~~ for steel as 8.4×10^4 N/mm² and C for brass 4.2×10^4 N/mm². Find also the maximum shear stress in each material and common angle of twist in a length of 4 m.

GIVEN :



For steel :

$D = 60$ mm

$T = 1000$ N-m

$= 1000 \times 10^3$ N-mm

Torque equally shared.

$T_s = 500 \times 10^3$ N-mm

$T_b = 500 \times 10^3$ N-mm

$C_s = 8.4 \times 10^4$ N/mm²

$C_b = 4.2 \times 10^4$ N/mm²

For brass :

$D_o = ?$

$D_i = 60$mm

Solution :

To find D_o :

w.k.T $\theta_s = \theta_b$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{T_s L_s}{J_s C_s} = \frac{T_b L_b}{J_b C_b} \qquad \left(\begin{matrix} L_s = L_b \\ T_s = T_b \end{matrix}\right)$$

$$J_b C_b = J_s C_s$$

$$J_b = \frac{\pi}{32} (D_o^4 - D_i^4)$$

$$J_b = \frac{\pi}{32} (D_o^4 - 60^4)$$

$$J_b = 0.098 D_o^4 - 12.717 \times 10^5$$

$$J_s = \frac{\pi}{32} (D^4)$$

$$= \frac{\pi}{32} \times 60^4 \Rightarrow 12.71 \times 10^5$$

$$(0.098 D_o^4 - 12.717 \times 10^5)(4.2 \times 10^4) = (12.71 \times 10^5)(8.4 \times 10^4)$$

$$4116 D_o^4 - 5.34 \times 10^{10} =$$

$$D_o = 78.96 N \simeq 79 mm$$

τ in each material :

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{R}$$

$$\frac{T_s}{-} = \frac{\tau_s}{R_s} \quad - P/.$$

(16.5)

$$\frac{T_b}{J_b} = \frac{\tau_b}{R_b} \rightarrow D_o/2$$

$$700 \times 10^3 = \frac{\pi}{16} \times \tau (60)^3$$

$$\tau_s = 11.79$$

(H.w) A composite shaft consist of copper rod of 30mm diameter enclosed in a steel tube of external diameter 50mm and 10mm thick. The shaft is required to transmit a torque of 1000Nm determine the shear stress developed in copper and steel if both the shaft have equal length and welded to a plate at each end. So that the twist are equal. Take modulus of rigidity of steel is twice of copper.

$$\frac{T_s}{J_s} = \frac{C_s \theta_s}{L_s}$$

$$\frac{T_c}{J_c} = C_c \theta_c$$

22/8/14. COMBINED BENDING AND TORSION :

W.K.T $\frac{M}{I} = \frac{\sigma}{y}$

① Solid shaft :

Major principle stresses $= \frac{16}{\pi D^3} \left(M + \sqrt{M^2 + T^2} \right)$

Minor principle stress $= \frac{16}{\pi D^3} \left(M - \sqrt{M^2 + T^2} \right)$

Max. shear stress $\tau = \frac{16}{\pi D^3} \sqrt{M^2 + T^2}$

② Hollow shaft :

Major principle stresses $= \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(M + \sqrt{M^2 + T^2} \right)$

Minor principle stresses $= \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(M - \sqrt{M^2 + T^2} \right)$

Max. shear stress $\tau = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(\sqrt{M^2 + T^2} \right)$

③. $\tan 2\theta = \frac{T}{M}$

5. strain energy in solid shaft, $U = \frac{\tau^2}{4C} V \quad \left(V = \frac{\pi}{4} D^2 L \right)$

Strain energy in Hollow shaft, $U = \frac{\tau^2}{4 C D_o^2} (D_o^2 + D_i^2) V$

$\left(V = \pi (D_o^2 - D_i^2) L \right)$

where D is Diameter of the shaft.

T is torque.

M is bending moment.

θ angle of oblique plane.

C Modulus of rigidity.

V Volume of the shaft.

U Strain energy.

(1) A solid shaft of diameter 80mm subjected to a twisting moment of 8 MN mm and bending moment of 5 MN mm at a point determine principle stresses and position of the plane on which they act also find strain energy stored taking $C = 8 \times 10^4 \; N/mm^2$

GIVEN :

$T = 8 \times 10^6$ N-mm , $D = 80$mm

$M = 5$ MN-mm

$= 5 \times 10^6$ N-mm

$C = 8 \times 10^4 \; N/mm^2$

Sol :

To find principle stresses :

$$\sigma_{major} = \frac{16}{\pi D^3}\left(M + \sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left(5\times10^6 + \sqrt{(5\times10^6)^2 + (8\times10^6)^2}\right)$$

$$= 143.6 \; N/mm^2$$

$$\sigma_{min} = \frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left((5\times10^6) \mp \sqrt{(5\times10^6)^2+(8\times10^6)^2}\right)$$

$$= (9.95\times10^{-6})\left(5\times10^6 - \sqrt{(5\times10^6)^2+(8\times10^6)^2}\right)$$

$$\sigma_{min} = -44.12 \text{ N/mm}^2$$

ii) Position of plane "θ"

$$\tan 2\theta = \frac{T}{M}$$

$$2\theta = \tan^{-1}\left(\frac{T}{M}\right)$$

$$2\theta = \tan^{-1}\left(\frac{8\times10^6}{5\times10^6}\right)$$

$$\theta = 28^\circ 59'$$

iii) strain energy stored 'U':

$$U = \frac{\tau^2}{4C} V.$$

$$\tau = \frac{16}{\pi D^3}\sqrt{M^2+T^2}$$

$$= \frac{16}{\pi\times 80^3}\sqrt{(8\times10^6)^2+(5\times10^6)^2}$$

$$= 93 \text{ N/mm}^2$$

If 'L' is not given assume $L = 1\,m$

$= 1000\,mm.$

$$U = \cancel{138} \frac{93^2}{4 \times 8 \times 10^4} \times V$$

$$V = \frac{\pi}{4}(D^2 \times L)$$

$$= \frac{\pi}{4}(80^2 \times 1000)$$

$$= 50.24 \times 10^5 \text{ mm}^3$$

$$\therefore U = \frac{93^2}{4 \times 8 \times 10^4} \times 50.24 \times 10^5$$

$$= 138 \times 10^3 \text{ N-mm}/\cancel{s}$$

$$= 138 \text{ N-m}/\cancel{s} \qquad (J = N\text{-}m)$$

$$= 138 \text{ J}$$

(2). The maximum allowable shear stress in a hollow shaft external dia is twice the internal dia, τ is 80 N/mm². Determine the diameter of the shaft if it is subjected to a torque of 4×10^6 N-mm and Bending moment of 3×10^6 N-mm also find strain energy. Take $C = 4.5 \times 10^4$ N/mm² and length of the bar as 2.5 m

GIVEN :

$D_o = 2D_i$

$\tau = 80 \text{ N/mm}^2$, $L = 2.5 \times 10^{-3}$ mm.

$T = 4 \times 10^2$ N-mm

$M = 3 \times 10^6$ N-mm²

Solution :

$$\sigma_{max} = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(M + \sqrt{M^2 + T^2} \right)$$

$$= \frac{16 \times 2D_i}{\pi ((2D_i)^4 - D_i^4)} \left((3\times10^6) + \sqrt{(3\times10^6)^2 + (4\times10^2)^2} \right)$$

$$= \frac{0.679}{D^3}$$

$$\tau = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \sqrt{M^2 + T^2}$$

$$80 = \frac{16 D_i \times 2}{\pi ((2D_i)^4 - D_i^4)} \sqrt{(3\times10^6)^2 + (4\times10^2)^2}$$

$$D_i = 34.88 \simeq 35 \text{ mm}$$

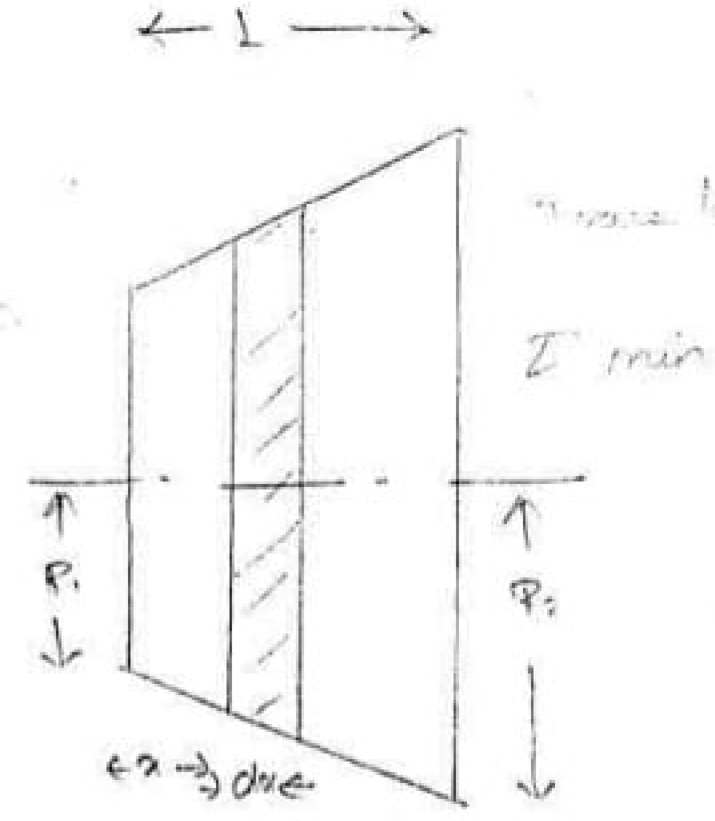
$$D_o = 69.76 \simeq 70 \text{ mm}$$

Strain energy :

$$U = \frac{\tau^2}{4 C D_o^2} (D_o^2 + D_i^2) V$$

23/8/14. (171)

TORSION OF TAPERING SHAFT :



$$T = \frac{\pi}{16}\tau_1 D_1^3 = \frac{\pi}{16}\tau_2 D_2^3$$

$$= \frac{\pi}{16}\tau_x D_x^3$$

$$\theta = \frac{32T}{\pi C} \times \frac{1}{3K}\left(\frac{1}{D_1^3} - \frac{1}{D_2^3}\right) \text{radians}$$

$$K = \frac{D_2 - D_1}{L} \text{ (constant)}$$

(1) Determine the angle of twist and max. shear stress developed in a shaft which tapers uniformly from a Diameter of 160 mm to a diameter of 240 mm. The length of the shaft is 2m and transmit a torque of 48 KNm. Take $C = 80$ GN/m²

GIVEN :

$D_1 = 160$ mm

$D_2 = 240$ mm

$L = 2000$ mm

$T = 48$ KN-m

$= 48 \times 10^6$ N-m

$C = 80$ GN/m²

$= 80 \times 10^9 \times \frac{1}{10^6}$ N/mm²

Solution :

i). Angle of twist 'θ':

$$\theta = \frac{32T}{\pi C} \times \frac{1}{3k}\left(\frac{1}{D_1^3} - \frac{1}{D_2^3}\right)$$

$$= \frac{32 \times 48 \times 10^6}{\pi \times 80 \times 10^3} \times \frac{1}{3 \times k}\left(\frac{1}{160^3} - \frac{1}{140^3}\right)$$

$$k = \left(\frac{D_2 - D_1}{L}\right) \Rightarrow \left(\frac{2\cancel{4}0 - 160}{2000}\right) = \frac{\cancel{0.062}}{0.04}$$

$$= \frac{32 \times 48 \times 10^6}{\pi \times 80 \times 10^3} \times \frac{1}{3 \times 0.04}\left(2.44 \times 10^{-7} - 3.644 \times 10^{-7}\right)$$

$$= \frac{6114.64}{\cancel{0.008}\ 0.12}\left(-1.2 \times 10^{-7}\right)$$

$$\theta = 0.0087 \text{ radians}$$

$$= 0.501^\circ.$$

ii) Max τ developed:

WKT, $T = \frac{\pi}{16} \tau D^3$

Max τ is in min dia) $T = \frac{\pi}{16} \times \tau \times D_1^3$

$$48 \times 10^6 = \frac{\pi}{16} \times \tau \times (160)^3$$

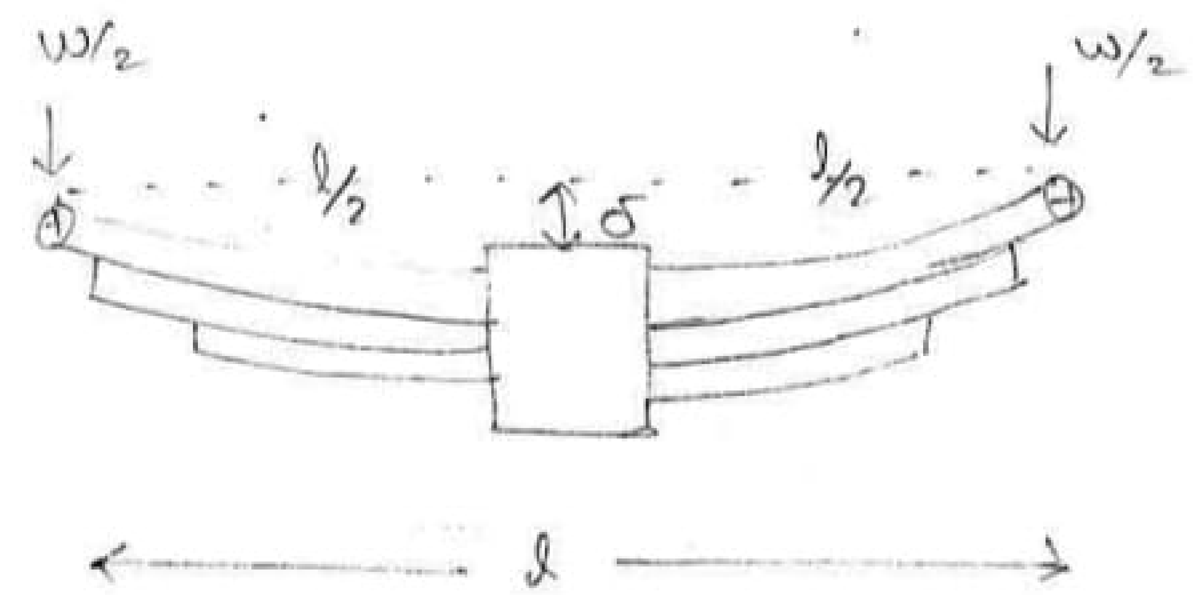
$$\tau = 59.96 \text{ N/mm}^2$$

SPRINGS :

1. Leaf spring.
2. Helical spring.

LEAF SPRING :

X



B.M = Moment of Resistance —— ①

B.M at centre $= \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$

Moment of ~~Inertia~~ Resistance $= \frac{M}{I} = \frac{\sigma}{y}$

$$I = \frac{bt^3}{12} \; ; \; y = t/2$$

$$M = \frac{\sigma \times I}{y} = \frac{\frac{\sigma \times bt^3}{12}}{t/2}$$

$$M = \frac{\sigma b t^2}{6}$$

Total Moment of Resistance, $M = n \times \frac{\sigma b t^2}{6}$

n – no. of spring plates

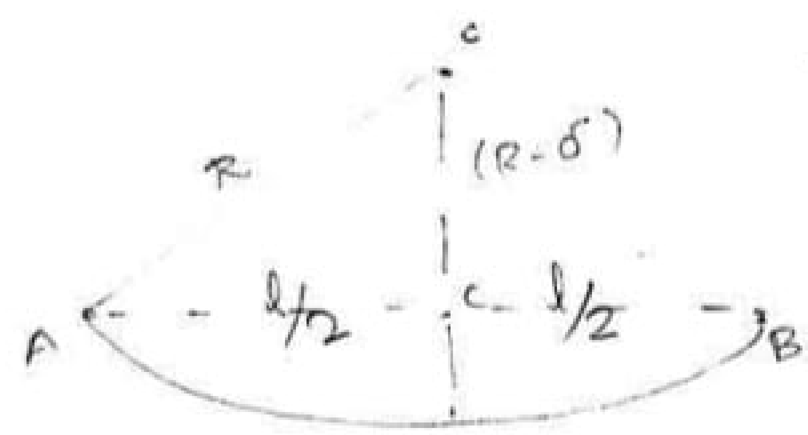
sub in eqn 1

$$\frac{WL}{4} = \frac{n\sigma b t^2}{6}$$

$$\sigma = \frac{\overset{3}{\cancel{6}}}{{}_{2}\cancel{4}} \frac{WL}{nbt^2}$$

$$\boxed{\sigma = \frac{3WL}{2nbt^2}}$$

EXPRESSION FOR CENTRAL DEFLECTION OF LEAF SPRING:



$$AO^2 = AC^2 + CO^2$$

$$R^2 = \left(\frac{l}{2}\right)^2 + (R-\delta)^2$$

$$R^2 = \frac{l^2}{4} + R^2 + \delta^2 - 2R\delta$$

($\delta^2 <<$ ∴ neglecting δ^2)

$$2R\delta = \frac{l^2}{4}$$

$$\boxed{\delta = \frac{l^2}{8R}}$$

w.k.T

$$\frac{\sigma}{Y} = \frac{E}{R}$$

$$R = \frac{EY}{\sigma} \quad (Y = t/2)$$

$$R = \frac{Et}{2\sigma}$$

$$\delta = \frac{l^2}{48\left(\frac{Et}{2\sigma}\right)}$$

$$\boxed{\delta = \frac{\sigma l^2}{4Et}}$$

where,

E = Young's modulus.

L = Length of the spring.

T = Thickness of the spring

σ = Bending stress.

① A laminated spring 0.9m long is made up of plates each 5cm wide and 1cm thick. If the bending stress is limited to 120 N/mm² . How many plates req. to carry a central load of 2.65 KN. If E = 2×10^5 N/mm² . what is the deflection under the load.

GIVEN:

$L = 0.9\times10^3$ mm

$b = 5\times10^2$ mm

$T = 1\times10^2$ mm

$\sigma = 120$ N/mm²

$N = ?$

$P = 2.65$ kN $= 2.65\times10^3$ N

$E = 2\times10^5$ N/mm²

Solution :

1). No. of plates :

$$\sigma = \frac{3wl}{2nbt^2}$$

$$120 = \frac{3 \times 2.65 \times 10^3 \times 0.9 \times 10^3}{2 \times n \times 5 \times 10^2 \times (1 \times 10^2)^2}$$

$$n = 5.96$$

$$n \simeq 6 \text{ plates.}$$

ii) $$\delta = \frac{\sigma l^2}{4Et}$$

$$= \frac{120 \times (0.9 \times 10^3)^2}{4 \times 2 \times 10^5 \times 1 \times 10^2}$$

$$\delta = 1.215$$

$$= 12.15 \text{ mm}$$

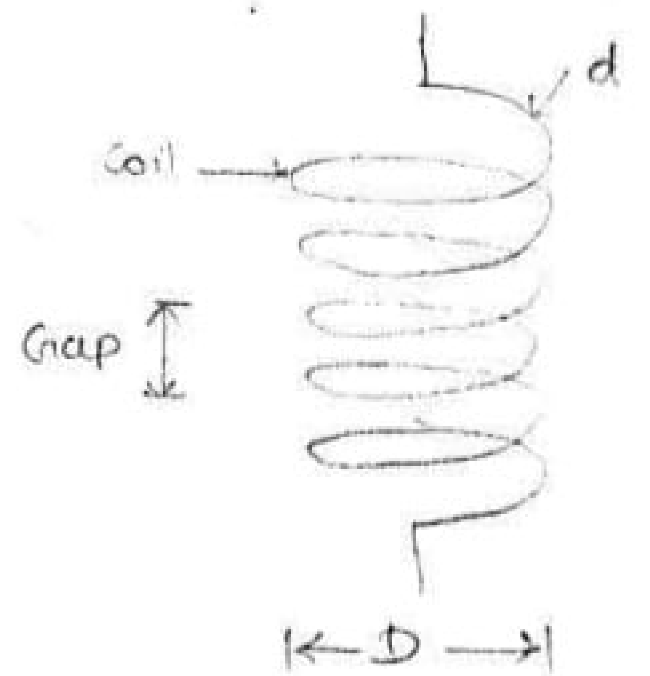
HELICAL SPRINGS :

Two types :

i). open coil springs

ii) closed coil springs

Expression for maximum shear stress induced in the wire:



Twisting moment,

$$T = W \times R$$

Also, $T = \frac{\pi}{16} \tau d^3$

$$W \times R = \frac{\pi}{16} \tau d^3$$

$$\boxed{\tau = \frac{16 WR}{\pi d^3}}$$

$d \Rightarrow$ diameter of spring wire.

$n \Rightarrow$ Number of coils.

$R \Rightarrow$ Mean radius of spring coil.

$W \Rightarrow$ Axial load on the spring

$\theta \Rightarrow$ Angle of twist in spring wire.

$\delta \Rightarrow$ deflection of spring due to axial load

$L \Rightarrow$ Length of the wire.

Expression for deflection of spring :

workdone = Energy stored

workdone $= \frac{1}{2} W \delta$

Energy stored $U = \frac{\tau^2}{4C}$ volume

$$\text{Volume} = \frac{\pi}{4} d^2 \times l$$

l = Circumference of coil $\times$ no. of coils.

$= 2\pi R \times n$

$$U = \left(\frac{16WR}{\pi d^3}\right)^2 \times \left(\frac{\pi}{4} d^2 \times 2\pi R n\right)$$

$$U = \frac{32 W^2 R^3 n}{C d^4}$$

$$\frac{1}{2} W \delta = \frac{32 W R^2 n}{C d^4}$$

$$\boxed{\delta = \frac{64 W R^3 n}{C d^4}}$$

Expression for stiffness of spring : (S)

$$S = \frac{W}{\delta}$$

$$= \frac{W}{\frac{64 W R^3 n}{C d^4}}$$

$$\therefore S = \frac{Cd^4}{64R^3n}$$

Solid length of the spring is nothing but length of the spring when adjacent coils are in contact with each other without any gap.

Solid length = no. of coils x dia. of wire:

= $n \times d$.

A close

(C). A closely coile helical spring made of 10 mm diameter steel wire. has 15 coils of 100 mm mean diameter. The spring is subjected to an axial load of 100 N. calculate i) maximum shearstress induced. ii) Deflection iii) stiffness of the spring. Take $C = 8.16 \times 10^4$ N/mm²

GIVEN :

d = 10mm ; 5mm
n = 15 coils.
P = 100N.

Sol :

i) Max. shear stress :

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 100 \times 50}{\pi \times 10^3}$$

τ = 2.54 x10 ⇒ 25.4

ii) Stiffness :

$$\delta = \frac{64 W R^3 n}{C d^4} \qquad \delta$$

$$= 64 \times 100 \times$$

$$S = \frac{C d^4}{64 R^3 n} \Rightarrow \frac{8.16 \times 10^4 \times (10)^4}{64 \times (}$$

(2) A closely coiled helical spring is to carry a load of 500 N if mean coil diameter is 10 times that of wire diameter. calculate the diameters if the maximum shear stress is 80 N/mm2. Also find the no. of coils if the stiffness of the spring is 20 N/mm and $C = 8.4 \times 10^4$ N/mm2. Also find the frequency of vibration for the load carried by it.

GIVEN:

$C = 8.4 \times 10^4$ N/mm^2

$W = 500$ N

$D = 10d$

$\tau = 80$ N/mm^2

Solution :

To find D & d :

W.K.T.

$$\tau = \frac{16\,WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times \cancel{10d}\ 5d.}{3.14 \times d^3}$$

$\therefore d^2 = \cancel{318.47}\ \ 159.23$ 6

$d = \cancel{17.84\ mm}\ \ 12.61\ mm$

$R = D/2$

$D = 10d$

$D = 10 \times \cancel{17.84}\ \ 12.61$

$D = \cancel{178.45\ mm}\ \ 126.18\ mm$

$\therefore R = \cancel{89.22\ mm}.\ \ 63.09\ mm$

ii) To find no. of coils :

$$\delta = \frac{64\,WR^3 n}{Cd^4}$$

$$25 = \frac{64 \times 500 \times (63.09)^3 \times n}{8.4 \times 10^4 \times (12.61)^4}$$

$n = 6.6 \simeq 7$

(8). A closely coil helical spring has a stiffness of 10 N/mm². Its length when fully compressed with adjacent coils touching each other is 40 cm. Determine the wire diameter and mean coil diameter if the ratio is 1/10.

ii) If the gap between any two adjacent coil is 0.2 cm what maximum load can be applied before the spring becomes solid (ie) adjacent coils touch.

iii) what is the max. corresponding shear stress in the spring. $C = 0.8 \times 10^5$ N/mm²

<u>GIVEN</u>:

$S = 10$ N/mm²

$S_L = 40 \times 10 \Rightarrow 400$ mm

$d = ?$

$D = ?$

$\frac{d}{D} = \frac{1}{10}$

$10d = D$

$5d = R$

$D = 2R$

Gap between coil $= 0.2$ cm $= 2$ mm

$C = 0.8 \times 10^5$ N/mm²

i). To find D & d :

$$S = \frac{Cd^4}{64R^3n}$$

$$10 = \frac{0.8 \times 10^5 \times d^4}{64 \times (5d)^3 \times n}$$

$S_L = n \times d$

$400 = n \times d$

$$10 = \frac{0.8 \times 10^5 \times d^4}{64 (5d)^3 \times \frac{400}{d}}$$

$d = 20 mm$

$D = 200 mm$

ii) Max. Load (W) when spring becomes sold :

$$\sigma = S = \frac{W}{\delta}$$

W.K.T , Total gap = Max. compression = Max. deflecti

Total gap = gap b/w coils x no. of coils

solid length = $n \times d$

$$n = \frac{400}{d} = \frac{\cancel{400}^{20}}{\cancel{20}} = 20$$

Total gap $= \delta = 20 \times 2$

$= 40 mm$

$W = 400 N$

iii) Shear stress (τ) :

$$\tau = \frac{16 W R}{\pi d^3}$$

01/9/14 (184)

UNIT-IV

DEFLECTION OF BEAMS

Deflection of Beams :

It is the vertical magnitude of bending profile of beam w.r to neutral axis.

Properties of Deflection :

The magnitude of deflection depends upon the elastic properties of material.

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{R = \frac{EI}{M}}$$

Methods of determining deflection:

1. Double Integration method: (beam [illegible] single load acts)
2. Macauly's method. (X)
3. Moment area method. ([illegible] load acts)
4. Conjugate beam method.

Relation between Deflection, slope, Bending moment and shear Force :

Deflection $= y$

Slope $= \dfrac{dy}{dx} = \theta$.

$$B.M = EI \frac{d^2y}{dx^2}$$

$$S.F = EI \frac{d^3y}{dx^3}$$

DOUBLE INTEGRATION METHOD :

In this method we use the formula.

$$M = EI \frac{d^2y}{dx^2}$$

(or)

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

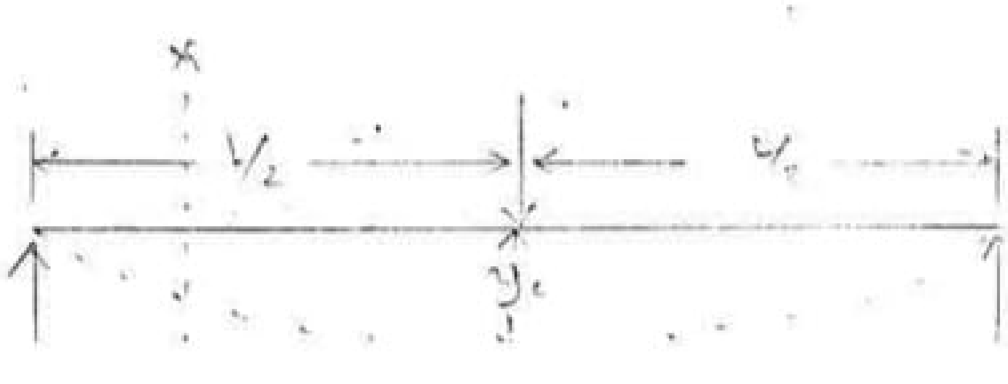
where, $\dfrac{dy}{dx}$ = slope $= \theta$.

It is obtained by single integration.

y = deflection.

obtained by double integration.

Deflection of simply supported beam carrying point load at the centre:



$\therefore R_A + R_B - W = 0$

$R_A + R_B = W$

Taking moment about B,

$R_A L - W\frac{L}{2} = 0$

$\therefore R_A L = W\frac{L}{2}$

$R_A = \frac{W}{2}$

$\therefore R_A = R_B = \frac{W}{2}$

Taking moment about XX

$M_{xx} = R_A \cdot x$

$M = M_{xx} = \frac{W}{2} x$

i). Slope :

W.K.T $EI \frac{d^2y}{dx^2} = \frac{Wx}{2}$ —— ①

Integrating eqn ①.

$EI \frac{dy}{dx} = \frac{Wx^2}{4} + C_1 = \frac{Wx^2}{4} + C_1$

To find C_1, we should eliminate $\frac{dy}{dx}$.

W.K.T, at max. deflection (i.e) $x = l/2$

Slope $\left(\frac{dy}{dx}\right) = 0$

$\therefore x = l/2$; $\frac{dy}{dx} = 0$

$0 = \frac{WL^2}{4 \times 4} + C_1$; $\boxed{C_1 = -\frac{WL^2}{16}}$

$$\boxed{EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}} \quad \text{--- (2)}$$

The above eqn can be used to find deflection at any point of the beam.

To find maximum slope:

$$\theta = \frac{dy}{dx} \quad ; \quad x = 0$$

$$\therefore \; EI\,\theta_A = -\frac{WL^2}{16}$$

$$\boxed{\theta_A = -\frac{WL^2}{16EI}} \qquad [\theta_A = \theta_B]$$

To find deflection :

Integrating eqn (2)

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}$$

$$EIy = \frac{Wx^3}{12} - \frac{WL^2}{16}x + C_2$$

At x=0

To find C_2 w.r.t $y=0$ at $x=0$.

$$\therefore \; \boxed{C_2 = 0}$$

$$EIy = \frac{Wx^3}{12} - \frac{WL^2x}{16} + 0$$

To find max. deflection at $x = l/2$, $y = y_c$

$$EIy_c = \frac{W(L/2)^2}{12} - \frac{WL^2(L/2)}{16}$$

$$= \frac{WL^3}{12 \times 8} - \frac{WL^3}{32}$$

$$EIy_c = \frac{WL^3}{96} - \frac{WL^3}{32}$$

$$\boxed{y_c = -\frac{WL^3}{48EI}}$$ (-ve downward deflection)

Note :

WKT, $y_c = \frac{WL^3}{48EI}$, $\theta = \frac{WL^2}{16EI}$

$$= \frac{WL^2 \times L}{16 \times 3EI}$$

$$y = \theta \times \frac{L}{3}$$

1. A beam 6m long simply supported at its end is carrying a point load of 50 kN at its centre. The moment of inertia of the beam is 78×10^6 mm4. and E is 2.1×10^5 N/mm2. Calculate deflection at the centre of the beam, and slope at the supports.

GIVEN:

$L = 6 \times 10^3$ mm

$W = 50 \times 10^3$ N

$E = 2.1 \times 10^5$ N/mm2

Solution:

$$\text{deflection}, y_c = \frac{WL^3}{48\,EI}$$

$$= \frac{50\times10^3 \times (6\times10^3)^3}{48 \times 2.1\times10^5 \times 78\times10^6}$$

$$= \cancel{3.81\times10^{-7}}$$

$$= 13.73\ \text{mm}$$

$$\theta = \frac{WL^2}{16\,EI} = \frac{50\times10^3 \times (6\times10^3)^2}{16\times 2.1\times10^5 \times 78\times10^6}$$

$$= 0.0068 \times \frac{180}{\pi}$$

$$\cancel{\theta_A = \theta}.$$

$$\theta_A = 0.393^\circ = \theta_B$$

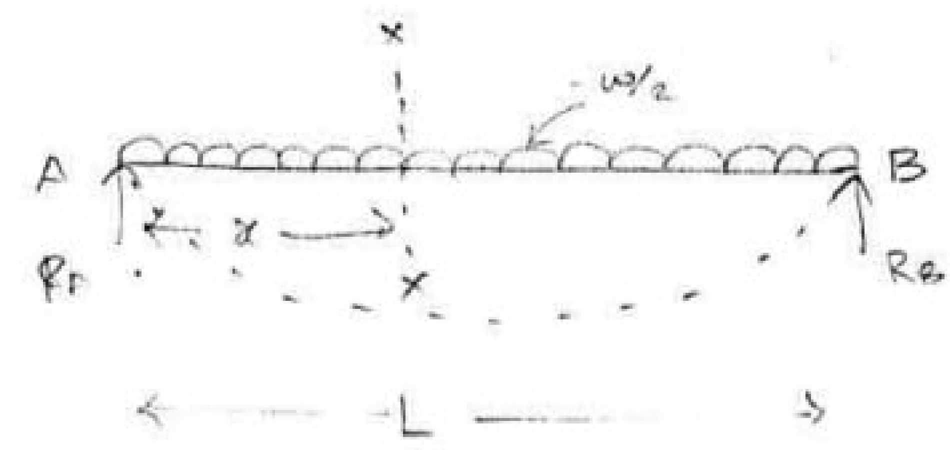
(Downward deflection)

(2). A beam 3 m long simply supported at its end carrying a point load W at the centre if the slope at the ends of the beam should not exceed 1° find the deflection at the centre of the beam.

GIVEN:

3/9/14

DEFLECTION OF SIMPLY SUPPORTED BEAM WITH UDL:



$$R_A = \frac{WL}{2}, \quad R_B = \frac{WL}{2}$$

Moment about XX :

$$M_{xx} = R_A \times x \qquad wx \cdot \frac{x}{2}$$

$$= \frac{WL}{2} \cdot x - \frac{Wx^2}{2}$$

$$M_{xx} = \frac{WLx}{2} - \frac{Wx^2}{2} = EI\frac{d^2y}{dx^2}$$

Slope $\frac{dy}{dx} = \theta$

$$\theta_A = \theta_B = \frac{-WL^2}{24EI} = \frac{-wL^3}{24EI}$$

UDL in terms of N/m

Max. Deflection y_c UDL in terms of N

$$y_c = \frac{-5WL^3}{384EI} = \frac{-5wL^4}{384EI}$$

① A beam of uniform rectangular section 200 mm wide × 300 mm deep is simply supported at its end it carries a UDL of 9 kN/m run over the entire span of 5 m if $E = 1\times10^4\ N/mm^2$. i) Find the slope at the supports.

ii) Max. deflection :

GIVEN :

$b = 200mm$

$d = 300mm$

$w = 9\ kN/m = 9\times10^3\ N/m$

$L = 5m = 5000\ mm$

$E = 1\times10^4\ N/mm^2$

$W = w \times L$

$= 9\times10^3 \times 5$

$= 45\times10^3\ N.$

Solution:

i) $\theta = \frac{-WL^2}{24EI} = \frac{-45\times10^3 \times 5^2}{24\times(1\times10^4)\times(I)}$

$I = \frac{bd^3}{12} = \frac{200\times300^3}{12} = \cancel{166.66\times10^5}$

$= 45\times10^7$

$\therefore \theta = \frac{-45\times10^3\times5000^2}{24\times(1\times10^4)\times \cancel{166.66\times10^5}\ 45\times10^7}$

$= \cancel{0.28126}$

$= -0.0104$ rad

$= 0.0104$ rad

ii) $y_c = \frac{-5WL^3}{384EI} \Rightarrow \frac{-5\times(45\times10^3)(5000)^3}{384\times(1\times10^4)(45\times10^7)}$

$= 16.27$ mm.

(192)

(2) A beam of length 5 m and uniform rectangular section is simply supported at its end. It carries a uniformly distributed load of 9 kN/m as seen over the entire length. Calculate width and depth of the beam if permissible bending stress is 7 N/mm² and central deflection is not to exceed 1 cm. Take $E = 1 \times 10^4$ N/mm².

GIVEN :

$L = 5 \times 10^3$ mm

$w = 9$ kN/m

$W = 9 \times 10^3 \times 5 = 45000$ N

$b = ?$

$d = ?$

$\sigma = 7$ N/mm²

$y_c = 1 \times 10^2$ mm.

$E = 1 \times 10^4$ N/mm²

Solution:

W.K.T,

$$y_c = \frac{5}{384} \cdot \frac{WL^3}{EI} \quad \text{(Downward deflection)}$$

$$1 \times 10^2 = \frac{5}{384} \times \frac{45000 \times (5 \times 10^3)}{(1 \times 10^4)(I)}$$

$$\left(I = \frac{bd^3}{12} \right)$$

$$I = 7.3242 \times 10^{8} \text{ mm}^4$$

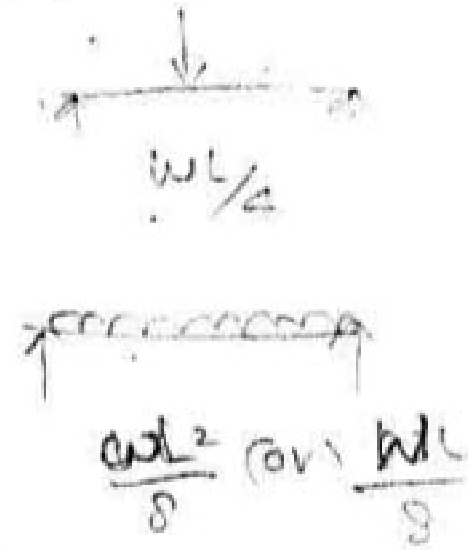
$$I = \frac{bd^3}{12}$$

$$bd^3 = 8789.6 \times 10^{6} \text{ mm}^4 \quad \text{— (1)}$$

Also:

w.k.T

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$



$$\frac{WL}{8} \times \frac{12}{bd^3} = \frac{7}{d/2}$$

$$\cancel{4\times} \ \frac{45000 \times 5000}{8} \times \frac{12}{8789.6 \times 10^{6}} = \frac{7}{d/2}$$

$$\frac{27 \times 10^{8}}{8 \times 8789.6 \times 10^{6}} = \frac{14}{d}$$

$$\boxed{d = 364.6 \text{ mm}}$$

$$bd^3 = 8789.6 \times 10^{6}$$

$$b(364)^3 = 8789.6 \times 10^{6}$$

$$\boxed{b = 182.2 \text{ mm}}$$

(3). A beam of length 5 m and of uniform rectangular section is supported at its ends and carries UDL over entire length. Calculate the depth of the section. If the max. bending stress is 8 N/mm2 and central deflection is 10mm. Take $E = 1.2 \times 10^4 \ N/mm^2$.

GIVEN :

$L = 5 \times 10^3 \ mm$

$d = ?$

$\sigma = 8 \ N/mm^2$

$y_c = 10 \ mm$

$E = 1.2 \times 10^4 \ N/mm^2$

Solution :

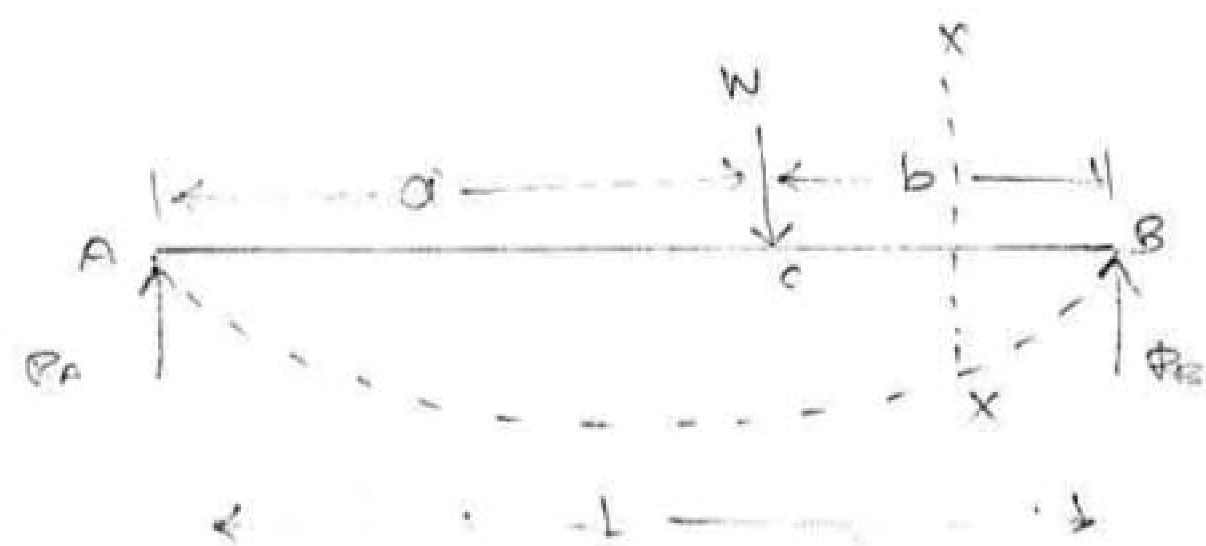
$$y_c = \frac{5}{384} \frac{WL^3}{EI}$$ (downward deflection)

$$10 = \frac{5}{384} \frac{W \times (5 \times 10^3)^3}{(1.2 \times 10^6)(I)}$$

$$W = 7.3728 \times 10^{-5} \times I$$

MACAULAY'S METHOD:

DEFLECTION OF SIMPLY SUPPORTED BEAM WITH A POINT LOAD:



$$R_A + R_B - W = 0$$

$$R_A + R_B = W \quad \text{--- } i$$

Moment about B:

$$(R_A \times L) - (W \times b) = 0$$

$$\boxed{R_A = \frac{Wb}{L}}$$

$$\therefore R_A + R_B = W$$

$$\frac{Wb}{L} + R_B = W$$

$$R_B = W - \frac{Wb}{L}$$

$$R_B = \frac{WL - Wb}{L}$$

$$R_B = \frac{W(L-b)}{L}$$

$$\boxed{R_B = \frac{Wa}{L}}$$

Taking moment between A and C : at a distanc x :

$$M_{xx} = R_A \times x$$
$$= \frac{Wb}{L} \times x$$

The above equation holds good between O to A distance. and for C & B. It is given by Taking section between B and C.

$$M_{xx} = R_A x \;\vdots\; -W(x-a)$$

up to dotted line can be used for finding deflection. for any point in section AC. But any pt. in section CB. add the expression beyond dotted line also.

w.r.t

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^2y}{dx^2} = \frac{Wb}{L} x \;\vdots\; -W(x-a) \quad \text{— ①}$$

Integrate eqn ①.

$$EI \frac{dy}{dx} = \frac{Wb}{L} \cdot \frac{x^2}{2} + C_1 \;\vdots\; -\frac{W(x-a)^2}{2} \quad \text{— ②}$$

Integrate eqn ②.

$$EI y = \frac{Wb}{2L} \cdot \frac{x^3}{3} + C_1 x + C_2 \;\vdots\; -\frac{W}{2} \frac{(x-a)^3}{3} \quad \text{— ③}$$

To find C_1 & C_2 :

i) At $x=0$, $y=0$

ii) At $x=L$, $y=0$

Subs i) in eqn 3.

$$EI(0) = C_2 - \frac{wa^3}{6}$$

$$\cancel{C_2 = \frac{wa^3}{6}} \qquad \boxed{C_2 = 0}$$

$x=L$, $y=0$

$$EIy = \frac{Wb}{2L} \cdot \frac{x^3}{3} + C_1 x + C_2 \;\vdots\; - \frac{W}{2} \frac{(x-a)^3}{3}$$

$$= \frac{Wb}{2L} \cdot \frac{L^3}{3} + C_1 \times L + 0 - \frac{W}{2} \frac{(L-a)^3}{3}$$

$$= \frac{WbL^2}{6} + C_1 \times L - \frac{Wb^3}{6}$$

$$C_1 L = -\frac{WbL^2}{6} + \frac{Wb^3}{6}$$

$$C_1 L = -\frac{Wb}{6}(L^2 - b^2)$$

$$\boxed{C_1 = -\frac{Wb}{6L}(L^2 - b^2)}$$

To find slope at any point subs. C_1, C_2 in eqn 2.

$$EI\frac{dy}{dx} \rightarrow \theta = \frac{Wb}{L}\cdot\frac{x^2}{2} - \frac{Wb}{6L}(L^2-b^2) - \frac{W(x-a)^2}{2}$$

To find max. slope.

$x = 0$

$$\therefore EI\theta = -\frac{Wb}{6L}(L^2-b^2) - \frac{W(-a^2)}{2}$$

$$\boxed{\theta_A = \frac{-Wb}{6EIL}(L^2-b^2)}$$

To find deflection at any p.t:

Subs. C_1, C_2 in eqn. ③.

$$EIy = \frac{Wb}{2L}\cdot\frac{x^3}{3} - \left(\frac{Wb}{6L}(L^2-b^2)\right)x + 0$$

To find max. deflection:

$y = y_c$, $x = a$.

$$EIy_c = \frac{Wba^3}{6L} - \frac{Wb}{6L}(L^2-b^2)a + 0 - 0$$

$$= -\frac{Wab}{6L}(L^2-b^2-a^2)$$

$$= -\frac{Wab}{6L}((a+b)^2 - b^2 - a^2)$$

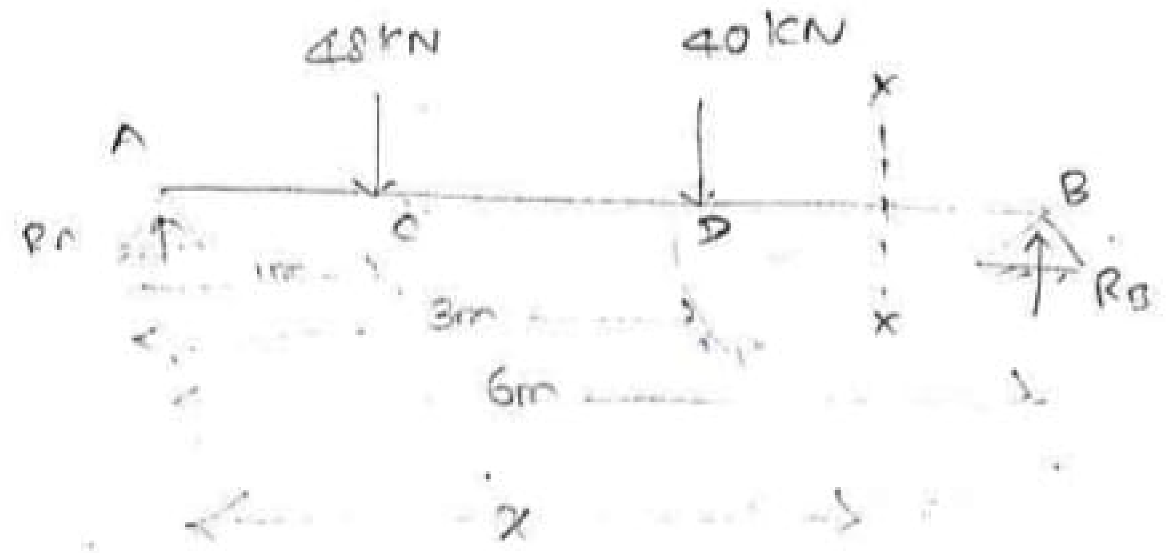
5/9/14 (200)

① A beam of length 6m is simply supported at its end carries two point loads of 48 kN and 40 kN at a distance of 1m and 3m respectively from the left support. i) Find deflection under each load

ii) Max. deflection.

iii) The point at which max. deflection occurs

Take $E = 2 \times 10^5$ N/mm² and $I = 85 \times 10^6$ mm⁴



$E \; \frac{\times 10^{-3} \times 10^6}{\times 10^3}$

$I = 10^{-12}$

GIVEN :

$L = 6m$.

$F = 48 kN, 40 kN$; $W_1 = 48 kN$

$E = 2 \times 10^5 N/mm^2$ $W_2 = 40 kN$

$I = 85 \times 10^6 mm^4$

Solution:

$$R_A + R_B = 48 + 40$$

$$R_A + R_B = 88$$

moment about A

$$\Sigma M_A \Rightarrow (48 \times 1) + (40 \times 3) - (R_B \times 6) = 0$$

$$48 + 120 - 6R_B = 0$$

$$-6R_B = -168$$

$$\boxed{R_B = 28 \text{ kN}}$$

$R_A = 88 - 28$

W.R.T $EI \frac{d^2y}{dx^2} = M_{xx}$

Taking the section xx in the region DB

$M_{xx} = +(60 \times x) - 48 \times (x-1) - 40 \times (x-3)) = 0$

$\Rightarrow 60x - 48x + 48 - 40x + 120 = 0$

$= -28x + 168 = 0$

$=$

$\therefore EI \frac{d^2y}{dx^2} = 60x - 48(x-1) - 40(x-3)$ —— ①

Integrating eqn ① :

$EI \frac{dy}{dx} = \frac{60x^2}{2} + C_1 - \frac{48(x-1)^2}{2} - \frac{40(x-3)^2}{2}$

$EI \frac{dy}{dx} = 30x^2 + C_1 - 24(x-1)^2 - 20(x-3)^2$ —— ②

(slope eqn)

Integrating eqn ② :

$EI\,y = \frac{30x^3}{3} + C_1x + C_2 - \frac{24(x-1)^3}{3} - \frac{20(x-3)^3}{3}$

$= 10x^3 + C_1x + C_2 - 8(x-1)^3 - 6.66(x-3)^3$ —— ③

To find C_1 & C_2 :

i) $x = 0$, $y = 0$

ii) $x = 6 = L$; $y = 0$

Sub (i) in eqn ③ up to 1st dotted line :

$$EIy = 10x^3 + C_1 x + C_2$$

$$\boxed{0 = C_2}$$

Sub ii) in eqn ③ up to last :

$$EIy = 10x^3 + C_1 x + C_2 \,\vdots\, -8(x-1)^3 \,\vdots\, -6.66(x-3)^3$$

$$0 = 10(6)^3 + 6C_1 + C_2 \,\vdots\, -8(5)^3 \,\vdots\, -6.66(3)^3$$

$$0 = 2160 + 6C_1 - 1000 - 179.82$$

$$0 = 980.18 + 6C_1$$

$$\boxed{C_1 = -163.33}$$

i) Deflection under each load :

a). Deflection at point C :

Since point C lies in 1st region take the 1st term of eqn ③.

$$EIy = 10x^3 + C_1 x + C_2$$

$$EIy = 10(1)^3 + (-163.33) + 0$$

$$EIy = -153.33 \text{ KN-m}^3$$ (convert in to N and mm)

$$EIy = -153.33 \times 10^3 \times 10^9 \text{ N-mm}^3$$

$$2 \times 10^5 \times 85 \times 10^6 \, y = -153.3 \times 10^3 \times 10^9$$

$$\boxed{y = -9.01 \text{ mm}}$$ (downward deflection)

b). Deflection at point D

Since D lies at 2nd. region take 2 terms from ⑧.

$$EIy = 10x^3 + C_1x + C_2 \;\vdots\; -8(x-1)^3$$

$$EIy = 10(3)^3 + (-163.33)(3) + 0 \quad -8(2)^3$$

$$EIy = 270 - 489.9 - 64$$

$$EIy = \cancel{-1219.9}\ kN\,m^3 \quad -283.9$$

$$2\times10^5\times85\times10^6 y = \cancel{-1219.9} \times 10^3 \times 10^9 \quad \times -283.9$$

$$\boxed{y = -16.7\ mm}$$

ii) To find max. Deflection (y_{max}):

To find max. deflection we need to find the point at which it occurs. w.k.T at max. deflection $\frac{dy}{dx} = 0$ = slope = 0.

Subs. $\frac{dy}{dx} = 0$ in eqn ⑦,

$$0 = 30x^2 + C_1 - 24(x-1)^2 \quad \cancel{-20(x-3)^2}$$

$$0 = 30x^2 + (-163.33) - 24(x^2+1-2x) \cancel{-20}$$

$$= 30x^2 - 163.33 - 24x^2 - 24 + 48x$$

$$= 6x^2 + 48x - 187.33$$

Deflection is max. where the load is acting ie. region C & D. Hence Take eqn ⑦ 2terms

$$x = 2.87\ m$$

Deflection at max. deflection: at (2.87m)

$$EIy = 10x^3 + C_1 x + C_2 - 8(x-1)^3$$

$$EIy = 10(2.87)^3 + \cancel{EI\theta}\ (-163.33)(2.87) + 0$$
$$- 8(2.87-1)^3$$

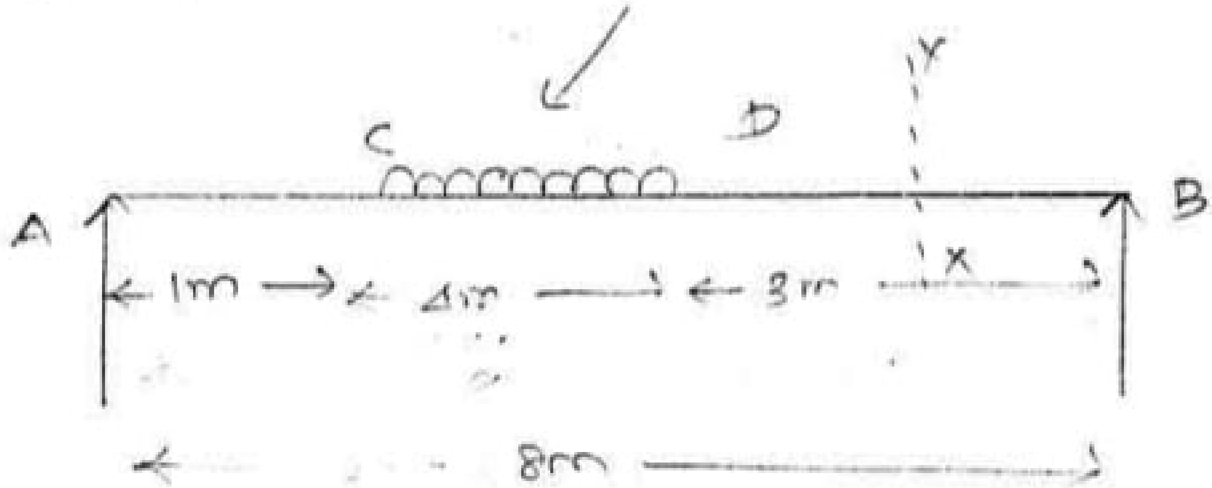
$$EIy = 236.39 - 468.75 - 52.31$$

$$EIy = -284.47 \text{ KN-m}^3$$

$$2\times10^5 \times 85\times10^6 \times y = -284.47 \times 10^3 \times 10^9$$

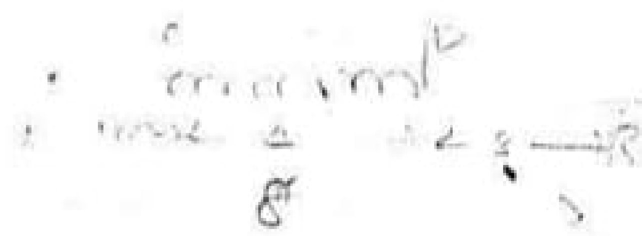
$$\boxed{y = -16.74 \text{ mm}}$$

(2). Same question :



$E = 2\times10^5\ N/mm^2$

$I = 4.3 \times 10^8\ mm^4$

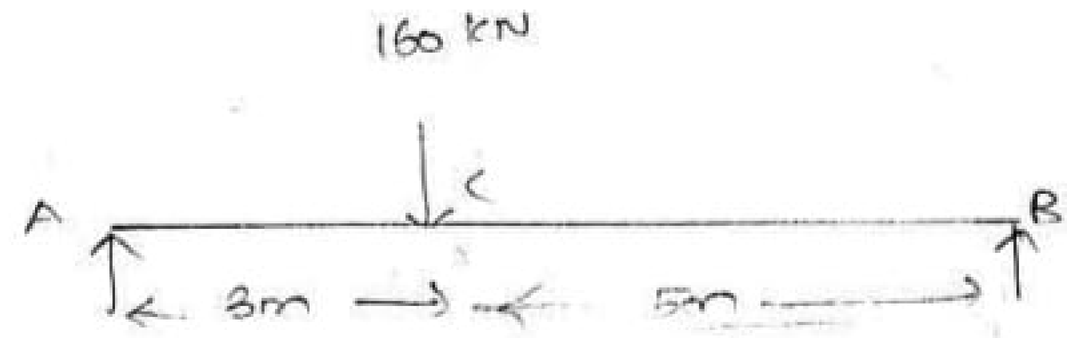


find i) Deflection at midpoint.

ii) Max. deflection point

iii) Max. deflection

GIVEN :



$$R_A + R_B = 160$$

Moment about B.

$$(R_A \times 8) - (160 \times 5) = 0$$

$$8R_A = 800$$

$$\boxed{R_A = 100\,KN}$$

$$\boxed{R_B = 60\,KN}$$

1) Taking moment about section xx in region DB :

$$M_{xx} = (R_A \times x) - \underbrace{\overset{20}{\cancel{40}}(x-1)}_{Force}\underbrace{\frac{(x-1)}{\cancel{2}}}_{Distance} + \overset{20}{\cancel{40}}(x-5)\left(\frac{x-5}{\cancel{2}}\right)$$

W.K.T.

$$EI \cdot \frac{d^2y}{dx^2} = 100x - 20(x-1)^2 + 20(x-5)^2 \quad \text{— ①}$$

Integrating ①

$$EI\frac{dy}{dx} = \overset{50}{\cancel{100}}\frac{x^2}{\cancel{2}} + C_1 - \frac{20(x-1)^3}{3} + \frac{20(x-5)^3}{3}$$

$$EI\frac{dy}{dx} = 50x^2 + C_1 - \frac{20(x-1)^3}{3} + \frac{20(x-5)^3}{3} \quad \text{— ②}$$

Integrating ②,

$$EIy = \frac{50x^3}{3} + C_1x + C_2 - \frac{20(x-1)^4}{4\times3} + \frac{20(x-5)^4}{4\times3}$$

$$EIy = \frac{50x^3}{3} + C_1x + C_2 - \frac{5(x-1)^4}{3} + \frac{5(x-5)^4}{3} \quad \text{—— ③}$$

To find C_1 & C_2 :

i) $x=0,\ y=0$ ii) $x=8,\ y=0$

Sub i) in ③ up to 1st term

$$EIy = \frac{50x^3}{3} + C_1x + C_2$$

$$0 = 0 + 0 + C_2$$

$$\boxed{C_2 = 0}$$

Sub ii) in ③ up to 3rd term :

$$0 = \frac{50(8)^3}{3} + 8C_1 + 0 - \frac{5}{3}(7)^4 + \frac{5}{3}(3)^4$$

$$0 = 8533.3 + 8C_1 - 4001.6 + 135$$

$$0 = 8C_1 + 4666.7$$

$$\boxed{C_1 = -583.33}$$

Deflection at mid point :

at ($x = 4$),

Sub $x=4$ in ③ up to 2nd term

$$EIy = \frac{50x^3}{3} + C_1 x + C_2 \; \vdots - \frac{5}{3}(x-1)^4$$

$$EIy = \frac{50(4)^3}{3} + C_1 4 + 0 - \frac{5}{3}(3)^4$$

$$EIy = 1066.6 + 4C_1 - 135$$

$$EIy = 931.6 + 4C_1$$

$$EIy = 931.6 + 4(583.33)$$

$$EIy = \cancel{3264.92 \text{ KN m}^4} \quad -1401.72 \text{ KN m}^3$$

$$2\times10^5 \times 4.3\times10^8 = 3264.92 \times 10^3 \times 10^{12}$$

$\cancel{\times y}$

$$EI\, y = -1401.72 \text{ KN m}^3$$

$$2\times10^5 \times y \times 4.3\times10^8 = -1401.72 \times 10^3 \times 10^9$$

$$\boxed{y = -16.29 \text{ mm}}$$

ii) Max deflection point (x):

At y_{max}, $\frac{dy}{dx} = 0$. it acts in the load load region CD. Take eqn ② upto 2nd dotted line.

$$EI\frac{dy}{dx} = 50x^2 + C_1 - \frac{20}{3}(x-1)^3$$

$$0 = 50(-16.29)^2 + (-583.33) - \frac{20}{3}(-15.29)^3$$

$$= 13268.205 - 583.33 + 23830.39$$

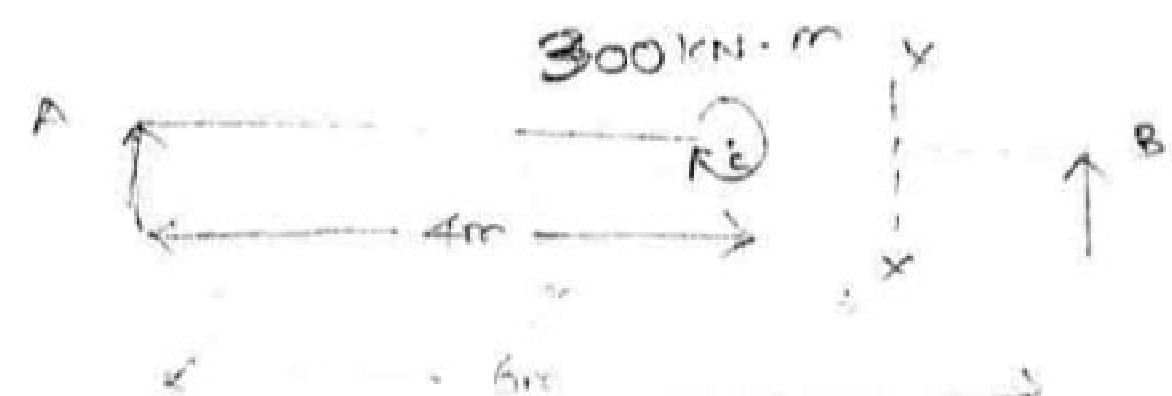
$=$

208

$$0 = 50x^3 - 583.33 - 6.67(x-1)^3$$

$$\boxed{x = 3.83}$$

(3.) A horizontal beam A.B simply supported at A and B 6m apart. The beam is subjected to clockwise couple of 300 kNm at a distance of 4 m from the left end. If $E = 2\times10^5 N/mm^2$, $I = 2\times10^8 mm^4$. Determine deflection at the point where the couple is acting and the max. deflection.



Solution :

$$R_A + R_B = 0$$

Moment about B :

$$\Sigma M_B = (R_A \times 6) + 300 = 0$$

$$6R_A = -300$$

$$\boxed{R_A = -50 \text{ kN}}$$

$$\boxed{R_B = 50 \text{ kN}}$$

$M_{xx} = (R_A \times x) + 300$

$= R_A x + 300(x-4)^0$

$$EI \frac{d^2y}{dx^2} = -50x + 300(x-4)^0 \quad \text{--- (1)}$$

Integrating,

$$EI \frac{dy}{dx} = \frac{-50x^2}{2} + C_1 + \frac{300(x-4)^1}{1}$$

$$EI \frac{dy}{dx} = -25x^2 + C_1 + 300(x-4) \quad \text{--- (2)}$$

Integrating,

$$EI y = -\frac{25x^3}{3} + C_1 x + C_2 + \frac{300(x-4)^2}{2}$$

$$EI y = -8.33x^3 + C_1 x + C_2 + 150(x-4)^2 \quad \text{--- (3)}.$$

To Find C_1 & C_2 :

Subs, i) $x=0, y=0$ ii) $x=6, y=0$

Sub i) in (3) upto 1st term

$EI y = -8.33x^3 + C_1 x + C_2$

$\boxed{0 = C_2}$

Subs ii) in (3) upto full :

$EI y = -8.33x^3 + C_1 x + C_2 + 150(x-4)^2$

$0 = -8.33(6)^3 + C_1 6 + 150(2)^2$

$0 = -1799.28 + 6C_1 + 600$

i) Deflection at $x = 4$: (ie) C :

Sub $x = 4$ in ③ up to 1st term.

$$EIy = -8.33x^3 + C_1 x + C_2$$

$$EIy = -8.33(4)^3 + (200)(4) + C_2$$

$$EIy = 533.12 + 800 + C_2$$

$$2\times10^5 \times 2\times10^8 \times y = 1333.12 \times 10^3 \times 10^9$$

$$y = 6.66\,mm$$

ii) Max. deflection :

Max. deflection occurs in region AC hence taking upto 1st dotted line :

$$EI\frac{dy}{dx} = -\frac{50x^2}{2} + C_1 \qquad \left(\frac{dy}{dx} = 0\right)$$

$$0 = -25x^2 + 200$$

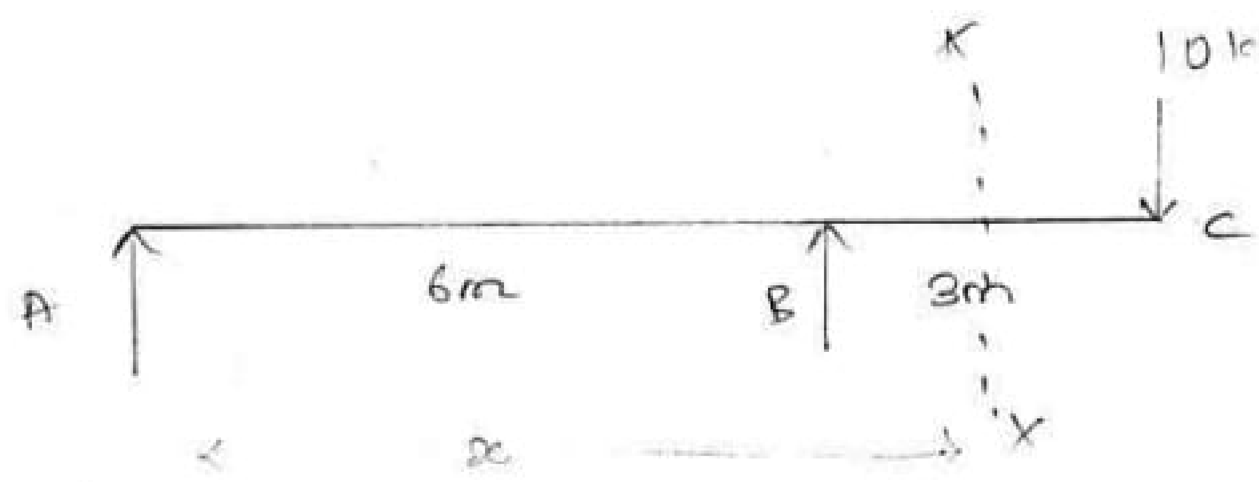
$$\boxed{x = \sqrt{8}} \simeq 2.8$$

$$EIy = -8.33x^3 + C_1 x + C_2 \text{ 1st}$$

$$2\times10^5 \times 2\times10^8 \times y = -8.33(2.8)^3 + (200\times2.8) +$$

$$\boxed{y_{max} = 9.4\,mm}$$

4.



$E = 2 \times 10^5 \ N/mm^2$

$I = 5 \times 10^8 \ mm^4$

Find:

i) Slopes over each support.

ii) Deflection at right end.

iii) Max. deflection.

Solution:

$R_A + R_B = 10$

Taking moment about A

$R_B \times 6 - 10 \times 9 = 0$

$R_B 6 = 90$

$\boxed{R_B = 15 kN}$

$R_A + 15 = 10$

$\boxed{R_A = -5 kN}$

11/9/14

Moment area method : → Flexural Rigidity! It is defined as the product of modulus of elasticity and the moment of inertia.

Flexural Rigidity = EI

$$\text{Slope},\ \theta = \frac{A}{EI}$$

$$\text{Deflection},\ y = \frac{A\bar{x}}{EI}$$

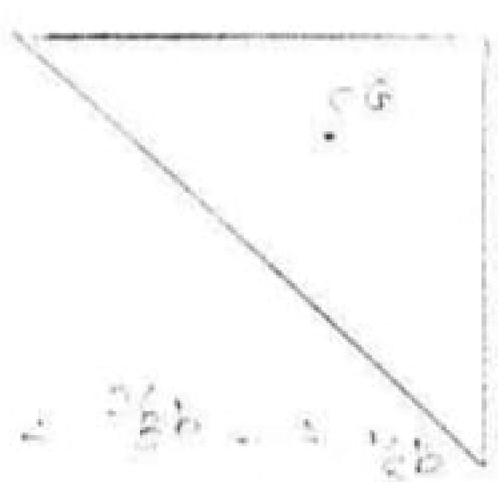
(2)

Area, $A = \frac{2}{3}ab$

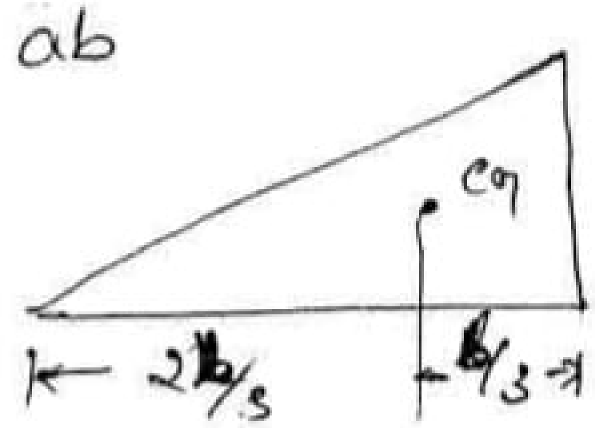
(3)

Area, $A = \frac{1}{3}ab$

(1)



Area, $A = \frac{1}{2}ab$



Procedure for moment area method :

(1) Draw the Bending moment diagram

(2) After drawing Bending moment diagram consider the area of bending moment diagram upto the point for which deflection is to be calculated.

(3) Determine the centroidal distance ($\bar{x}$) of Bending moment diagram under consideration.

(4). knowing the area (A) and centroidal distance ($\bar{x}$) calculate slope and deflection using the formula.

Protel Note:

EI is called as flexural rigidity.

If EI value is given substitute in the formula or keep as it is.

PROBLEMS :

(1). Determine maximum slope at the support and maximum deflection under the load. A rectangular simply supported beam 100mm bread and 200 mm depth. having a span of 10m. The beam carries a central concentrated load of 20 kN. If E is 2×10^5 N/mm^2.

GIVEN :

$b = 100mm$

$d = 200mm$

$Load = 20\times10^3 N$

$E = 2\times10^5 N/mm^2$

Solution :

Reactions at A, B:

$R_A + R_B = 20\times10^3$

Moment about B,

$R_A \times 100 = 1000\,000$

$R_A = 10000$ N

$\boxed{R_A = 10 \text{ kN}}$

$10 + R_B = 20.$

$\boxed{R_B = 10 \text{ kN}}$

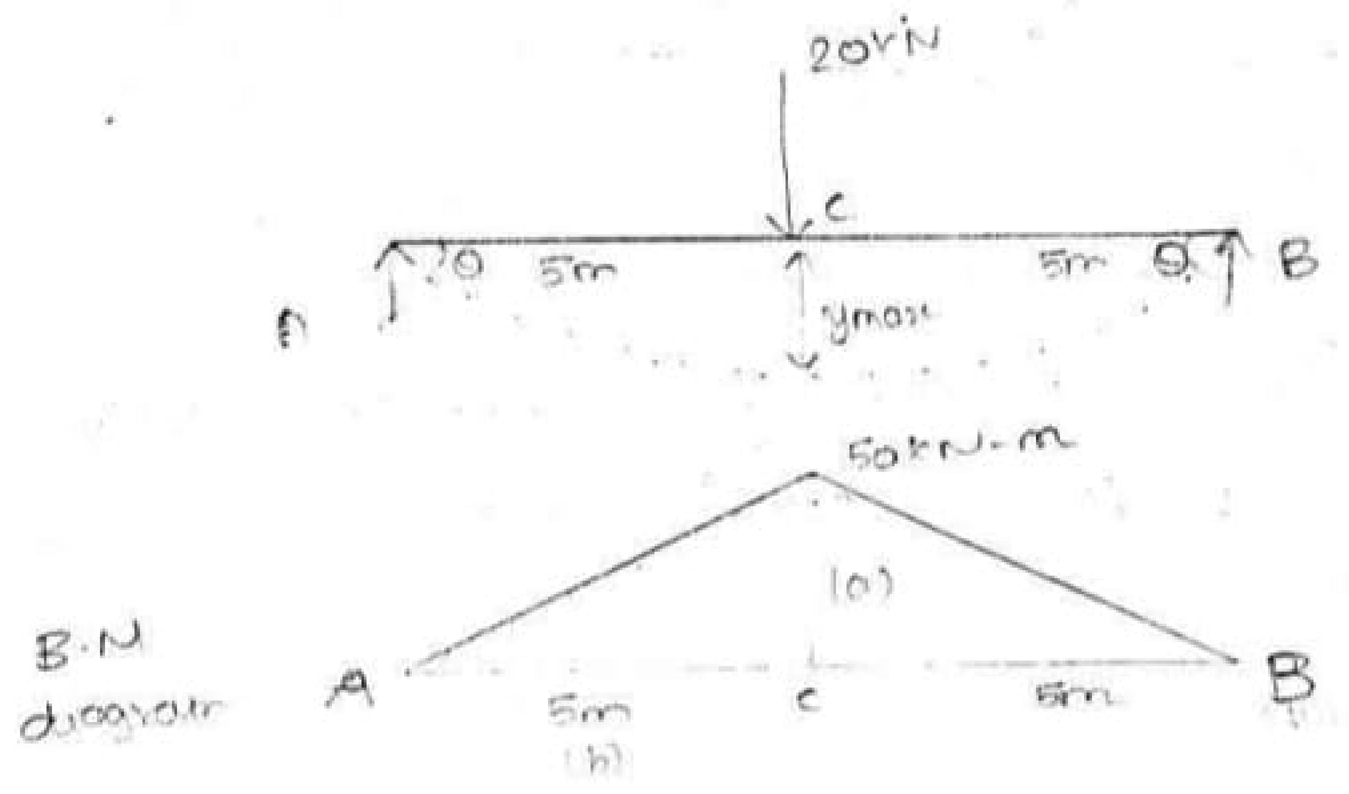
Bending Moment :

B.M at A = 0

B.M at B = 0

B.M at C = $R_A \times 5$

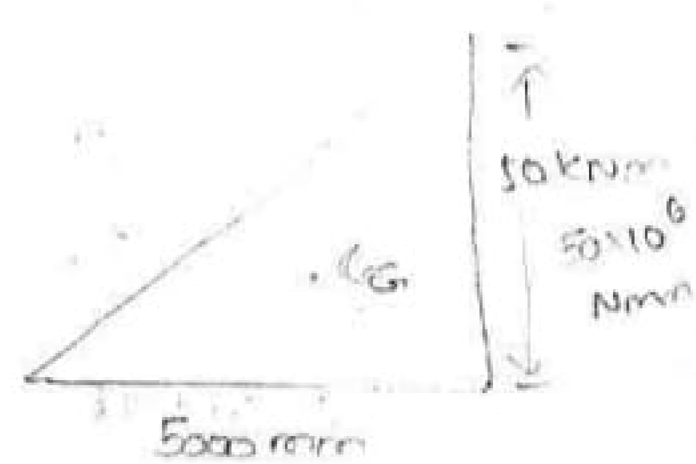
$= 10 \times 5 = 50$ kN-m



To find area of shaded portion

$\bar{X} = \frac{2}{3}(5000)$

$= 3333.33$



Area $= \frac{1}{2} bh = \frac{1}{2} \times 5000 \times 50 \times 10^6$

$= 1.25 \times 10^{11}$ N-mm²

$$\text{Slope} = \frac{A}{EI}$$

$$I = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = 6.66 \times 10^7$$

$$\text{Slope} = \frac{1.25 \times 10^{11}}{2 \times 10^5 \times I}$$

$$\text{Slope} = \frac{2.5 \times 10^{16}}{I} = \cancel{37.5 \times 10^7}$$

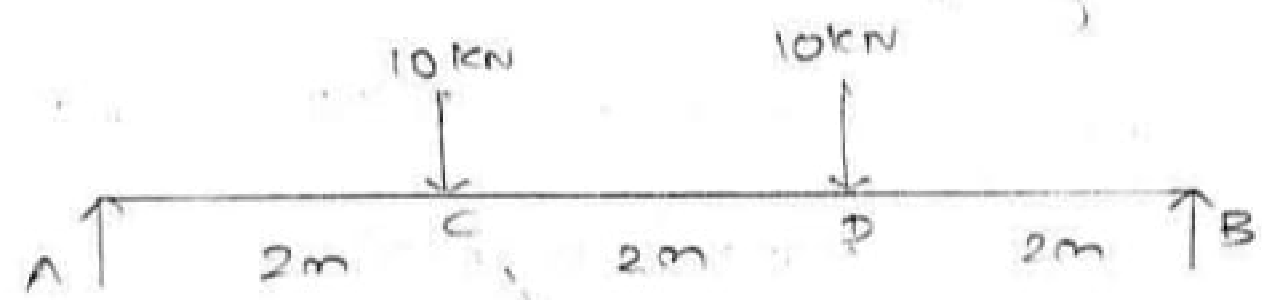
$$\boxed{= 9.38 \times 10^{-3} \text{ rad}}$$

Deflection : $y = \frac{Ax}{EI}$

$$= \frac{1.25 \times 10^{11} \times 3333.3}{2 \times 10^5 \times 6.66 \times 10^6}$$

$$\boxed{= 31.28 \text{ mm}}$$

② A simply supported beam carries two concentrated load as shown. Determine a max. deflection and also deflection under each load assume EI is constant



To find reactions :

$R_A + R_B = 10 + 10$

$R_A + R_B = 20$

Moment about B.

$(-10 \times 2)(-10 \times 4) + (R_A \times 6) = 0$

$-20 - 40 + 6R_A = 0$

$6R_A = 60$

$\boxed{R_A = 10}$ KN

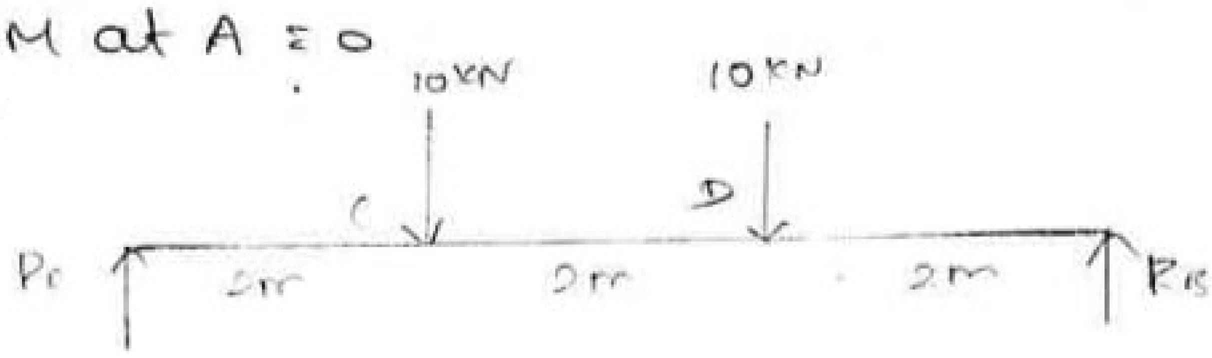
Bending moment : ~~at B~~ :

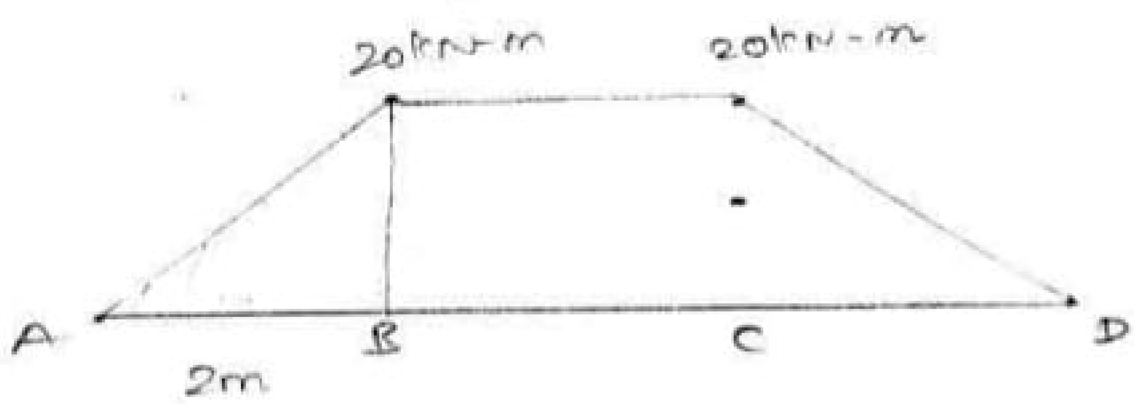
B.M at B = 0

B.M at D = $\cancel{(10\times2)} = \cancel{-20\text{ kN}}$ $(R_A\times4)(-10\times2) = 20\text{ kN-m}$

B.M at C = $(R_A\times2) = 20\text{ kN-m}$

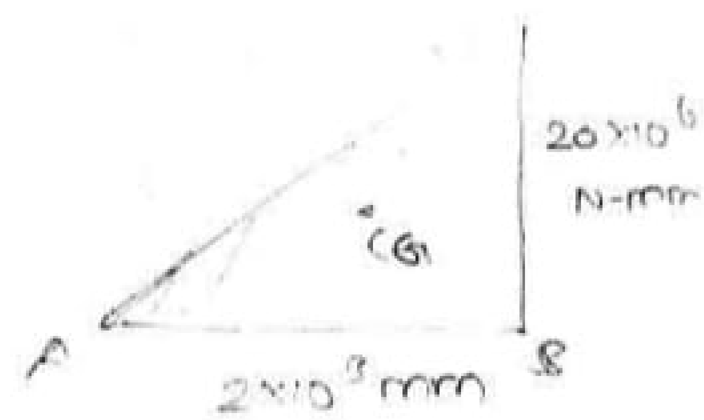
B.M at A = 0





$\bar{x} = \frac{2}{3}b \Rightarrow \frac{2}{3}\times 2\times10^3$

$\bar{x} = 1333.33$



Area,

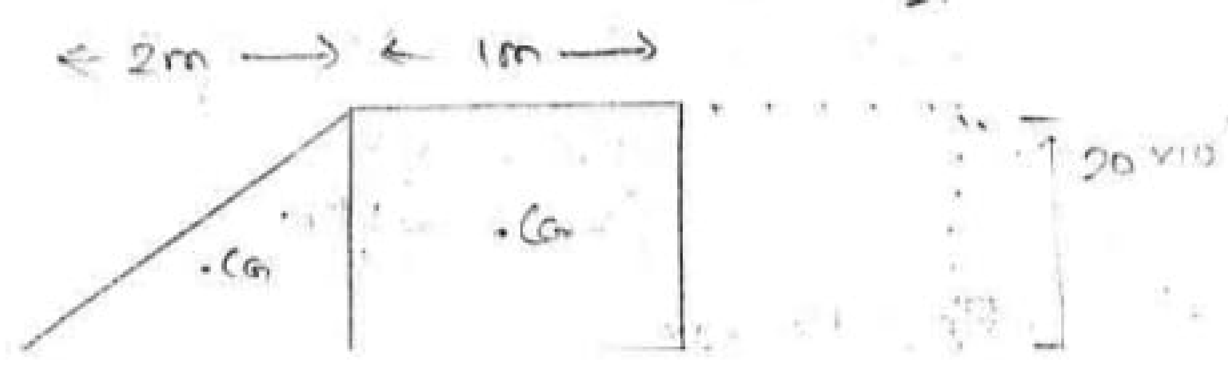
$A = \frac{1}{2}bh \Rightarrow \frac{1}{2}\times(2\times10^3)(20\times10^6)$

$= 2\times10^{10}$ N-mm²

* Deflection at C, D $= \frac{A\cdot\bar{x}}{EI}$

$= \frac{2.66\times10^{13}}{EI}$ mm

* To find Max. deflection : (Y_{max})



$$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \left(\tfrac{1}{2} a_1 b_1\right)\left(\tfrac{2}{3} b_1\right) + (a_2 \times b_2) \times \left(2 + \tfrac{1}{2} b_2\right)$$

all in mm

$$= (2\times10^{10})(1333.33) + (1) \times \left(2 \times 10^3 + \tfrac{1}{2}\times 1\right)$$

$$= (2.66 \times 10^{13}) + (0.5)$$

$$= \cancel{2.66 \times 10^{13}} \; 76.66 \times 10^{12}$$

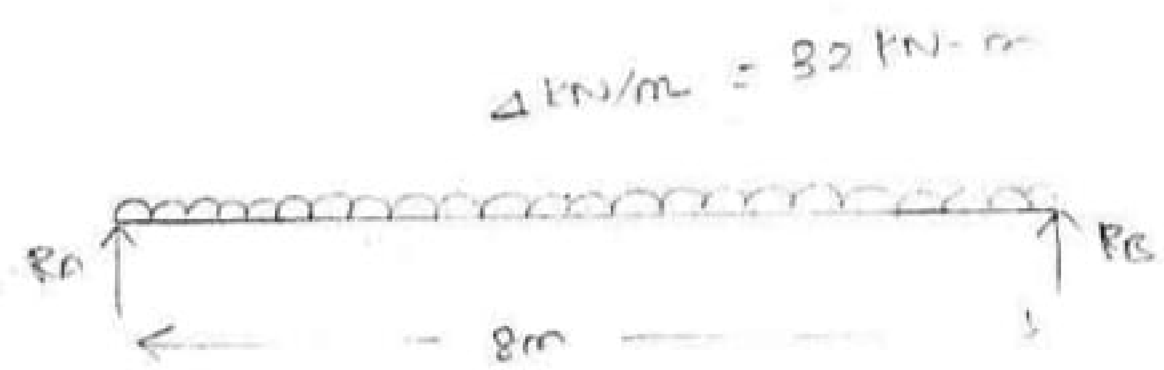
* Max deflection $= \dfrac{A\bar{x}}{EI}$

$$= \cancel{\frac{2.66 \times 10^{13}}{EI}}$$

$$y_{max} = \frac{76.66 \times 10^{12}}{EI}$$

(3) Determine the max slope at the support and deflection at the centre of a simply supported beam carrying a uniformly distributed load of 4 kN/m over a span of 8 m if E is 2×10^5 N/mm² and the cross section of the beam is 100 mm diameter.

<u>GIVEN</u>:



The reactions :

$$R_A + R_B = 32 \text{ kN·m}$$

~~Moment about~~ B :

$$(-R_A \times 8) + (32 \times 4) = 0$$

$$\boxed{R_A = 16\text{kN}}$$

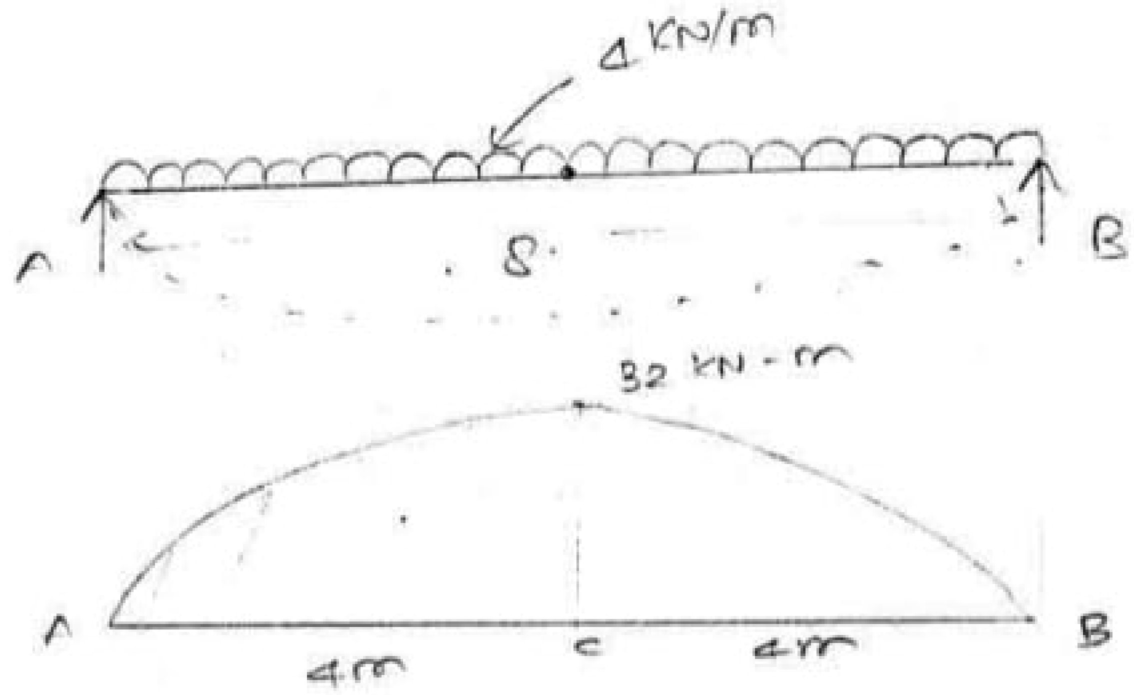
Bending moment :

B.M at B = 0

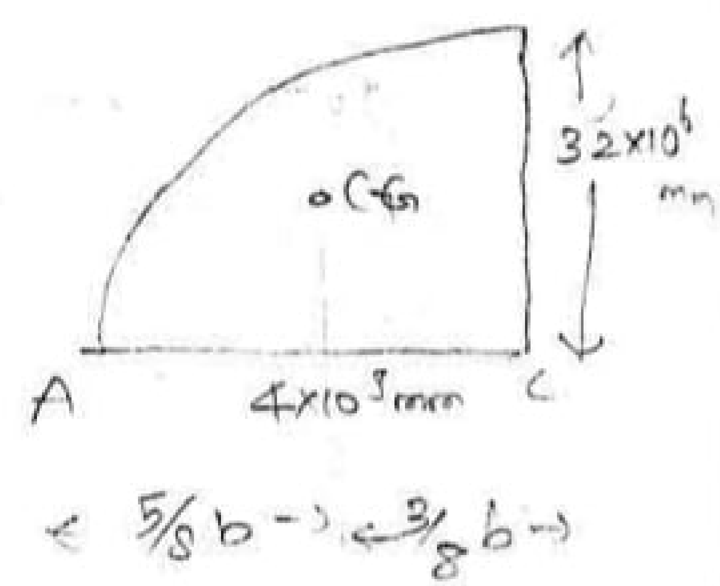
B.M at A = 0

B.M at c = $(R_A \times 4) - (16 \times \frac{4}{2})$

$= 4R_A - (16 \times 2) \Rightarrow 32$ kN-m



To find $\bar{x}$ and Area,



$\bar{X} = \frac{5}{8} \times 4 \times 10^3$

$= 2500$

Area $A = \frac{2}{3} ab$

$= \frac{2}{3} \times 4 \times 10^3 \times 32 \times 10^6$

$= 8.53 \times 10^{10}$

$I = \frac{\pi}{4} d^4 = \frac{3.14}{4} \times 100 = 4.9 \times 10^6 \text{ mm}^4$

$\theta = \frac{A}{EI} = \frac{8.53 \times 10^{10}}{2 \times 10^5 \times 4.9 \times 10^6} = 0.0870$ rad.

$Y = \frac{A\bar{x}}{EI} = \frac{8.5 \times 10^{10} \times 2500}{2 \times 10^5 \times 4.9 \times 10^6} = 217.6$ mm

12/9/14.

DEFLECTION OF CANTILEVERS:

Slope, $\frac{dy}{dx} = 0$ at $x = 0$

Deflection, $y = 0$ at $x = 0$

(slope & deflection are zero at fixed end)

DOUBLE INTEGRATION METHOD:

i) Deflection of cantilever with pt load at free end:

$\theta_B = \frac{WL^2}{2EI}$ (at $x = L$)

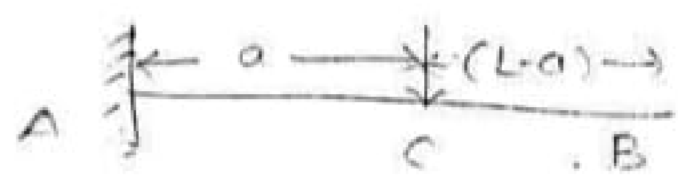
$y = \frac{WL^3}{3EI}$ (downward deflection) (at $x = L$)

y =

$\times 10^6$ mm

ii) y of cantilever with point load at a distance 'a' from fixed end

$\theta_C = \theta_B = \frac{Wa^2}{2EI}$

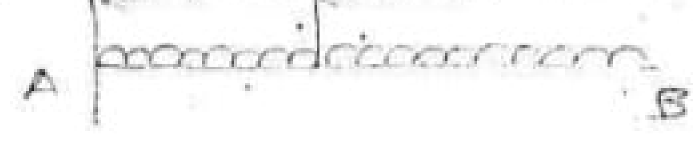


$y_B = y_C + \theta_C (L-a)$

$= \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L-a)$

iii) Deflection of cantilever with UDL:

$\theta_B = -\frac{wL^3}{6EI} = -\frac{WL^2}{6EI}$



$y_B = \frac{wL^4}{8EI} = \frac{WL^3}{8EI}$

UDL for a distance 'a' from the fixed end

$$\theta_C = \theta_B = \frac{Wa^3}{6EI}$$

$$y_B = y_C + \theta_C (L-a)$$

$$= \frac{Wa^4}{8EI} + \frac{Wa^3}{6EI}(L-a)$$

UDL for a distance 'a' from free end.

$$\theta_B = \frac{WL^3}{6EI} - \frac{W(L-a)^3}{6EI}$$

$$y_B = \frac{WL^4}{8EI} - \left[\frac{W(L-a)^4}{8EI} + \frac{W(L-a)^3}{6EI} \cdot a\right]$$

① A cantilever of length 3m carries two point loads of 2kN at the free end and 4kN at a distance of 1m from the free end take, $E = 2\times10^5\ N/mm^2$, $I = 10^8\ mm^4$. Find deflection at free end

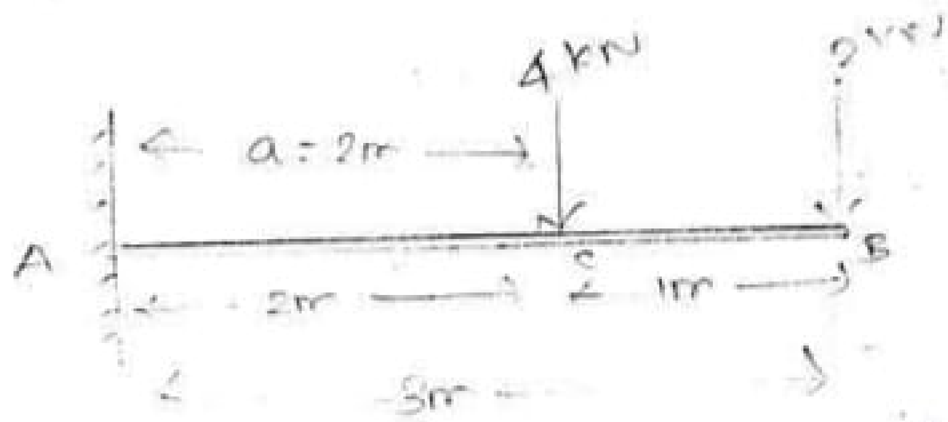
GIVEN:

$L = 3m = 3\times10^3\ mm$

$P_1 = 2\times10^3\ N$

$P_2 = 4\times10^3\ N$

$E = 2\times10^5\ N/mm^2$, $I = 10^8\ mm^4$



Solution:

Deflection at free end (y):

$$y = y_B + y_C$$

y_c - Deflectaction at a distance from free end.

$$y_c = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L-a)$$

$$= \frac{2\times10^3 \times 2000^3}{3\times2\times10^5\times10^8} + \frac{2\times10^3\times2000^2}{2\times2\times10^5\times10^8}(3000-2000)$$

$$= 0.266 + 2\times10^{-4}(1000)$$

$$= 0.266 + 0.2$$

$$= 0.933 \text{ mm}$$

y_B : ~~Wa~~ deflection at free end.

$$y_B = \frac{WL^3}{3EI} = \frac{4\times10^3 \times 3000^3}{3\times10^5\times2\times10^8}$$

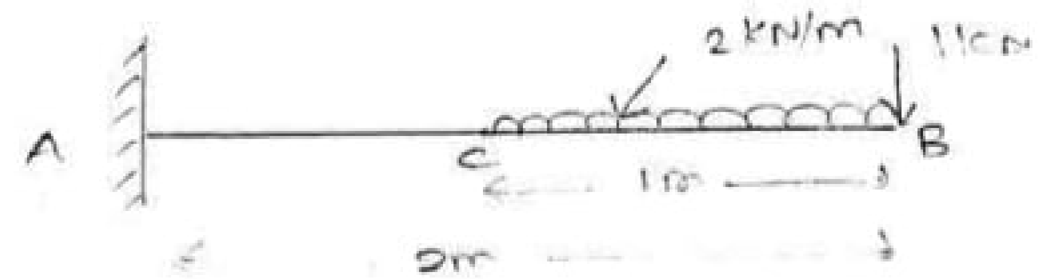
$$= 0.9 \text{ mm}$$

$$Y = Y_B + Y_C$$
$$= 1.833 \text{ mm}$$

② A cantilever of length 2m carries a UDL of 2.5 kN/m run for a length of 1.25 m from the fixed end and a point load of 1 kN at free end. Find the deflection at the free end if the section is rectangular 12 cm wide & 24 cm deep take $E = 1\times10^4$ N/mm^2.

y_B = 0.9334 mm

(3). A cantilever of length 2m carries a UDL of $2\,kN/m$ ~~per/m~~. over a length of 1m from the free end and a point load of 1kN at the free end. Find the slope and deflection at the free end. If $E = 2.1 \times 10^5\ N/mm^2$ and $I = 6.667 \times 10^7\ mm^4$.



GIVEN:

$L = 2m$

$E = 2.1 \times 10^5\ N/mm^2$.

$I = 6.667 \times 10^7\ mm^4$

Solution:

$\theta = \theta_B + \theta_C$

$$\theta_B = \frac{WL^2}{2EI} = \frac{10^3 \times (2 \times 10^3)^2}{2 \times 2.1 \times 10^5 \times 6.667 \times 10^7}$$

For the load at free end

$$= \cancel{7.142 \times 10^{-4}}\ \ 1.428 \times 10^{-4}$$

$$\theta_C = \frac{wL^3}{6EI} - \frac{w(L-a)^3}{6EI} \quad \text{(for UDL at free end)}$$

$$= \frac{2 \times 10^3 \times 2000^3}{6 \times 2.1 \times 10^5 \times 6.667 \times 10^7} - \frac{2\,(2000 - 1000)^3}{6 \times 2.1 \times 10^5 \times 6.667 \times 10^7}$$

$$= 1.904 \times 10^{-4} - 2.38 \times 10^{-5}$$

$$= 1.665 \times 10^{-4}$$

$\therefore \theta = \theta_B + \theta_C$

$= \ldots \times 10^{-4}$ radians

Deflection, $y = y_B + y_c$

$$y_B = \frac{WL^3}{3EI} \quad \text{(point load at free end)}$$

$$= \frac{10^3 \times (2\times10^3)^3}{3\times2.1\times10^5\times6.667\times10^7} = 0.1904$$

$$y_c = \frac{wL^4}{8EI} - \left[\frac{w(L-a)^4}{8EI} + \frac{w(L-a)^3}{6EI}a\right] \leftarrow \text{(UDL at free end)}$$

$$= \frac{2\times(2000)^4}{8\times2.1\times10^5\times6.667\times10^7} - \left[\frac{2(2000-1000)^4}{8\times2.1\times10^5\times6.667\times10^7} + \frac{2(1000)^4\times1000}{6\ \cancel{8}\times2.1\times10^5\times6.667\times10^7}\right]$$

$$= 0.2857 - [0.0178 + 0.0238]$$

$$= 0.$$

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CANTILEVER:

Moment area method:

$$\theta = \frac{A}{EI}, \quad y = \frac{A\bar{x}}{EI}$$

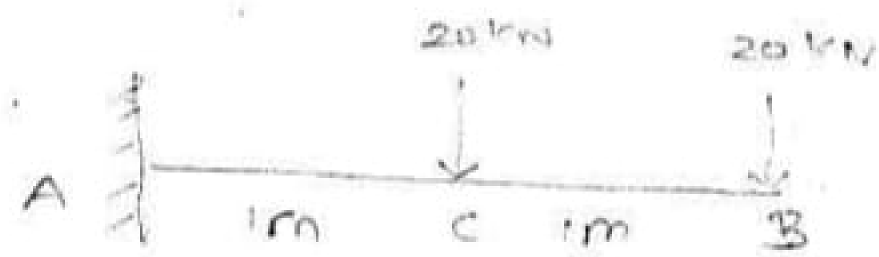
1) A cantilever of length 2 m carries a point load of 20 kN at the free end and another load of 20 kN at its centre. If $E = 10^5\ N/mm^2$ and $I = 10^8\ mm^4$ for the cantilever. Determine by moment area method the slope and deflection at the free end.

GIVEN:

$L = 2 \times 10^3\ mm$

$E = 105\ N/mm^2$

$I = 10^8\ mm^4$



Solution:

B.M at B = 0

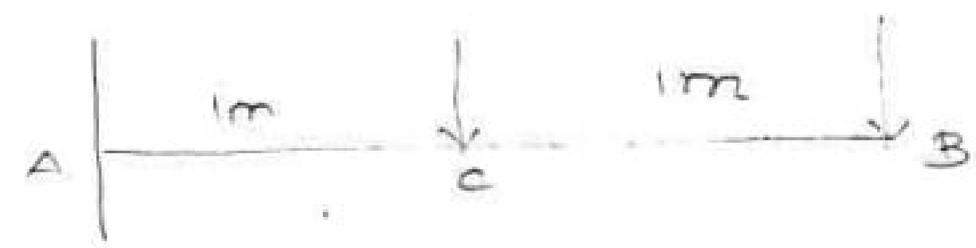
B.M at C $= -20 \times 10^3 \times 1 \times 10^3$

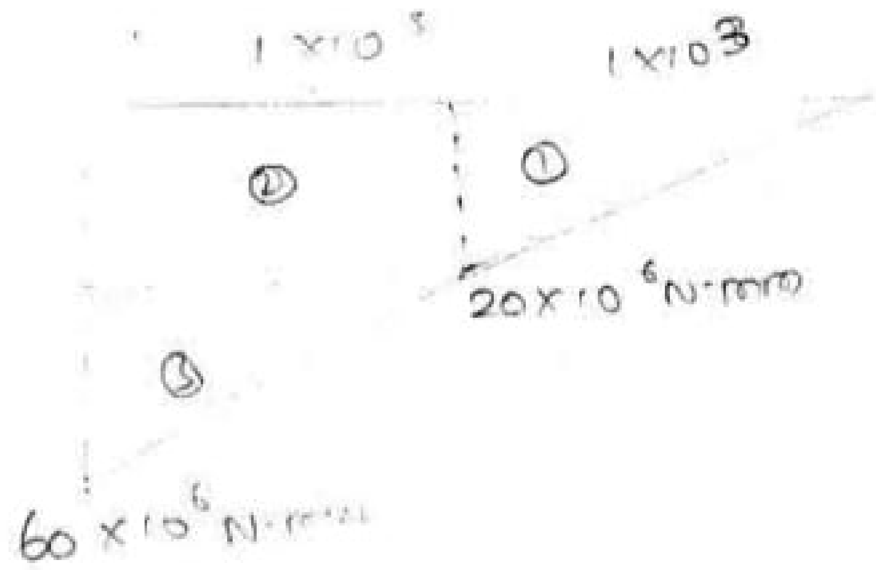
$= -20 \times 10^6$ N-mm

B.M at A $= -(20 \times 10^3 \times 2 \times 10^3) - (20 \times 10^3 \times 1 \times 10^3)$

$= -40 \times 10^6 - 20 \times 10^6$

$= -60 \times 10^6$ N-mm





W.K.T,

$$\theta = \frac{A}{EI} \qquad y = \frac{A\bar{x}}{EI}$$

$$A = A_1 + A_2 + A_3$$

$$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

$$A_1 = \frac{1}{2}ab$$

$$= \frac{1}{2} \times 20\times10^6 \times 1\times10^3$$

$$= 10^{10}\ N.mm^2$$

$$X_1 = \frac{2}{3}b = \frac{2}{3}\times1\times10^3$$

$$= 666.6\ mm^2.$$

$$A_2 = a\times b$$

$$= 20\times10^6 \times 1\times10^3$$

$$= 20\times10^9$$

$$\bar{x}_2 = 1\times10^3 + \frac{1}{2}b$$

$$= \left(1\times10^3 + \frac{1\times10^3}{2}\right)$$

$$= 1500\ mm$$

$$A_3 = \frac{1}{2} ab$$
$$= \frac{1}{2} \times 40 \times 10^6 \times 1 \times 10^3$$
$$= 2 \times 10^{10} \text{ mm}$$
$$\bar{X}_3 = (1 \times 10^3 + \tfrac{2}{3} b)$$
$$= \left(1 \times 10^3 + \frac{2}{3}(1 \times 10^3)\right)$$
$$= 1666.6 \text{ mm}$$

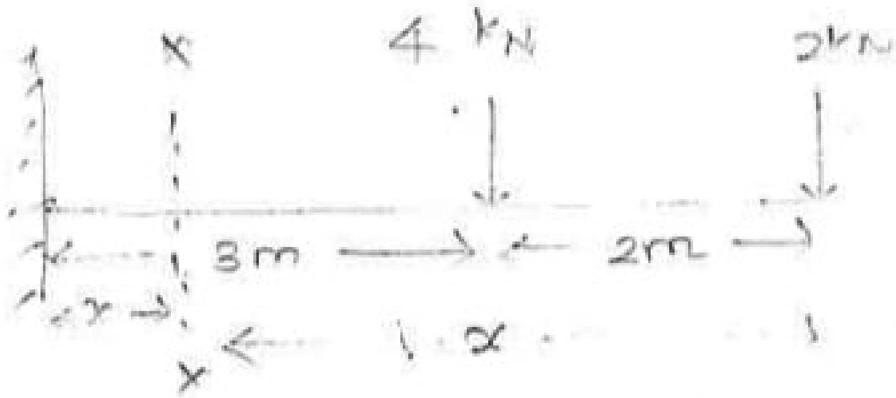
$$A\bar{X} = A_1\bar{x}_1 + A_2\bar{X}_2 + A_3\bar{X}_3$$
$$= (666.6 \times 10^{10}) + (3 \times 10^{13}) + (3.33 \times 10^{13})$$
$$A\bar{x} = 6.99 \times 10^{13}$$

$$y = \frac{A\bar{x}}{EI} = \frac{6.99 \times 10^{13}}{10^5 \times 10^8}$$
$$y = 6.99 \text{ mm}$$

CANTILEVER BEAM :

Macaulay's method :

1.



$E = 2.1 \times 10^{12}$ N/mm²
$I = 5 \times 10^7$ mm⁴

WKT :

$$M_{xx} = EI \frac{d^2y}{dx^2}$$
$$M_{xx} = -2x \;\vdots\; -4 \times (x-2)$$

$$EI\frac{d^2y}{dx^2} = -2x \;\vdots\; -4(x-2) \quad \text{—} ①$$

$\int$ ean ①

$$EI\frac{dy}{dx} = -\frac{2x^2}{2} + C_1 \;\vdots\; -\cancel{4}\frac{(x-2)^2}{\cancel{2}} \quad - ②$$

$\int$ ean. ②.

$$EIy = -\frac{x^3}{3} + C_1x + C_2 \;\vdots\; -\frac{2(x-2)^3}{3} \quad \text{—} ③$$

Find C_1 & C_2

and i) $x=0 \quad y=0$

ii) $x=0 \quad \frac{dy}{dx}=0$

Sub cond (i) in ③

$$\boxed{C_2 = 0}$$

Sub cond ii) in ean ②

$$\boxed{C_1=0}$$

To find slope (use eqn ②) at free end.
$(x=5m)$

$$EI\frac{dy}{dx} = -x^2 + C_1 \;\vdots\; -2(x-2)^2$$

To find y at free end use eqn. ③ $(x=5m)$

$$EIy = -\frac{x^3}{3} + C_1x + C_2 \;\vdots\; -\frac{2(x-2)^3}{3}$$

$$= \cancel{-\frac{5^3}{3}} + \cancel{C_1(5)} \cancel{+0} - \frac{2(3)^3}{3}$$

$=$

$$EI \frac{dy}{dx} = 43 \text{ kN-m}^2$$

$$= 43 \times 10^9 \text{ N-mm}^2$$

$$\frac{dy}{dx} = \theta = \frac{-43 \times 10^9}{EI} = -0.00409 \text{ radians}$$

$$EIy = -\frac{x^3}{3} + C_1 x + C_2 \Big| -\frac{2(x-2)^3}{3}$$

$$EIy = -\frac{(5)^3}{3} + 0 + 0 \quad -\frac{2(3)^3}{3}$$

$$EIy = -59.66$$

$$y = -\frac{59.66 \times 10^3 \times 10^9}{EI}$$

$$= -5.68 \text{ mm}$$

$$\boxed{y = 5.68 \text{ mm}}$$ (downward deflection)

CONJUGATE BEAM METHOD : (EI not uniform)

This method is used when flexural rigidity (EI) is not uniform throughout the length of the beam.

1) Slope at any section is equal to shear force.

2). Deflection at any section is equal to bending moment.

Procedure :

* Draw the B.M diag for given beam.

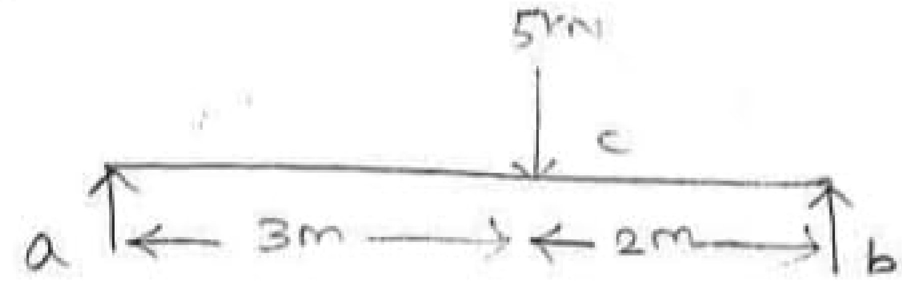
* Draw the load diagram of the conjugate beam by using the relation $\frac{BM}{EI}$.

* Find the reactions of conjugate beam to find the slope.

* Find the B.M of conjugate beam to find the deflection.

1. A simply supported beam of length 5m carries a pt. load of 5 kN at a distance of 3m from the left end. If $E = 2\times10^5 N/mm^2$ and $I = 10^8 mm^4$. Determine the slope at the left supp pnt and deflection under the point load using conjugate beam method.

GIVEN :



$R_a + R_b = 5 kN$

Moment about B.

$(R_a \times 5) - (5\times 2) = 0$

$R_a \times 5 = 10$

$\boxed{R_a = 2 kN}$

$\boxed{R_b = 3 kN}$

B.M at a $= 0$

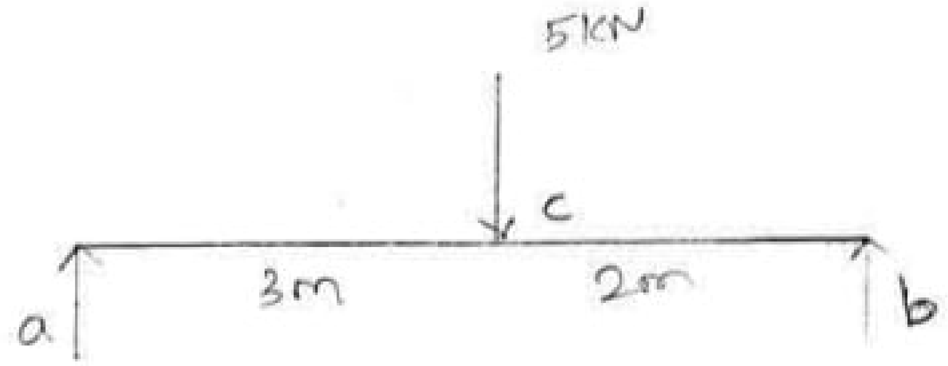
B.M at b $= 0$

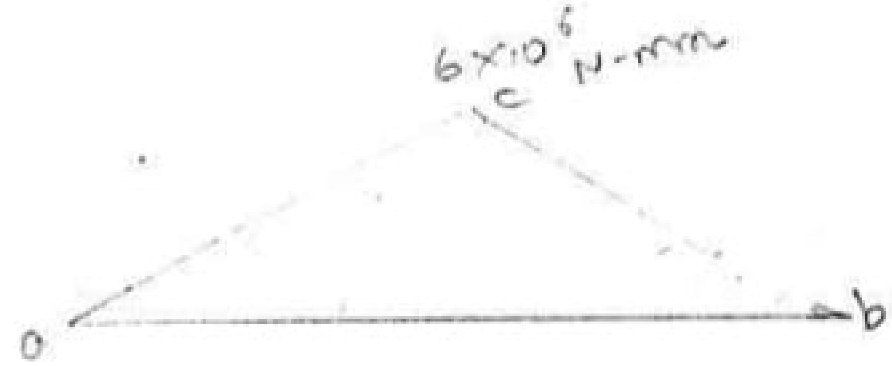
B.M at c $= R_a \times 3$

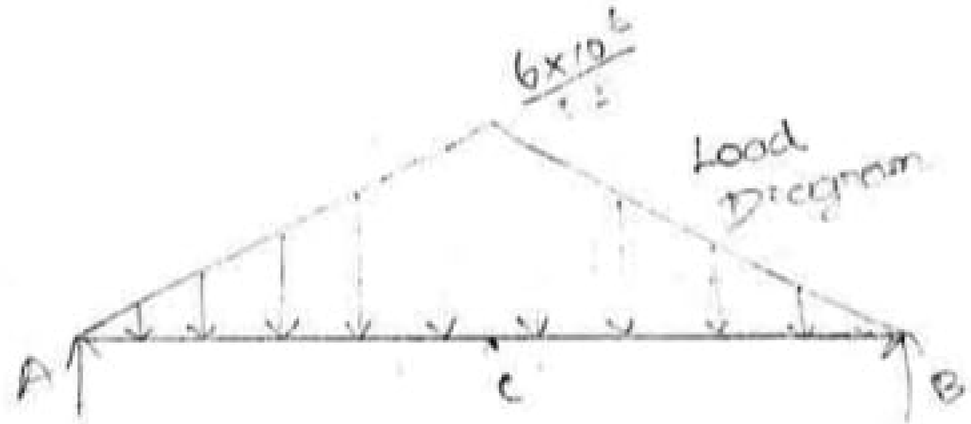
$= 2 \times 3$

$= 6$ KN-m

$= 6 \times 10^6$ N-mm

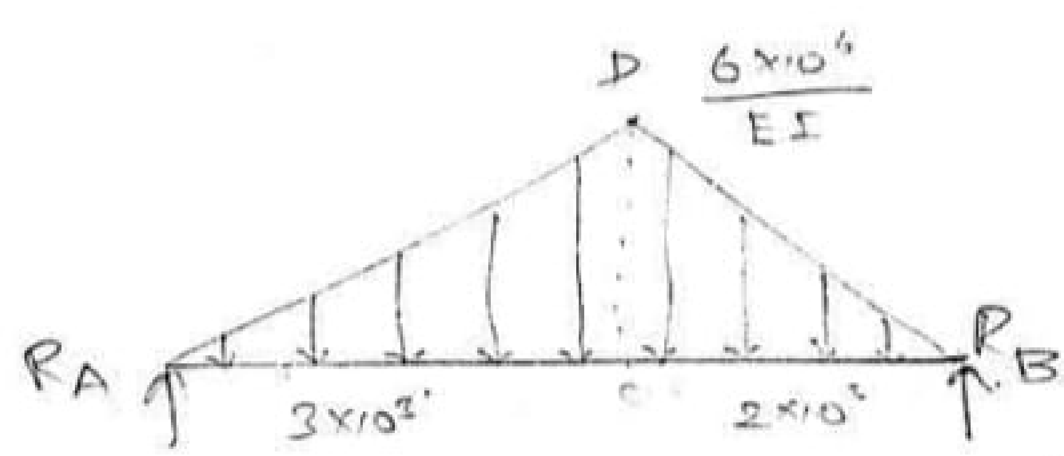






~~To find Reac~~

To find Reactions R_A & R_B



$R_A + R_B =$ Total load

$= (\text{Area of } \triangle\ ABD)$

$= \frac{1}{2} AB \times CD$

$= \frac{1}{2} \times 5 \times 10^3 \times \frac{6 \times 10^6}{}$

B.M about B:

$R_A \times 5\times10^3$ – (Area of ΔACD × C.G of ACD) –

(Area of ΔBCD × C.G of ΔBCD from B)

Load on BCD

$$R_A \times 5\times10^3 = \left(\frac{1}{2}\times 3\times10^3 \times \frac{6\times10^6}{EI}\right)\times\left(2\times10^3+\frac{1}{3}3\times10^3\right) +$$
$$\left(\frac{1}{2}\times 2\times10^3\times\frac{6\times10^6}{EI}\right)\times\left(\frac{2}{3}\times 2\times10^3\right)$$

$$R_A = \frac{7\times10^9}{EI} : \cancel{\frac{7\times10^9}{2\times10^5\times10^8}} =$$

$$R_B = \frac{8\times10^9}{EI}$$

According to conj. Beam method:

$$\text{Slope} = \theta = \frac{dy}{dx} = \text{Shear force at the section.}$$

Slope at left support = Shear force at the section

Slope at left support = Shear force at A = R_A

$$= \frac{7\times10^9}{EI}\ \text{rad}$$

$$\theta = 0.00035\ \text{radians.}$$

Deflection at load C:

y = B.M at C.

B.M at C = $R_A \times 3\times10^3$ – (Area of Δ ACD (or) Load on ACD × C.G of ACD from 'C')

$$= \frac{7\times10^9}{EI}\times 3\times10^3 - \left(\frac{1}{2}\times 3\times10^3\times\frac{6\times10^6}{EI}\times\frac{1}{3}3\times10^3\right.$$

② A simply supported beam AB of length 4 m carries a point load of 100 kN. at its centre C. The value of I for the left half is 1×10^8 mm^4. and for the right half portion is $I = 2 \times 10^8$ mm^4. Find the slope at two support and deflection under the load. Take $E = 200$ GN/m^2

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UNIT-V

THIN CYLINDERS, SPHERES & THICK CYLINDERS

Vessels of spherical and cylindrical forms are used for storing fluids under pressure
ex: Steam boilers, air compressors, tanks., etc.

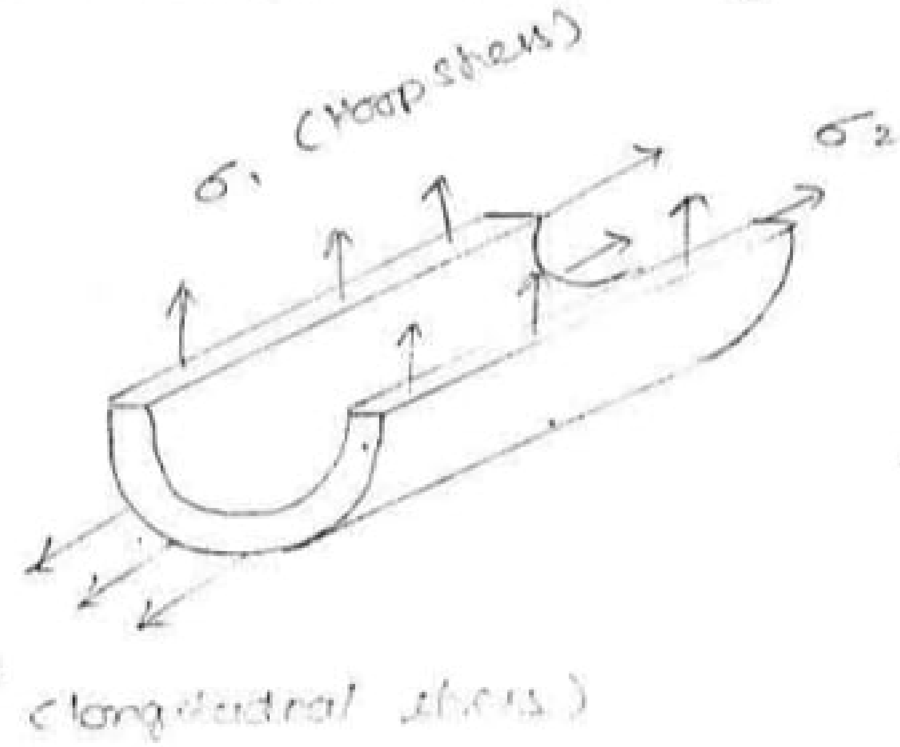
When it is a gas the pressure is constant in all parts of the vessel but in liquids the pressure is low at the top and increases with the depth.

THIN CYLINDER :

Thickness of the cylinder plate in relation to the internal diameter is

$$\frac{t}{d} = \frac{1}{20}$$

The maximum capacity is 30 MN/m²



TYPES OF STRESSES IN CYLINDRICAL SHEELS :

1) Circumferencial stress (or) Hoop stress.
2). Longitudinal stress
3). Radial stress. (negligible)

FORMULAS :

1). Hoop stress (or) circumferial stress :

$$\sigma_1 = \frac{Pd}{2t}$$

$$\sigma_2 = \frac{Pd}{4t} \text{ (or) } \frac{1}{2}\sigma_1$$

Max. shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{Pd}{8t}.$$

Note : ① If max. permissible stress is given it has to be considered as circumferencial stress for finding thickness.

② Hoop stress (or) longitudnal stress is directly proportional to diameter and inversely proportional to thickness.

EFFICENCY OF A JOINT :

1). circumferencial stress (or) Hoop stress :

$$\sigma_1 = \frac{Pd}{2t \times \eta_l}$$

$$\sigma_2 = \frac{Pd}{4t \times \eta_c} \text{ (or) } \frac{1}{2}\sigma_1$$

Max. shear stress. $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

NOTE :

(1). Efficiency of a joint means the efficiency of longitudinal joint.

(2) If efficiency of a joint is given the thickness is found by using circumferential stress equation.

where,

P - Internal fluid pressure.

T = Thickness of wall the cylinder.

D - Internal dia. of cylinder.

η_l - Longitudinal efficiency.

η_c - Circumferential joint efficiency

PROBLEMS :

(1) A water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m³ Find the thickness of the metal required for the water main. given the permissible stress as 20 N/mm².

GIVEN :

$d = 80 \times 10$ mm.

$h = 100 \times 10^3$ mm.

$\rho = 9810 \times 10^{-9}$ N/mm³

$\sigma = 20$ N/mm² $(\sigma_1 \text{ or } \sigma_2)$

Solution :

* To find 't' :

Hoop stress , $\sigma_1 = \frac{Pd}{2t}$

Pressure , $P = w \times h$

$= 9810 \times 10^{-9} \times 100 \times 10^3$

$= 0.981 \ N/mm^2$

$\sigma_1 = \frac{Pd}{2t}$

$20 = \frac{0.981 \times 80 \times 10}{2 \times t}$

$t = 19.62$ ✓

When t is greater σ is small (σ should be small), ∴ t = 19.62 is taken.

$\sigma_2 = \frac{Pd}{4t}$

$20 = \frac{0.981 \times 80 \times 10}{4 \times t}$

$t = \cancel{9.81}$ mm
19.62

$t \propto \frac{1}{\sigma}$

If $t = \cancel{9.81}$ 19.62, σ_1 is greater than permissible stress. Hence $\boxed{t = 19.62 \text{ mm}}$

(2) A boiler shell is to be made of 15mm thick plate having limiting tensile stress of 120 N/mm². If the efficiency of the longitudinal and circumferencial joints are 70% and 30%. Determine the maximum permissible diameter of the shell for an internal pressure of 2 N/mm².

ii) Find permissible intensity of internal pressure of the shell when the diameter is 1.5 m.

GIVEN :

$t = 15 mm$

$\sigma = 120 N/mm^2$ $(\sigma_1 (or) \sigma_2)$

$\eta_l = 70\%$

$\eta_c = 30\%$

$P = 2 N/mm^2$

Solution :

TO find 'd' if $P = 2 N/mm^2$

$\sigma_1 = \frac{Pd}{2t}$ $\qquad$ $\sigma_1 = \frac{Pd}{2t\eta_l}$

$120 = \frac{2 \times d}{2 \times 15}$ $\qquad$ $120 = \frac{2 \times d}{2 \times 15 \times 0.7}$

$d =$

$d = 1260 mm$

longitudinal stress. $\sigma_2 = \frac{Pd}{4t\eta_c}$

$120 = \frac{2 \times d}{4 \times 15 \times 0.3}$

$d = 1080 mm.$

The safe diameter is 1080 mm.

since dia & stress

For checking : Sub $d = 1260 mm$

$\sigma_2 = \frac{Pd}{4t\eta_c} = 140 N/mm^2$ not safe.

Hence $d = 1080 mm$

re

ii). D = 1.5 mm

$$\sigma_1 = \frac{Pd}{2t\,\eta_l}$$

$$P = \frac{\sigma \times 2t \times \eta_l}{d}$$

$$= \frac{120 \times 2 \times 15 \times 0.70}{1.5 \times 10^3}$$

$$P = 1680 \times 10^{-3}$$

$$P = 1.680 \ N/mm^2$$

iii

$$\sigma_2 = \frac{Pd}{4t \times \eta_L}$$

$$P = \frac{\sigma_2 \times 4 \times t \times \eta_c}{d}$$

$$P = 1440 \times 10^{-3} \ N/mm^2$$

$$P = 1.44 \ N/mm^2$$

$\therefore$ Safe pressure, $P = 1.44 \ N/mm^2$

26/9/14

EFFECT OF INTERNAL PRESSURE ON THE DIMENSION OF THIN CYLINDER:

CIRCUMFENTIAL STRAIN (ε_1)

$$\varepsilon_1 = \frac{\text{change in dia}}{\text{original dia}} = \frac{\delta d}{d}$$

$$\varepsilon_1 = \frac{\delta d}{d} = \frac{\sigma_1}{F} - \frac{\mu \sigma_2}{E} = \frac{Pd}{2tE}\left(1 - \mu/2\right)$$

Longitudnal strain (ε_2):

$$\varepsilon_2 = \frac{\text{change in length}}{\text{original length}} = \frac{\delta L}{L}$$

$$= \frac{\delta L}{L} = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E} = \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

volumetric strain : (ε_v):

$$\varepsilon_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

$$= \frac{\delta V}{V} = 2\varepsilon_1 + \varepsilon_2 = \frac{Pd}{2Et}\left(\frac{5}{2} - 2\mu\right)$$

where, $V = \frac{\pi}{4} d^2 L$.

① A cylindrical vessel whose ends are closed by means of rigid flanged plates made of steel 3mm thick the length and internal diameter of vessel are 50cm and 25cm. Determine the hoop stress and longitudnal stress in the cylinder due to internal fluid pressure of 3N/mm². Also calculate increase in length, diameter and volume of the vessel. Take $E = 2 \times 10^5$ N/mm². And poisson ratio $\mu = 0.3$.

GIVEN:

$t = 3$ mm

$L = 500$ mm

$d_1 = 250$ mm.

P ~~σ~~ $R = 3$ N/mm²

$E = 2 \times 10^5$ N/mm²

Solution :

$$\sigma_1 = \frac{Pd}{2t}$$

$$3 = \frac{P \times 250}{2 \times 3}$$

$$P = 0.072 \text{ W}$$

$$\sigma_2 = \frac{Pd}{4t} \quad (\text{or}) \ \frac{1}{2}\sigma_1$$

$$\sigma_2 = \frac{0.072 \times 250}{4 \times 3}$$

$$\sigma_2 = 1.5 \ N/mm^2$$

Hoop stress : $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \Rightarrow \frac{3 - 1.5}{2} = 0.75 \ N/mm^2$

i). Hoop stress,

$$\sigma_1 = \frac{Pd}{2t} \Rightarrow \frac{3 \times 250}{2 \times 3} = 125 \ N/mm^2.$$

$$\sigma_2 = \frac{Pd}{4t} = \frac{3 \times 250}{4 \times 3} = 62.5 \ N/mm^2$$

ii) Increase in length: (δl)

W.K.T $\quad \varepsilon_2 = \frac{\delta L}{L} = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E}$

(or)

$$\frac{\delta L}{L} = \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\frac{\delta L}{500} = \frac{3 \times 250}{2 \times 3 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

$$\delta L = 0.0625 \text{ mm}$$

iii) change in dia (δd):-

$$\epsilon_1 = \frac{\delta d}{d} = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$$

$$\frac{\delta d}{250} = \frac{125}{2\times10^5} - \frac{0.3\times62.5}{2\times10^5}$$

$$\delta d = 0.133 \text{ mm}$$

iv). change in volume (δV)

$$\epsilon_v = \frac{\delta v}{v} = 2\epsilon_1 + \epsilon_2$$

(or)

$$\frac{\delta v}{v} = \frac{Pd}{2tE}\left(5/2 - 2\mu\right)$$

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times 250^2 \times 500$$

$$= 245.31 \times 10^5 \text{ mm}^3$$

$$\therefore \frac{\delta v}{245.31\times10^5} = \frac{3\times250}{2\times3\times2\times10^5}\left(\frac{5}{2} - 2(0.3)\right)$$

$$\delta V = 29.13 \times 10^3 \text{ mm}^3$$

Not required

(2). A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure, Length = 1.2 m, Ext dia = 20 cm thickness of metal = 8 mm. Find the value of pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of 25 cm³ of liquid is pumped into the cylinder. Take $E = 2.\cancel{1} \times 10^5\ N/mm^2$ and $\mu = 0.33$.

GIVEN :

$L = 1.2m \Rightarrow 1.2 \times 10^3\ mm$

$d_2 = 20cm \Rightarrow 200\ mm$

$t = 8\ mm.$

$\partial v = 25\ cm^3$

$= 25 \times 10^3\ mm^3.$

Solution :

Internal diameter = 184 mm.

$$\frac{\delta v}{v} = \frac{Pd}{2Et}\left(\frac{5}{2} - 2\mu\right)$$

$$v = \frac{\pi}{4} \times 184^2 \times 1.2 \times 10^3$$

$$= 318.923 \times 10^5\ mm^3$$

$$\frac{25000}{318.923 \times 10^5} = \frac{P \times 184}{2 \times 2.1 \times 10^5 \times 8}\left(\frac{5}{2} - 2 \times 0.33\right)$$

$$7.83 \times 10^{-4} = P(5.476 \times 10^{-5})(1.84)$$

$$P = 7.77\ N/mm^2$$

(i) Hoop Stress : (σ_1) :

$$\sigma_1 = \frac{Pd}{2t} \Rightarrow \frac{7.77 \times 184}{2 \times 8}$$

$$\sigma_1 = 89.35\ N/mm^2$$

0cm

244

③ A cylindrical shell 90 cm long and 20 cm internal diameter having thickness as 8 mm is filled with fluid at atmospheric pressure if an additional volume of 43 cm^3 of fluid is pumped into the cylinder. Find the

i) pressure exerted on the cylinder, ~~hoop stress~~

ii) The Hoop stress and Longitudinal stress.

iii) Change in dia & change in length.

Take $E = 2 \times 10^5$ N/mm² and $\mu = 0.35$.

GIVEN:

$L = 900$ mm.

$d = 200$ mm.

$t = 8$ mm.

$\delta V = 43 \times 10^3$ mm³

$E = 2 \times 10^5$ N/mm²

$\mu = 0.35$.

Solution:

$$V = \frac{\pi}{4} \times d^2 l.$$

$$= \frac{\pi}{4} \times 200^2 \times 900$$

$$V = 282.600 \times 10^5 \text{ mm}^3$$

$$\frac{\delta V}{V} = \frac{Pd}{2Et}\left(\frac{5}{2} - 2\mu\right)$$

$$\frac{43 \times 10^3}{282.6 \times 10^5} = \frac{P \times 200}{2 \times 2 \times 10^5 \times 8}(2.5 - 0.7)$$

$$P = 13.525 \text{ N/mm}^2$$

Hoop stress,

$$\sigma_1 = \frac{pd}{2t} \Rightarrow \frac{13.52 \times 200}{2 \times 8}$$

$$\sigma_1 = 169.06 \text{ N/mm}^2$$

iii) change in dia :

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$\frac{\delta d}{d} = \frac{13.52 \times 200}{2 \times 8 \times 2 \times 10^5}\left(1 - \frac{0.35}{2}\right)$$

$$\delta d = 6.97 \times 10^{-4} \times 200$$

$$\delta d = 0.139 \text{ mm}$$

Change in length :

$$\frac{\delta l}{l} = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\frac{\delta l}{900} = \frac{13.52 \times 200}{2 \times 8 \times 2 \times 10^5}(0.5 - 0.35).$$

$$\delta l = 0.114 \text{ mm}.$$

4. A thin copper cylinder 90 cm long, 40 cm external diameter and value thickness 6 mm has its both ends closed. by rigid flanges It is initially full of oil at atmospheric pressure. calculate the additional volume of oil which must be pumped into it. inorder to rise the oil pressure to 5 N/mm². For copper. $E = 1 \times 10^5$ N/mm² and $\mu = 1/3$. Take Bulk modulus of oil as 2.6×10^3 N/mm²

246

GIVEN :

$l = 900\ mm$

$d = 400\ mm$

$t = 6mm$

$d = D - 2t$

$= 388\ mm$

$P = 5\ N/mm^2$

$E = 1 \times 10^5\ N/mm^2$

$\mu = 1/3$

Bulk modulus $K = 2.6 \times 10^3\ N/mm^2$

Solution :

To find additional volume of oil to raise 'P' by $5\ N/mm^2$ (δV)

Additional space created in cylinder

= Increase in volume of cylinder shell + Decrease in volume of oil.

$\delta V = \delta V_1 + \delta V_2$

i) Increase in volume of cylinder (δV_1)

$$\frac{\delta V_1}{V} = \frac{Pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times 400 \times 900$$

$$= 282.6 \times 10^3\ mm^3$$

$$\frac{\delta V_1}{282.6 \times 10^3} = \frac{5 \times \cancel{400}\ 388}{2 \times 6 \times 1 \times 10^5}(2.5 - 0.66)$$

$$\delta V_1 = \cancel{316.38 \times 10^3\ mm^3}$$

$$316.382 \times 10^3\ mm^3$$

ii) Decrease in volume of oil (δV_2).

$$K = \frac{\text{Increase in pressure}}{\text{volumetric strain}} \quad \text{(Direct strain)}$$

$$K = \frac{P}{\left(\frac{\delta V_2}{V}\right)}$$

$$2.6 \times 10^3 = \frac{5}{\frac{\cancel{316.38 \times 10^3}\ \delta V_2}{282.6 \times 10^3}}$$

$$\delta V_2 = 204.56 \times 10^3 \, mm^3$$

$$\delta V = \delta V_1 + \delta V_2$$

$$= (316.38 \times 10^3) + (204.56 \times 10^3)$$

$$\delta V = 520.9 \times 10^3 \, mm^3$$

3. The volume of oil required to rise the internal fluid pressure by $5 N/mm^2$ = $520.9 \times 10^3 \, mm^3$

THIN CYLINDER VESSEL SUBJECTED TO INTERNAL FLUID PRESSURE AND TORQUE :

Major principle stress $= \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

Minor principle stress $= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

Max. shear stress, $\tau_{max} = \frac{1}{2}$ [Major principle stress − Minor principle stress]

Shear stress τ :

$$\boxed{\text{Torque} = \text{shear force} \times d/2}$$

shear force = shear stress × Area

Area $= \pi d \times t$

$$\boxed{\text{Shear } \cancel{\text{Force}} \text{ Stress} = \frac{2 \cdot T}{\pi d^2 \cdot t}}$$

where σ_1 = circumferencial stress

σ_2 = longitudnal stress.

τ = shear stress

d = internal diameter

t = Thickness of the shell

1. A thin cylindrical tube 80 mm internal dia and 5mm thick is closed at the ends and is subjected to an internal pressure of 6 N/mm² a torque of 2009×10^3 N/mm is also applied to the tube. Find the Hoop stress, longitudnal stress, Maximum and minimum principle stress and Max shear stress

GIVEN :

$d = 80 mm$

$t = 5mm$

$P = 6 N/mm^2$

$T = 2009 \times 10^3$ N mm.

Solution :

Hoop stress $\sigma_1 = \frac{Pd}{2t}$

$= \frac{6 \times 80}{2 \times 5} = 48 N/mm^2$

$\sigma_2 = \frac{Pd}{4t} = \frac{6 \times 80}{4 \times 5} = 24 N/mm^2$

Torque $= \text{Shear force} \times \frac{d}{2}$

Shear force $= \frac{2009 \times 2}{80}$

$= 50,225.$

$50.225 = \text{Shear } \cancel{\text{force}} \text{ stress} \times \pi d t$

$S.F = \frac{50,225}{\pi \times 80 \times 5}$.

$S.F = \cancel{0}.0399 N/mm^2$ $40 N/mm^2$

Major principle stress $= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{48 + 24}{2} + \sqrt{\left(\frac{24}{2}\right)^2 + \cancel{0.0399}(40)^2}$

$= 36 + 41.76$

$= 76 N/mm^2$ (tensile)

250

Minor principle stress $= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$= 36 - 41.76$

$= -5 \ N/mm^2$

Max. shear stress $= \frac{1}{2}[77 + 5]$

$= 41 \ N/mm^2$

(2). A steel cylinder of length 1.5 m and internal diameter 25cm and thickness 10 mm is filled with a liquid, find the value of pressure exerted by liquid on the walls of the cylinder and the hoop stress and the longitudnal stress if an additional volume of $30cm^3$ of liquid is pumped in to the cylinder. Take $E = 2.5 \times 10^5 \ N/mm^2$ and poisson ratio $\mu = 0.3$. Also find major, or minor & max shear stress if Torque applied is 3×10^5 Nmm.

GIVEN:

$l = 1.5 \times 10^3 mm$

$d = 250 mm$

$t = 10 mm.$

$\delta V = 30 \times 10^3 mm^3$

$E = 2.5 \times 10^5 \ N/mm^2$

$\mu = 0.3$

$T = 3 \times 10^5$ Nmm

Solution :

$$\sigma_1 = \frac{Pd}{2t}$$

$$V = \frac{\pi}{4} D^2 L \Rightarrow \frac{\pi}{4} \times (250)^2 \times (1.5 \times 10^3)$$

$$V = 7.3631 \times 10^7 \ mm^3.$$

$$\frac{\delta V}{V} = \frac{Pd}{2tE} \left(5/2 - \mu\right)$$

ϕV , E

$$\tau_{max} = \frac{Pd}{8t}$$

$$3 \times 10^5 = \frac{P \times 250}{8 \times 10}$$

$$P =$$

$$\text{Torque} = \text{Shear force} \times \frac{d}{2}$$

$$\frac{30 \times 10^3}{7.3631 \times 10^7} = \frac{P \times 250}{2 \times 10 \times 2.5 \times 10^3} \left(\frac{5}{2} - 2(0.3)\right)$$

$$\frac{30 \times 10^3}{7.3631 \times 10^7} = \frac{P \times 250}{2 \times 10 \times 2.5 \times 10^3} (2.5 - 0.3)$$

$$\frac{30 \times 10^3}{7.3631 \times 10^7} = 0.011 P$$

$$P = 0.0370$$

(251)

(2) A thin spherical shell of 2m dia and of thickness 10mm is filled with an incompressible liquid. Estimate the quantity of liquid that should be pumped in so as to raise ~~the quantity of liquid~~ pressure to 5N/mm2. Take E = 200 kN/mm² & $\mu = 0.3$

$$\frac{\delta v}{v} = \frac{3pd}{4tE}(1-\mu)$$

$$\delta v = 10.99 \times 10^{6}\ mm^{3}.$$

(1

THIN SPHERES :

①. circumferential stress, $\sigma_1 = \frac{Pd}{4t}$

②. Longitudinal stress, $\sigma_2 = \frac{Pd}{4t}$

③. Strain, $e = \frac{\text{change in dia}}{\text{original data}} = \frac{\delta d}{d}$

$$e = \frac{\delta d}{d} = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} = \frac{Pd}{4tE}(1-\mu)$$

④. volumetric strain $= \frac{\delta v}{V}$

$$\frac{\delta v}{V} = \frac{3Pd}{4tE}(1-\mu)$$

Volume $= \frac{4}{3}\pi r^3$ (or) $\frac{\pi}{6} d^3$

$\sigma = \rho r^2 \omega^2$

$\omega = \frac{2\pi N}{60}$

12

①. A steel spherical shell is made of 5mm thick plate and external diameter 27cm if the maximum permissible stress is 140 N/mm² and the pressure exerted due to the fluid is 6 N/mm². Find the thickness of the plate.
ii) Find the change in volume and change in diameter, $\mu = 0.3$ and $E = 2\times10^5$ N/mm²

GIVEN : $d = d_o - 2t$

$t = 2.8$mm

$\delta V = 14.22\times10^3$ mm³ $\quad V = 9.67\times10^6$ mm³

$\delta d = 0.129$mm.

(25a)

30/9/14.

THICK CYLINDERS :

* Hoop stress is maximum at the inner surface of the cylinder and minimum at the outer side. (σ_x · ϵ_x)

* Longitudinal stress and longitudinal strain are constant (σ_2 -Stress) (ϵ_2 - Strain)

* Radial stress is compressive.

① Radial stress at any point on the surface of cylinder:

$$P_x = \frac{b}{x^2} - a$$

② Hoop stress at any point on the surface of cylinder.

$$\sigma_x = \frac{b}{x^2} + a$$

Boundary conditions,

i) $x = r_i$, $P_x = P$ (fluid pressure)

ii) $x = r_o$, $P_x = 0$ (Atm pressure)

where,

a, b are constants and can be determined from the boundary conditions.

P_x is radial stress

σ_x is Hoop stress. (circumferencial stress)

r_i inner radius

r_o outer radius.

P fluid pressure.

1. Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick when pipe contains a fluid at a pressure of 8 N/mm² and also find the radial stresses inside and outside of the cylinder. Also draw the radial stress & Hoop stress distribution across the section.

GIVEN :

$$r_i = \frac{400}{2} \text{ mm} \Rightarrow 200 \text{ mm}$$

$$r_o = \frac{600}{2} \text{ mm} \Rightarrow 300 \text{ mm}$$

$$P = 8 \text{ N/mm}^2$$

Solution :

To find.

i) Min & max σ_x :

W.K.T, $\sigma_x = \frac{b}{x^2} + a$

i) $x = r_i = 200$, $P_x = P = 8$

ii) $x = r_o = 300$, $P_x = 0$

$$P_x = \frac{b}{x^2} - a$$

$$8 = \frac{b}{(200)^2} - a$$

$$a = \frac{b}{40,000} - 8$$

Subs ii) boundary condition.

$$0 = \frac{b}{(300)^2} - a$$

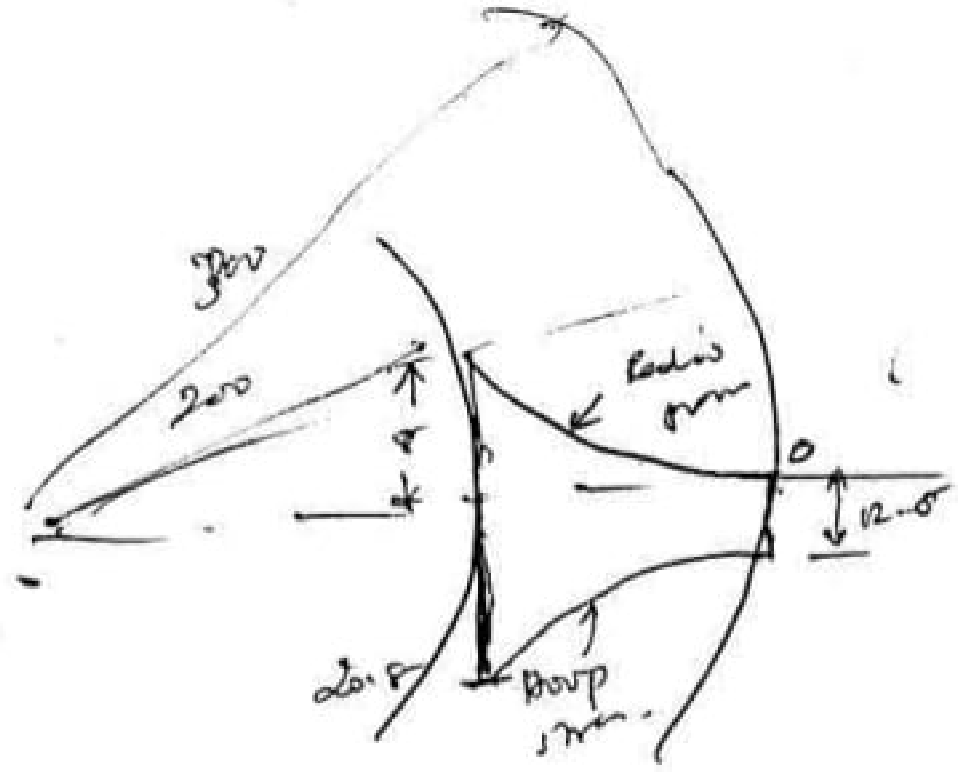
subs a value in above eqn.

$$0 = \frac{b}{90000} - \left(\frac{b}{40000} - 8\right)$$

$$0 = \frac{b}{} - \frac{b}{} + 8$$

$b = 576 \times 10^3$

$a = 6.4$



Max. Hoop stress,

$$\sigma_x = \frac{b}{x^2} + a$$

$x = r_i = 200$

$$\sigma_{200} = \frac{b}{(200)^2} + a$$

$$= 20.8 \ N/mm^2$$

Min. Hoop stress,

$$\sigma_{300} = \frac{b}{x^2} + a \quad (x = r_o = 300)$$

$$\sigma_{300} = 12.8 \ N/mm^2$$

(ii) Radial stresses (P_x) :

P_x at inner cylinder.

$$P_{200} = \frac{b}{x^2} - a \quad (x = 200)$$

$$P_{200} = 8 \ N/mm^2$$

$$P_{300} = \frac{b}{x^2} - a \quad (x = 300)$$

$$P_{300} = 0.$$

(9). Find the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N/mm² The maximum hoop stress in the section is not to exceed 35 N/mm². Find the value of P.

GIVEN :

$d_i = 160\ mm$

$r_i = 80\ mm$

$P = 8\ N/mm^2$

$\sigma_x = \sigma_{80} = 35\ N/mm^2$

(Hoop stress max. at inner stress surface of cylinder i.e. $x = r_i = 80mm$).

Solution :

To find thickness (t) :-

$$d_o = d_i + 2t$$

i) $x = r_i = 80$ $\qquad$ $P_x = P = 8$

ii) $x = r_o$ $\qquad$ $P_x = 0$

$$P_x = \frac{b}{x^2} - a$$

Sub. boundary condition (i)

$$8 = \frac{b}{(800)^2} - a$$

$$a = \frac{b}{6400} - 8$$

W.K.T,

$$\sigma_x = \frac{b}{x^2} + a \ (x = r_i)$$

$$35 = \frac{b}{(80)^2} + a$$

$$b = 137.6 \times 10^3$$

$$a = 13.5$$

W.K.T $\quad P_x = \frac{b}{x^2} - a$

Sub bound condition (ii)

$$0 = \frac{b}{(r_o)^2} - a$$

$$r_o = 100.05$$

$$\boxed{d_o = 201.9\ mm}$$

w.k.t $d_o = d_i + 2t$

$$t = \frac{d_o - d_i}{2} \qquad \boxed{t = 20.45 \text{ mm}}$$

9/10/14. COMPOUND CYLINDER :

Stresses in compound cylinders.

i). stresses in cylinders (without fluid pressure)

a). Inner cylinder.

$$P_x = \frac{b_1}{x^2} - a_1 \qquad \sigma_x = \frac{b_1}{x^2} + a_1$$

$$P^* = \frac{b_1}{r^{*2}} - a_1$$

Boundary condition.

i) $x = r_1 \qquad P_x = 0$

ii) $x = r^* \qquad P_x = P^*$

b) Outer cylinder :

$$P_x = \frac{b_2}{x^2} - a_2 \qquad \sigma_x = \frac{b_2}{x^2} + a_2$$

$$P^* = \frac{b_2}{r^{*2}} - a_2$$

Boundary condition.

i) $x = r_2 \qquad P_x = 0$

ii) $x = r^* \qquad P_x = P^*$

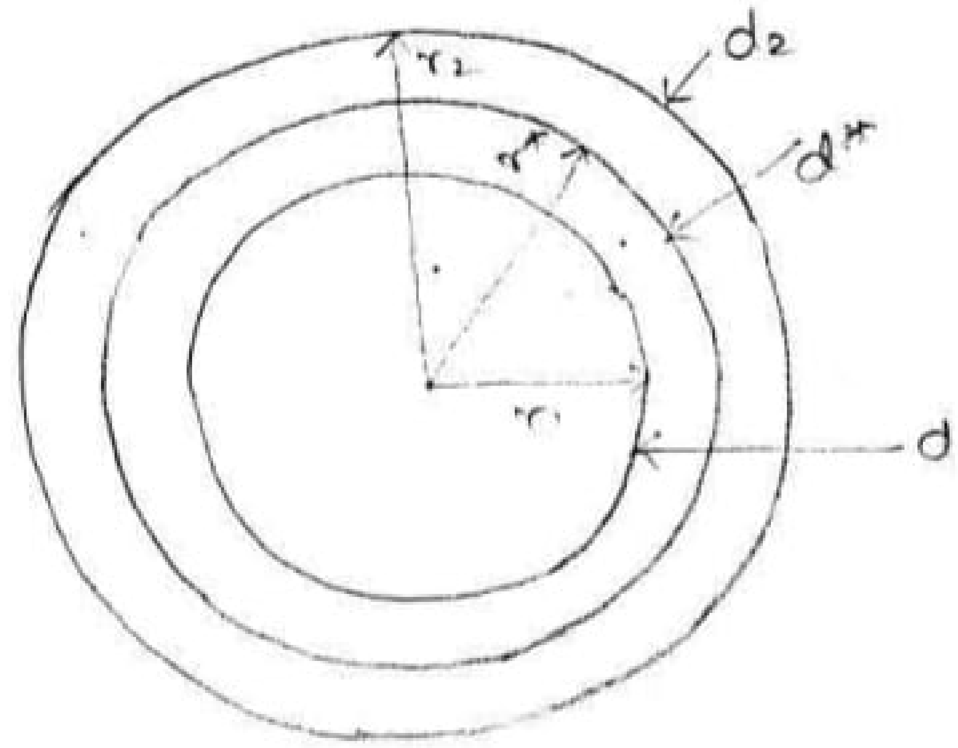
ii). stresses in compound cylinder due to fluid pressure :

$$P_x = \frac{B}{x^2} - A \qquad \sigma_x = \frac{B}{x^2} + A$$

Bound cond. i) $x = r_1$ $P_x = P$

ii) $x = r_2$ $P_x = 0$

iii) Diff in radii at junction $= \frac{2r^*}{E}(a_1 - a_2)$



r_1 = Inner cylinder radius

r^* = Junction radius

r_2 = Outer cylinder radius

p^* = Junction pressure

$a_1, a_2, b_1, b_2 \rightarrow$ constants

(1). A compound cylinder is made by shrinking a cylinder of external dia 300 mm and internal dia 250 mm over another cylinder of external dia 250 mm and internal dia 200 mm. The radial pressure at the junction after shrinking is 8 N/mm². Find the final stresses setup across the section when compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm2.

GIVEN :

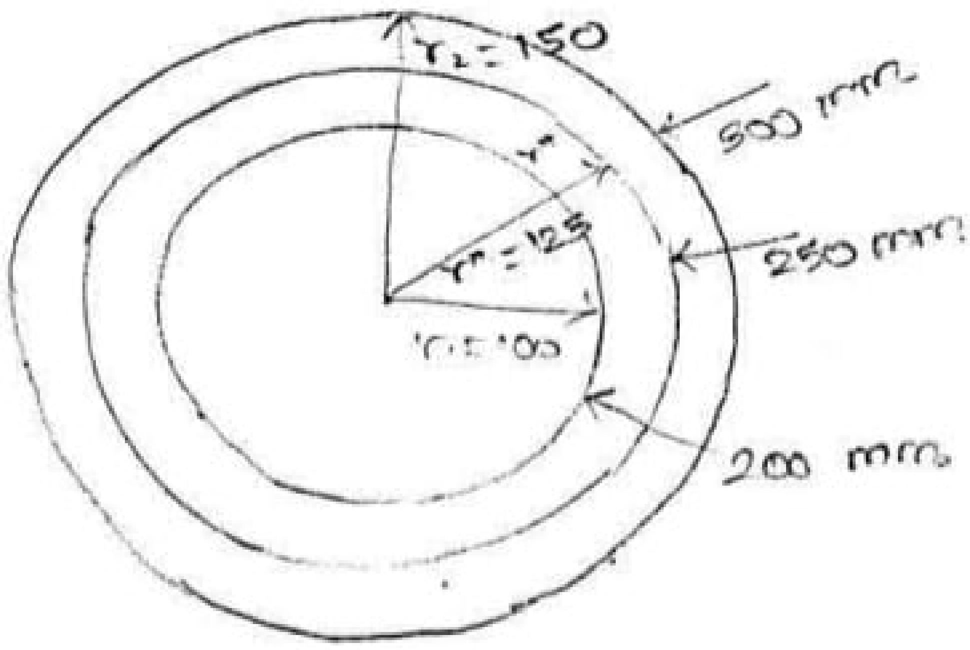
$r_2 = 150$ mm

$r^* = 125$ mm

$r_1 = 100$ mm

$p^* = 8$ N/mm²

$P = 84.5$ N/mm²



Solution :

To find net stress across section :

i) Stress without fluid pressure.

Inner cylinder :

$$\sigma_x = \frac{b_1}{x^2} + a_1$$

$$P_x = \frac{b_1}{x^2} - a_1$$

Boundary condition

i) $x = r_1 = 100$ mm $\quad P_x = 0$

ii) $x = r^* = 125$ mm $\quad P_x = P^* = 8$

$$0 = \frac{b_1}{100^2} - a_1$$

$$a_1 = \frac{b_1}{100^2}$$

ii) $$8 = \frac{b_1}{125^2} - a_1$$

$$8 = \frac{b_1}{125^2} - \frac{b_1}{100^2}$$

$$b_1 = -22.2 \times 10^4$$

$$a_1 = -22.2$$

$$\sigma_x = \frac{-22.2 \times 10^4}{} - +22.2$$

The hoop stress at inner and outer side of the inner cylinder is found by substituting ~~x =~~ $x = r_1 = 100$ mm and $x = r^* = 125$ mm

$$\sigma_{100i} = \frac{-22.2 \times 10^4}{100^2} - 22.2 \quad (x = 100)$$ in inner cylinder

$$= -44.4 \text{ N/mm}^2 \quad \text{(compression)}$$

$$\sigma_{125i} = \frac{-22.2 \times 10^4}{125^2} - 22.2 \quad (x = 125)$$

$$= -36.4 \text{ N/mm}^2 \quad \text{(compression)}$$

ii) outer cylinder :

$$\sigma_x = \frac{b_2}{x^2} + a_2$$

$$P_x = \frac{b_2}{x^2} - a_2$$

i) $x = r^* = 125$ $\qquad P_x = P^*$

ii) $x = r_2 = 150$ $\qquad P_x = 0$

~~$a_2 =$~~

$$\sigma_{125} = \frac{-22.2 \times 10^4}{125^2} - 22.2 \quad (\cancel{x} = 125)$$

$$= \cancel{14}4 \cdot$$

$$\boxed{a_2 = 18.1}$$

$$\boxed{b_2 = 40.9 \times 10^{4}}$$

~~$a_{125} = 44 \cdot$~~

The hoop stresses at inner and outer side of the outer cylinder is found by substituting $x = r_1 = 125$ and $x = r^* = 150$

$\sigma_{125.0} = 44.28\ N/mm^2$ (tensile)

$\sigma_{150.0} = 36.27\ N/mm^2$ (tensile)

iii) Stress due to fluid pressure :

when fluid is stored, the two cylinder is considered as one single cylinder and Hoop stresses are calculated.

$$\sigma x = \frac{B}{x^2} + A$$

$$P_x = \frac{B}{x^2} - A$$

i) $x = r_1 = 100 \quad P_x = P = 84.5\ N/mm^2$

ii) $x = r_2 = 150 \quad P_x = 0$

$$84.5 = \frac{B}{100^2} - A$$

$$A = \frac{B}{100^2} - 84.5$$

$$0 = \frac{B}{150^2} - A$$

$$A = \frac{B}{150}$$

$$\frac{B}{150^2} - \frac{B}{100^2} = 84.5$$

$$(100^2 - 150^2)\ B$$

$A = 67.6 \qquad B = 15.2 \times 10^5$

$\sigma_{100} = 219\ N/mm^2$

$\sigma_{150} = 135.15\ N/mm^2$

$\sigma_{125} = 164.88\ N/mm^2$

iii) To find Resultant stress (or) Net stress across section :

$$F_{100} = \sigma_{100} + \sigma_{100i} = 175 \ N/mm^2$$

$$F_{125} = \sigma_{125} + \sigma_{125i} = 128 \ N/mm^2$$

Outer cylinder,

$$F_{125} = \sigma_{125} + \sigma_{125o}$$

$$= 209.16 \ N/mm^2$$

$$F_{150} = \sigma_{150} + \sigma_{150}$$

$$= 171.4 \ N/mm^2$$